Sampling and Reconstruction of Visual Appearance: From Denoising to View Synthesis

CSE 274 [Fall 2022], Lecture 6

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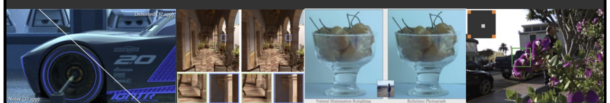

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Basics of Denoising, Frequency Analysis

Monte Carlo Rendering (biggest application)

- Basic idea of denoising
- Frequency analysis one key concept
- Presentation of key papers at next class
- Relevant to other applications as well

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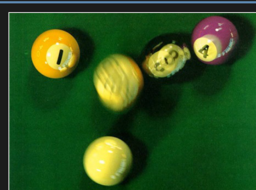
Cook et al. [1984] results

depth of field



motion blur

soft shadows

glossy reflection

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Motivation

- Distribution effects (depth of field, motion blur, global illumination, soft shadows) are slow. Many dimensions sample

- Ray Tracing physically accurate but slow, not real-time
- Can we adaptively sample and filter for fast, real-time?

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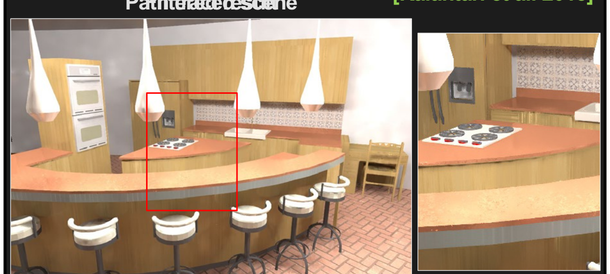
Sample result

Path traced scene / Filtered result

[Kalantari et al. 2015]



scene by Jo Ann Elliott

4 samples/pixel (40.8 sec)

using only post-process filter!

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Adaptive sampling + reconstruction

- **These algorithms use 2 kinds of noise reduction strategies, sometimes combined:**
  1. **Adaptive sampling algorithms**
     - **Use information from renderer to position new samples better to reduce noise**
  2. **Reconstruction (filtering) algorithms**
     - **Use information from renderer to remove MC noise directly**
- **Both methods have been explored in the past, but new algorithms make remarkable advances**

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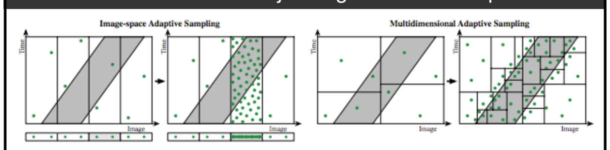
Multi-Dimensional Adaptive Sampling

- Hachisuka, Jarosz, … Zwicker, Jensen [MDAS 2008]
- Scenes with motion blur, depth of field, soft shadows
- Involves high-dimensional integral, converges slowly
- Exploit high-dimensional info to sample adaptively
- Sampling in 2D image plane or other dims inadequate
  - Need to consider full joint high-dimensional space



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Multi-Dimensional Adaptive Sampling

Motion Blur and Depth of Field 32 samples per pixel

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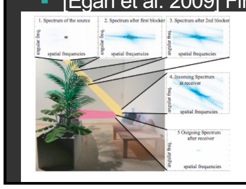
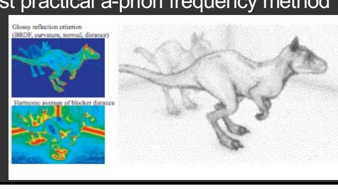
Resurgence (2008 -)

- Eurographics 2015 STAR report by Zwicker et al.
  - Papers below are key a-priori, frequency analysis methods
  - Many other approaches to be discussed in class
- [Durand et al. 2005] *Frequency analysis light transport*
  - Key theoretical ideas, but not initially very practical
- [Chai et al. 2000] Plenoptic Sampling (wedge spectrum)
- [Egan et al. 2009] First practical a-priori frequency method





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Background: Fourier Analysis

Analysis in the frequency (not spatial) domain

- Sum of sine waves, with possibly different offsets (phase)
- Each wave different frequency, amplitude

