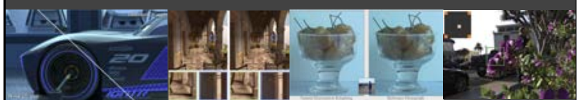
Sampling and Reconstruction of Visual Appearance

CSE 274 [Winter 2018], Lecture 2

Ravi Ramamoorthi

http://www.cs.ucsd.edu/~ravir



Motivation: BRDFs, Radiometry

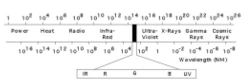
- Basics of Illumination, Reflection
- Formal radiometric analysis (not ad-hoc)
- Reflection Equation
- Monte Carlo Rendering next week

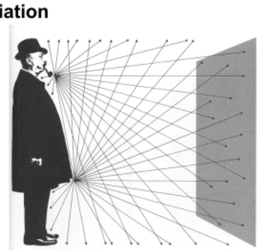
- Appreciate formal analysis in a graduate course, even if not absolutely essential in practice

Light

Visible electromagnetic radiation

Power spectrum





From London and Upton

Polarization

Photon (quantum effects)

Wave (interference, diffraction)

Radiometry

- Physical measurement of electromagnetic energy

- *Measure spatial (and angular) properties of light*
  - *Radiance, Irradiance*
  - Reflection functions: Bi-Directional Reflectance Distribution Function or BRDF
  - Reflection Equation
  - Simple BRDF models

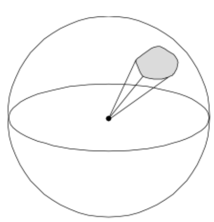
Angles and Solid Angles

- **Angle** $\theta = \frac{l}{r}$

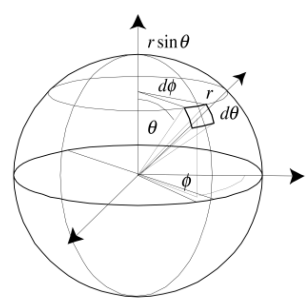
  ⇒ circle has 2π radians

- **Solid angle** $\Omega = \frac{A}{R^2}$

  ⇒ sphere has 4π steradians

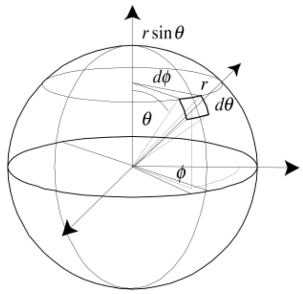


Differential Solid Angles



$$dA = (r\,d\theta)(r\sin\theta\,d\phi)$$
$$= r^2 \sin\theta\,d\theta\,d\phi$$

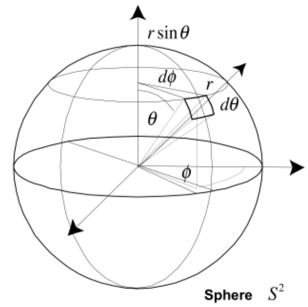
Differential Solid Angles



$$dA = (r\,d\theta)(r\sin\theta\,d\phi) = r^2\sin\theta\,d\theta\,d\phi$$

$$d\omega = \frac{dA}{r^2} = \sin\theta\,d\theta\,d\phi$$

Differential Solid Angles



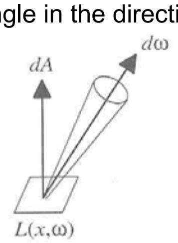
$$d\omega = \sin\theta\,d\theta\,d\phi$$

$$\Omega = \int_{S^2} d\omega = \int_0^{\pi}\int_0^{2\pi} \sin\theta\,d\theta\,d\phi = \int_{-1}^{1}\int_0^{2\pi} d\cos\theta\;d\phi = 4\pi$$

Sphere S^2

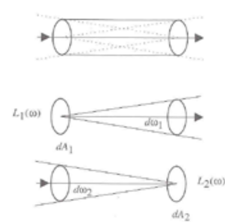
Radiance

- Power per unit projected area perpendicular to the ray per unit solid angle in the direction of the ray
- Symbol: L(x,ω) (W/m^2 sr)
- Flux given by
  dΦ = L(x,ω) cos θ dω dA