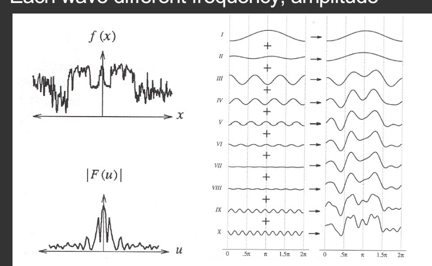

Figure 2.6 Wolberg

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Fourier Transform

- Tool for converting from spatial to frequency domain

$$f(x) = \sum_{u=-\infty}^{+\infty} F(u)e^{2\pi iux}$$

$$e^{2\pi iux} = \cos(2\pi ux) + i\sin(2\pi ux)$$

- Or vice versa $\quad i = \sqrt{-1}$
- One of most important mathematical ideas
- Computational algorithm: Fast Fourier Transform
  - One of 10 great algorithms scientific computing
  - Makes Fourier processing possible (images etc.)
  - Not discussed here, but look up if interested

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Fourier Transform

- Simple case, function sum of sines, cosines

$$f(x) = \sum_{u=-\infty}^{+\infty} F(u)e^{2\pi iux}$$

$$F(u) = \int_0^1 f(x)e^{-2\pi iux}dx$$

- Continuous infinite case

Forward Transform: $F(u) = \int_{-\infty}^{\infty} f(x)e^{-2\pi iux}dx$

Inverse Transform: $f(x) = \int_{-\infty}^{+\infty} F(u)e^{2\pi iux}du$

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Fourier Transform

- Simple case, function sum of sines, cosines

$$f(x) = \sum_{u=-\infty}^{+\infty} F(u) e^{2\pi iux}$$

$$F(u) = \int_0^1 f(x) e^{-2\pi iux} dx$$

- Discrete case

$$F(u) = \sum_{x=0}^{x=N-1} f(x)\left[\cos(2\pi ux/N) - i\sin(2\pi ux/N)\right], \qquad 0 \le u \le N-1$$

$$f(x) = \frac{1}{N}\sum_{u=0}^{u=N-1} F(u)\left[\cos(2\pi ux/N) + i\sin(2\pi ux/N)\right], \qquad 0 \le x \le N-1$$

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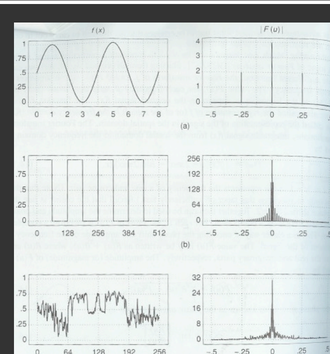
Fourier Transform: Examples 1

Single sine curve
(+constant DC term)



$$f(x) = \sum_{u=-\infty}^{+\infty} F(u) e^{2\pi iux}$$

$$F(u) = \int_0^1 f(x) e^{-2\pi iux} dx$$

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Fourier Transform Examples 2

Forward Transform: $F(u) = \int_{-\infty}^{\infty} f(x) e^{-2\pi iux} dx$

Inverse Transform: $f(x) = \int_{-\infty}^{+\infty} F(u) e^{2\pi iux} du$

- Common examples

$f(x)$	$F(u)$
$\delta(x - x_0)$	$e^{-2\pi iux_0}$
1	$\delta(u)$
e^{-ax^2}	$\sqrt{\pi / a}\, e^{-\pi^2 u^2 / a}$

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Fourier Transform Properties

Forward Transform: $F(u) = \int_{-\infty}^{\infty} f(x) e^{-2\pi iux} dx$

Inverse Transform: $f(x) = \int_{-\infty}^{+\infty} F(u) e^{2\pi iux} du$

- Common properties
  - Linearity: $F(af(x) + bg(x)) = aF(f(x)) + bF(g(x))$
  - Derivatives: [integrate by parts] $F(f'(x)) = \int_{-\infty}^{\infty} f'(x) e^{-2\pi iux} dx = 2\pi iuF(u)$
  - 2D Fourier Transform
    Forward Transform: $F(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y) e^{-2\pi iux} e^{-2\pi ivy} dxdy$
    Inverse Transform: $f(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{+\infty} F(u,v) e^{2\pi iux} e^{2\pi ivy} dudv$
- Convolution (next)

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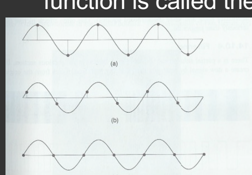
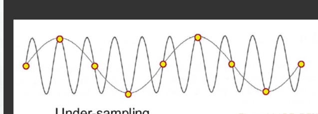
Sampling Theorem, Bandlimiting

- A signal can be reconstructed from its samples, if the original signal has no frequencies above half the sampling frequency – Shannon
- The minimum sampling rate for a bandlimited function is called the Nyquist rate



16 Sampling at the Nyquist rate (a) at peaks, (b) between peaks, (c) at zero. (Courtesy of George Wolberg, Columbia University.)



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Sampling Theorem, Bandlimiting

- A signal can be reconstructed from its samples, if the original signal has no frequencies above half the sampling frequency – Shannon
- The minimum sampling rate for a bandlimited function is called the Nyquist rate
- A signal is bandlimited if the highest frequency is bounded. This frequency is called the bandwidth
- In general, when we transform, we want to filter to bandlimit before sampling, to avoid aliasing

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Antialiasing

- Sample at higher rate
  - Not always possible
  - Real world: lines have infinitely high frequencies, can’t sample at high enough resolution
- Prefilter to bandlimit signal
  - Low-pass filtering (blurring)
  - Trade blurriness for aliasing

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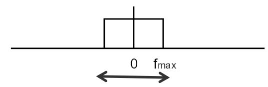
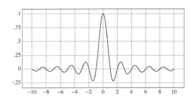
Ideal bandlimiting filter

- Formal derivation is exercise
  - Frequency domain



  - Spatial domain



if full width $f_{max} = 1$

$$Sinc(x) = \frac{\sin \pi x}{\pi x}$$

Figure 4.5 Wolberg

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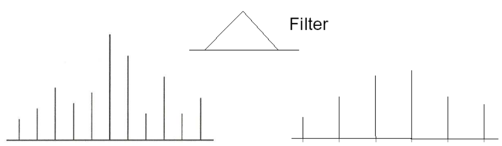
Convolution 1

- Spatial domain: output pixel is weighted sum of pixels in neighborhood of input image
  - Pattern of weights is the “filter”



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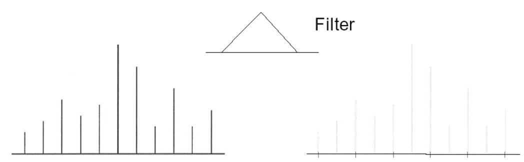
Convolution 2

- Example 1:



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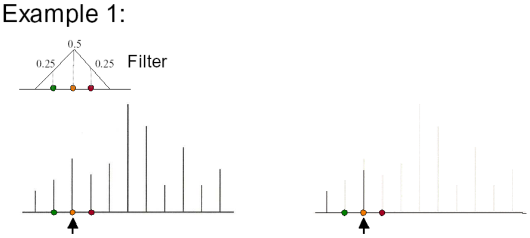
Convolution 3

- Example 1:



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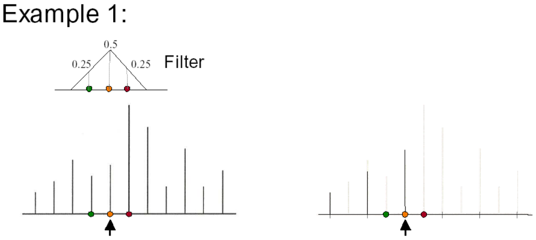
Convolution 4

- Example 1:



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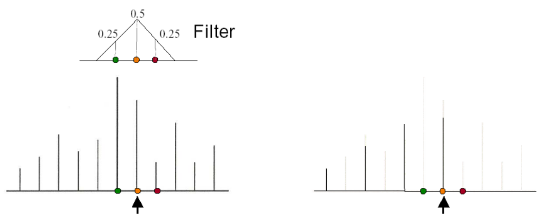
Convolution 5

- Example 1:



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Convolution in Frequency Domain

Forward Transform: $F(u) = \int_{-\infty}^{\infty} f(x) e^{-2\pi iux} dx$

Inverse Transform: $f(x) = \int_{-\infty}^{+\infty} F(u) e^{2\pi iux} du$

- Convolution (f is signal ; g is filter [or vice versa])

$$h(y) = \int_{-\infty}^{+\infty} f(x) g(y-x) dx = \int_{-\infty}^{+\infty} g(x) f(y-x) dx$$

$$h = f * g \;\; \text{or} \;\; f \otimes g$$

- Fourier analysis (frequency domain multiplication) $H(u) = F(u)G(u)$

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A Frequency Analysis of Light Transport