Radiance properties

- Radiance constant as propagates along ray
  - Derived from conservation of flux
  - Fundamental in Light Transport.



$$d\Phi_1 = L_1 d\omega_1 dA_1 = L_2 d\omega_2 dA_2 = d\Phi_2$$

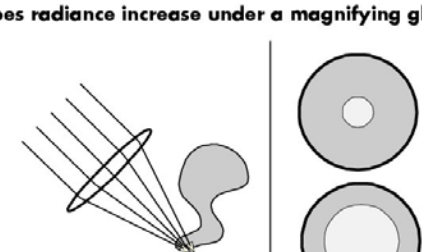
$$d\omega_1 = dA_2/r^2 \qquad d\omega_2 = dA_1/r^2$$

$$d\omega_1 dA_1 = \frac{dA_1 dA_2}{r^2} = d\omega_2 dA_2$$

$$\therefore L_1 = L_2$$

Quiz

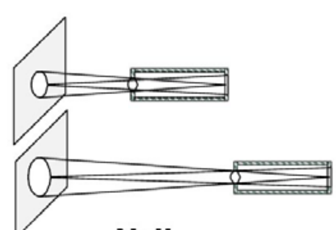
Does radiance increase under a magnifying glass?



No!!

Quiz

Does the brightness that a wall appears to the eye depend on the distance of the viewer to the wall?



No!!

Radiance properties

- Sensor response proportional to radiance (constant of proportionality is throughput)
  - – Far away surface: See more, but subtends smaller angle
  - – Wall equally bright across viewing distances

Consequences

- – Radiance associated with rays in a ray tracer
- – Other radiometric quants derived from radiance

Irradiance, Radiosity

- Irradiance E is radiant power per unit area
- Integrate incoming radiance over hemisphere
  - – Projected solid angle (cos θ dω)
  - – Uniform illumination: Irradiance = π [CW 24,25]
  - – Units: W/m^2
- Radiant Exitance (radiosity)
  - – Power per unit area leaving surface (like irradiance)

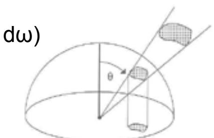
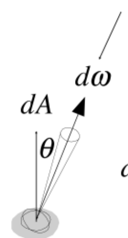


Figure 2.8: Projection of differential area.

Directional Power Arriving at a Surface



$$d^2\Phi_i(x,\omega) = L_i(x,\omega)\cos\theta\, dA\, d\omega$$

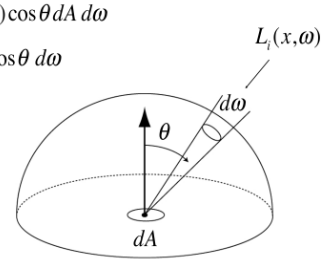
CS348B Lecture 4 Pat Hanrahan, 2007

Irradiance from the Environment

$$d^2\Phi_i(x,\omega) = L_i(x,\omega)\cos\theta\, dA\, d\omega$$

$$dE(x,\omega) = L_i(x,\omega)\cos\theta\, d\omega$$

Light meter



$$E(x) = \int_{H^2} L_i(x,\omega)\cos\theta\, d\omega$$

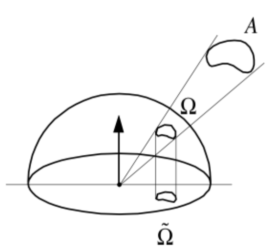
CS348B Lecture 4 Pat Hanrahan, 2007

Uniform Area Source

$$E(x) = \int_{H^2} L\cos\theta\, d\omega$$

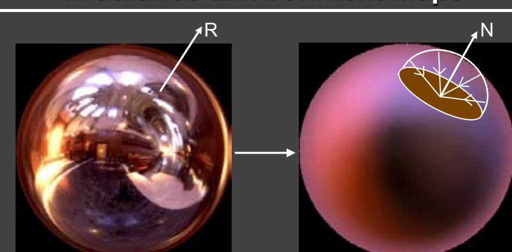
$$= L\int_{\Omega} \cos\theta\, d\omega$$

$$= L\tilde{\Omega}$$



CS348B Lecture 5 Pat Hanrahan, 2009

Irradiance Environment Maps



Incident Radiance (Illumination Environment Map)