F. Durand, MIT CSAIL
N. Holzschuch, C. Soler, ARTIS/GRAVIR-IMAG INRIA
E. Chan, MIT CSAIL
F. Sillion, ARTIS/GRAVIR-IMAG INRIA

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Illumination effects

- Blurry reflections:

From [Ramamoorthi and Hanrahan 2001]

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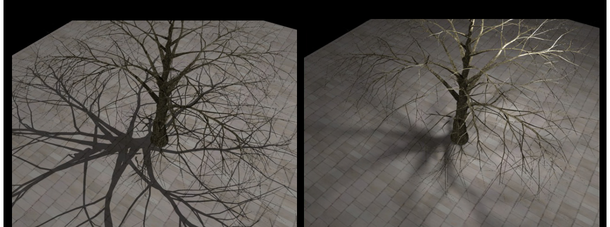
Illumination effects

- Shadow boundaries:



Point light source — Area light source

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Problem statement

- How does light interaction in a scene explain the frequency content?
- Theoretical framework:
  - Understand the frequency spectrum of the radiance function
  - From the equations of light transport

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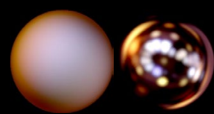
Unified framework:

- Spatial frequency (e.g. shadows, textures)

- Angular frequency (e.g. blurry highlight)



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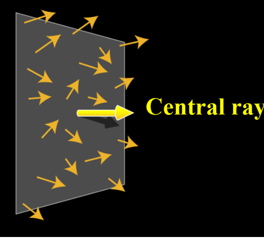
Local light field

- 4D light field, around a *central ray*



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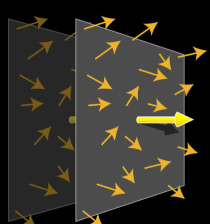
Local light field

- 4D light field, around a *central ray*
- We study its spectrum during transport



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Local light field

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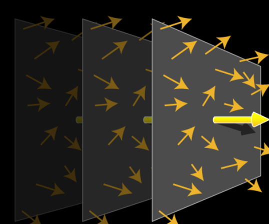

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Local light field

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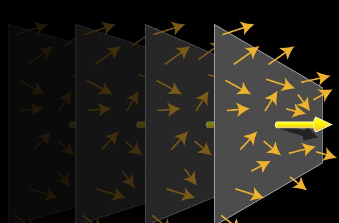

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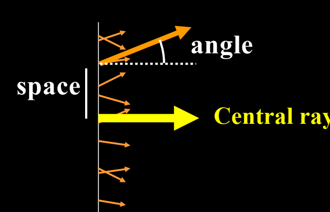
Local light field parameterization

- Space and angle



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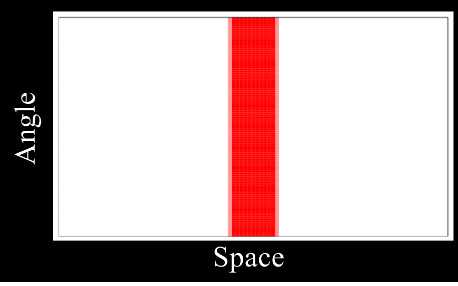
Local light field representation

- Density plot:



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Local light field Fourier spectrum

- We are interested in the Fourier spectrum of the local light field
- Also represented as a density plot

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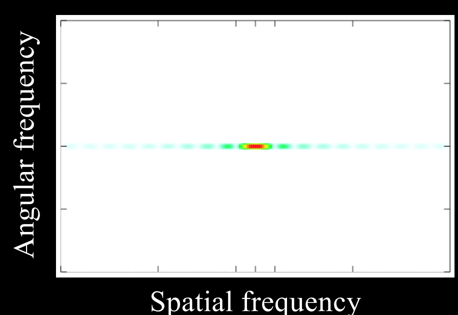
Local light field Fourier spectrum



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Fourier analysis 101

- Spectrum corresponds to blurriness:
  – Sharpest feature has size ~ $1/F_{max}$
- Convolution theorem:
  – Multiplication of functions: spectrum is convolved
  – Convolution of functions: spectrum is multiplied
- Classical spectra:
  – Box becomes sinc
  – Dirac becomes constant

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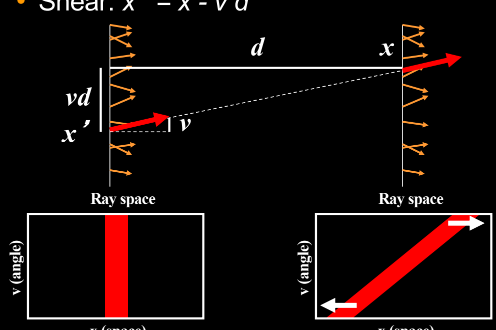
Transport

- Shear: $x' = x - v\,d$



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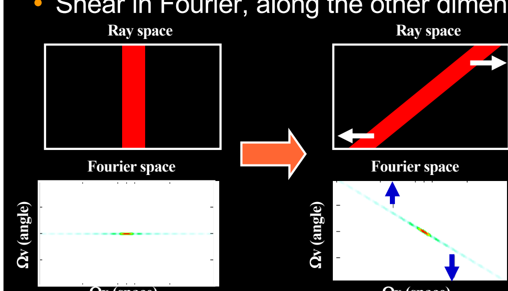
Transport in Fourier space

- Shear in primal: $x' = x - v\,d$
- Shear in Fourier, along the other dimension



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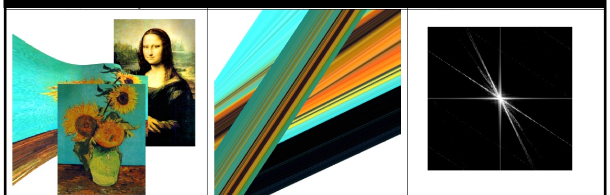
Transport becomes Shear

- This is consistent with light field spectra [Chai et al. 00, Isaksen et al. 00]



(d1)Scene image (d2) EPI (d3) Fourier transform of EPI

From [Chai et al. 2000]

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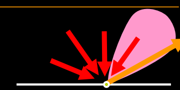
BRDF integration



- Ray-space: ***convolution***
  - Outgoing light: convolution of incoming light and BRDF
  - For rotationally-invariant BRDFs
- Fourier domain: ***multiplication***
  - Outgoing spectrum: multiplication of incoming spectrum and BRDF spectrum

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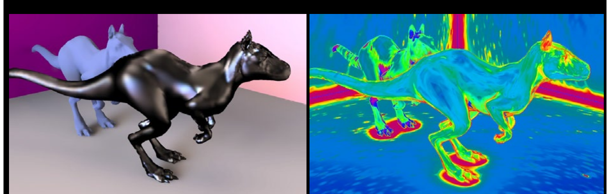
Adaptive shading sampling

- Per-pixel prediction of max. frequency (bandwidth)
  - Based on curvature, BRDF, distance to occluder, etc.
  - No spectrum computed, just estimate max frequency



Per-pixel bandwidth criterion

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Adaptive shading sampling

- Per-pixel prediction of max. frequency (bandwidth)
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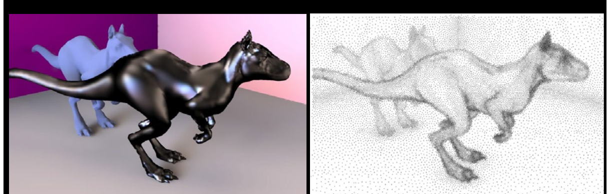

Shading samples

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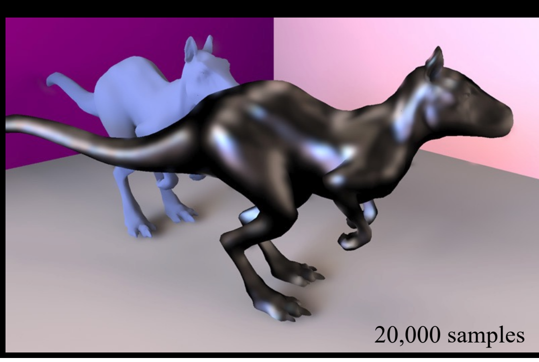
Uniform sampling



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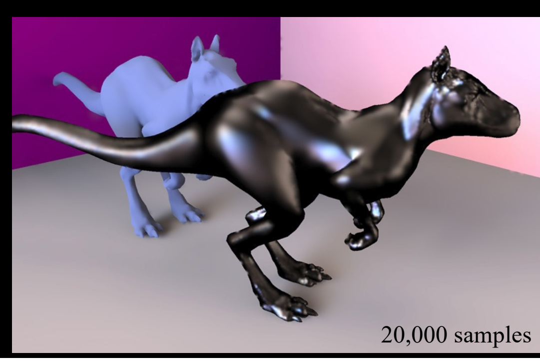
Adaptive sampling