Irradiance Environment Map

Radiometry

- Physical measurement of electromagnetic energy
- *Measure spatial (and angular) properties of light*
  - Radiance, Irradiance
  - *Reflection functions: Bi-Directional Reflectance Distribution Function or BRDF*
  - *Reflection Equation*
  - Simple BRDF models

Types of Reflection Functions

Ideal Specular

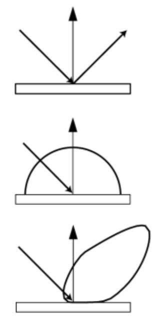
- **Reflection Law**
- **Mirror**

Ideal Diffuse

- **Lambert's Law**
- **Matte**

Specular

- **Glossy**
- **Directional diffuse**



Materials

Plastic **Metal** **Matte**

From Apodaca and Gritz, *Advanced RenderMan*

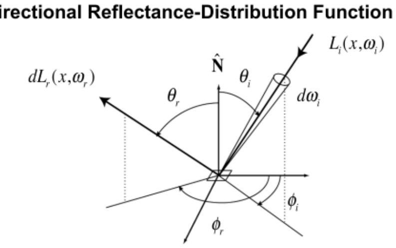
Spheres [Matusik et al.]

Building up the BRDF

- Bi-Directional Reflectance Distribution Function [Nicodemus 77]
- Function based on incident, view direction
- Relates incoming light energy to outgoing
- Unifying framework for many materials

The BRDF

Bidirectional Reflectance-Distribution Function



$$f_r(\omega_i \to \omega_r) \equiv \frac{dL_r(\omega_i \to \omega_r)}{dE_i} \quad \left[\frac{1}{sr}\right]$$

BRDF

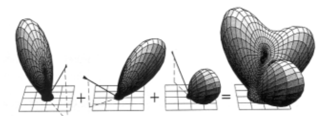
- Reflected Radiance proportional Irradiance
- Constant proportionality: BRDF
- Ratio of outgoing light (radiance) to incoming light (irradiance)
  - Bidirectional Reflection Distribution Function
  - (4 Vars) units 1/sr

$$f(\omega_i, \omega_r) = \frac{L_r(\omega_r)}{L_i(\omega_i)\cos\theta_i d\omega_i}$$

$$L_r(\omega_r) = L_i(\omega_i) f(\omega_i, \omega_r) \cos\theta_i d\omega_i$$

Properties of BRDF's

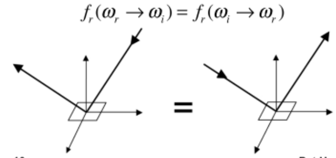
1. Linearity



From Sillion, Arvo, Westin, Greenberg

2. Reciprocity principle

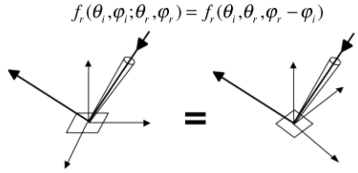
$$f_r(\omega_r \to \omega_i) = f_r(\omega_i \to \omega_r)$$



Properties of BRDF's

3. Isotropic vs. anisotropic

$$f_r(\theta_i, \varphi_i; \theta_r, \varphi_r) = f_r(\theta_i, \theta_r, \varphi_r - \varphi_i)$$