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Plenoptic Sampling

- Plenoptic Sampling. *Chai, Tong, Chan, Shum 00*
- Signal-processing on light field
- Minimal sampling rate for antialiased rendering
- Relates to depth range, Fourier analysis
- Fourier spectra derived for 2D light fields for simplicity. Same ideas extend to 4D
- Key paper in many newer methods on sheared and axis-aligned filtering for adaptive sampling

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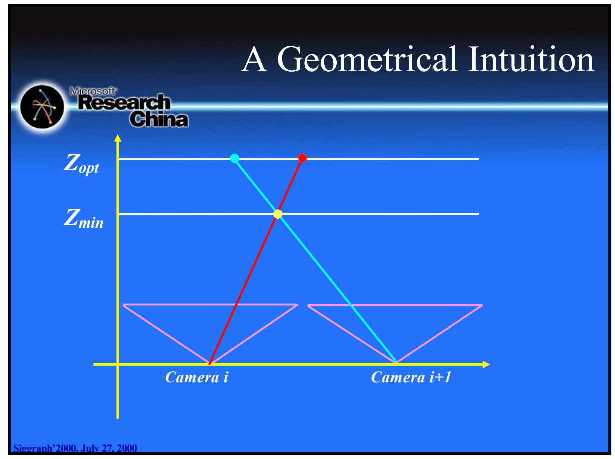

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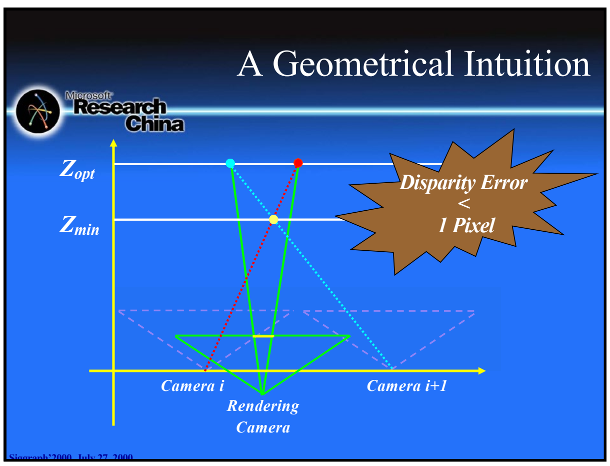

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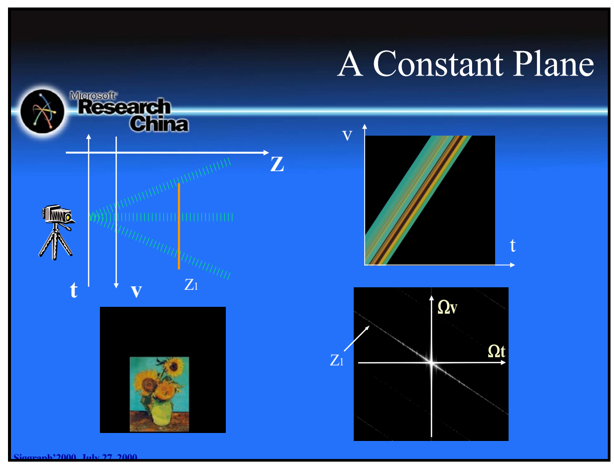

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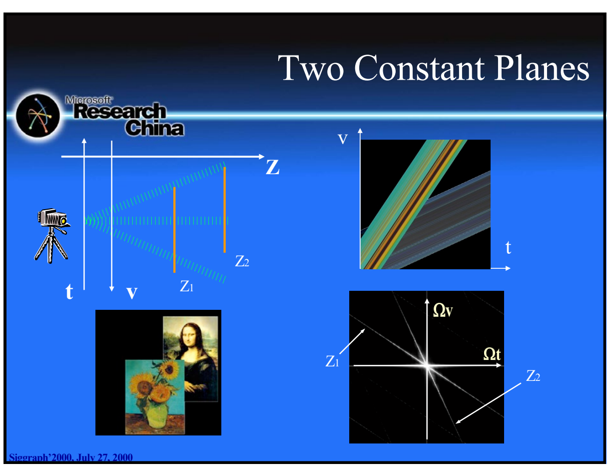

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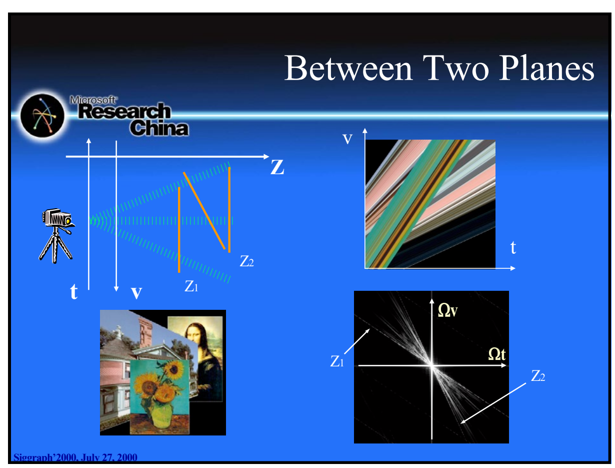

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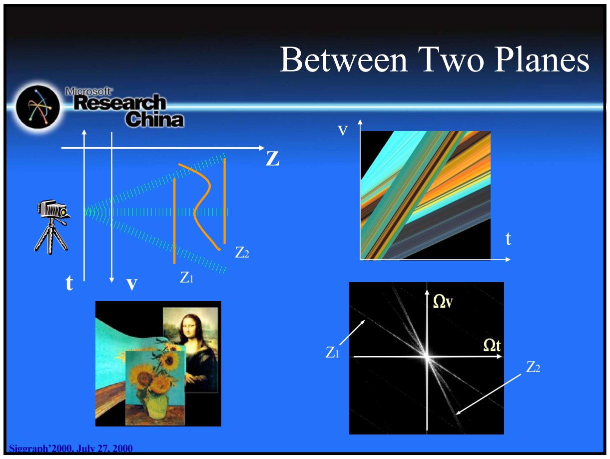

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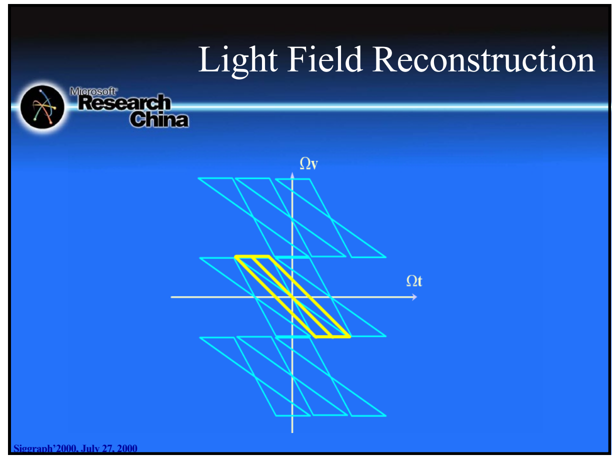

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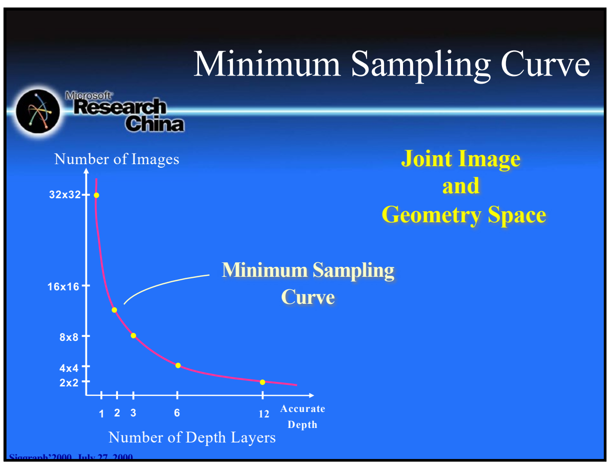