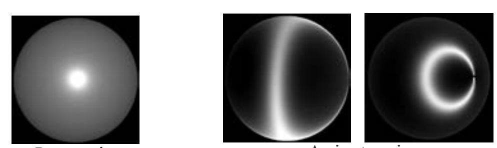
Reciprocity *and* isotropy

$$f_r(\theta_i, \theta_r, \varphi_r - \varphi_i) = f_r(\theta_r, \theta_i, \varphi_i - \varphi_r) = f_r(\theta_i, \theta_r, |\varphi_r - \varphi_i|)$$

4. Energy conservation

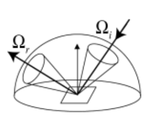
Isotropic vs Anisotropic

- Isotropic: Most materials (you can rotate about normal without changing reflections)
- Anisotropic: brushed metal etc. preferred tangential direction



Isotropic Anisotropic

Energy Conservation

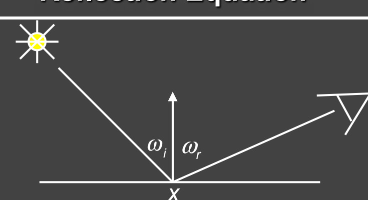


$$\frac{d\Phi_r}{d\Phi_i} = \frac{\int_{\Omega_r} L_r(\omega_r)\cos\theta_r \, d\omega_r}{\int_{\Omega_i} L_i(\omega_i)\cos\theta_i \, d\omega_i}$$

$$= \frac{\int_{\Omega_r}\int_{\Omega_i} f_r(\omega_i \to \omega_r) L_i(\omega_i)\cos\theta_i \, d\omega_i \cos\theta_r \, d\omega_r}{\int_{\Omega_i} L_i(\omega_i)\cos\theta_i \, d\omega_i}$$

$$\le 1$$

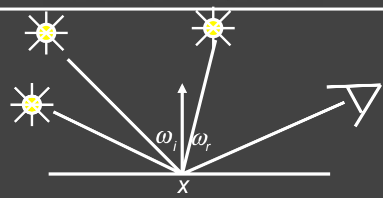
Reflection Equation



$$L_r(x,\omega_r) = L_e(x,\omega_r) + L_i(x,\omega_i) f(x,\omega_i,\omega_r)(\omega_i \bullet n)$$

Reflected Light (Output Image) | Emission | Incident Light (from light source) | BRDF | Cosine of Incident angle

Reflection Equation

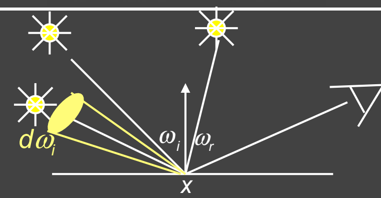


Sum over all light sources

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \sum L_i(x,\omega_i) f(x,\omega_i,\omega_r)(\omega_i \bullet n)$$

Reflected Light (Output Image) | Emission | Incident Light (from light source) | BRDF | Cosine of Incident angle

Reflection Equation



Replace sum with integral

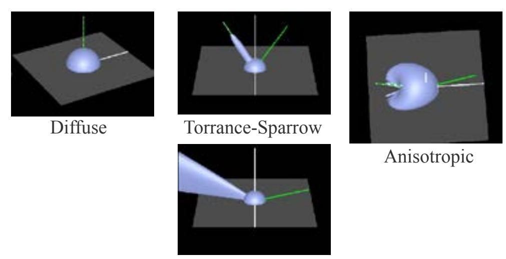
$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_\Omega L_i(x,\omega_i) f(x,\omega_i,\omega_r) \cos\theta_i d\omega_i$$

Reflected Light (Output Image) | Emission | Incident Light (from light source) | BRDF | Cosine of Incident angle

Radiometry

- Physical measurement of electromagnetic energy
- *Measure spatial (and angular) properties of light*
  - Radiance, Irradiance
  - Reflection functions: Bi-Directional Reflectance Distribution Function or BRDF
  - Reflection Equation
  - *Simple BRDF models*

Brdf Viewer plots

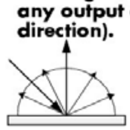


Diffuse | Torrance-Sparrow | Anisotropic

bv written by Szymon Rusinkiewicz

Ideal Diffuse Reflection

Assume light is equally likely to be reflected in any output direction (independent of input direction).



$$L_{r,d}(\omega_r) = \int f_{r,d} L_i(\omega_i)\cos\theta_i\, d\omega_i = f_{r,d}\int L_i(\omega_i)\cos\theta_i\, d\omega_i = f_{r,d} E$$

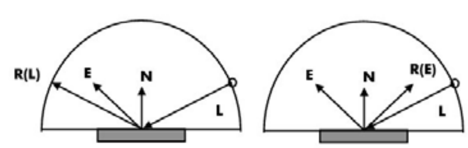
$$M = \int L_r(\omega_r)\cos\theta_r\, d\omega_r = L_r\int\cos\theta_r\, d\omega_r = \pi L_r$$

$$\rho_d = \frac{M}{E} = \frac{\pi L_r}{E} = \frac{\pi f_{r,d} E}{E} = \pi f_{r,d} \quad \Rightarrow \quad f_{r,d} = \frac{\rho_d}{\pi}$$

Lambert's Cosine Law $M = \rho_d E = \rho_d E_s \cos\theta_s$

CS348B Lecture 10 — Pat Hanrahan, Spring 2002

Phong Model



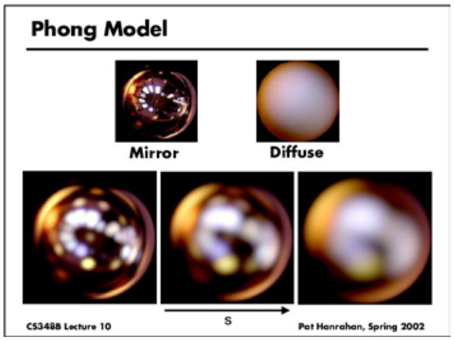
$(\hat{E} \bullet R(\hat{L}))^s$ $\quad$ $(\hat{L} \bullet R(\hat{E}))^s$

Reciprocity: $(\hat{E} \bullet R(\hat{L}))^s = (\hat{L} \bullet R(\hat{E}))^s$

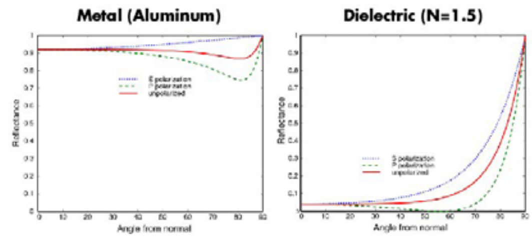
Distributed light source!

CS348B Lecture 10 — Pat Hanrahan, Spring 2002

Specular Term (Phong)



Fresnel Reflectance



Gold F(0)=0.82
Silver F(0)=0.95

Glass n=1.5 F(0)=0.04
Diamond n=2.4 F(0)=0.15

Schlick Approximation $F(\theta) = F(0) + (1 - F(0))(1 - \cos\theta)^5$

CS348B Lecture 10 Pat Hanrahan, Spring 2002

Experiment

Reflections from a shiny floor

From Lafortune, Foo, Torrance, Greenberg, SIGGRAPH 97

CS348B Lecture 10 Pat Hanrahan, Spring 2002

Torrance-Sparrow

$$f = \frac{F(\theta_i)G(\omega_i,\omega_r)D(\theta_h)}{4\cos(\theta_i)\cos(\theta_r)}$$

Fresnel term: allows for wavelength dependency

Geometric Attenuation: reduces the output based on the amount of shadowing or masking that occurs.

Distribution: distribution function determines what percentage of microfacets are oriented to reflect in the viewer direction.

How much of the macroscopic surface is visible to the light source

How much of the macroscopic surface is visible to the viewer

Other BRDF models

- Empirical: Measure and build a 4D table
- Anisotropic models for hair, brushed steel
- Cartoon shaders, funky BRDFs
- Capturing spatial variation
- Very active area of research