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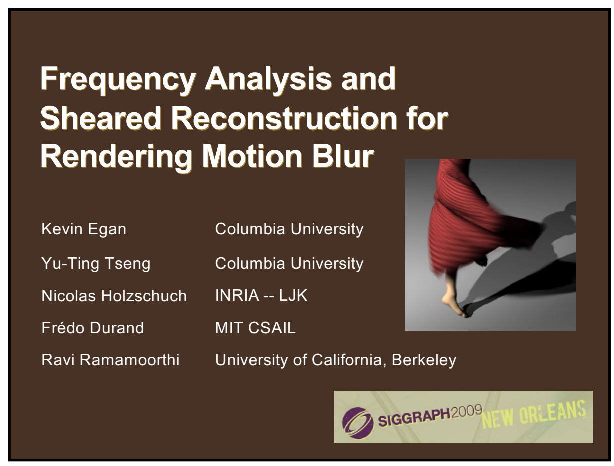

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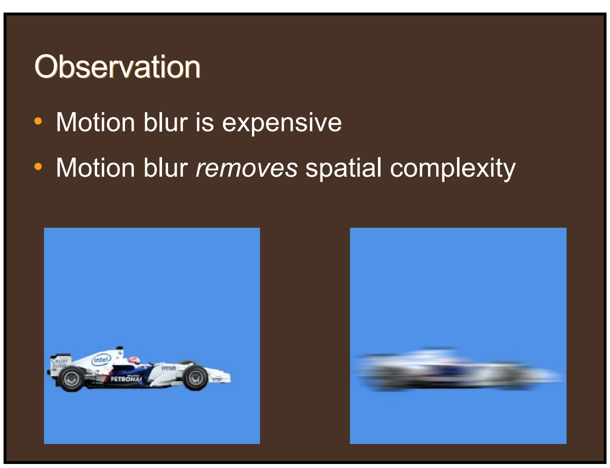

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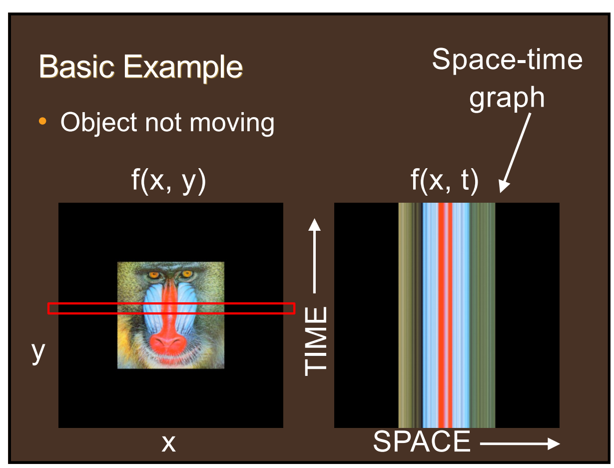

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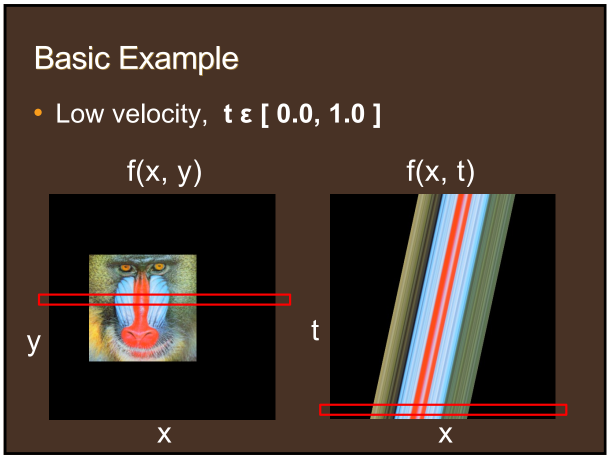

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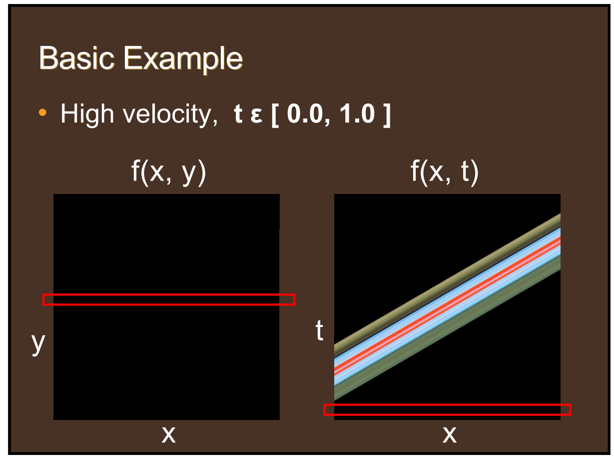

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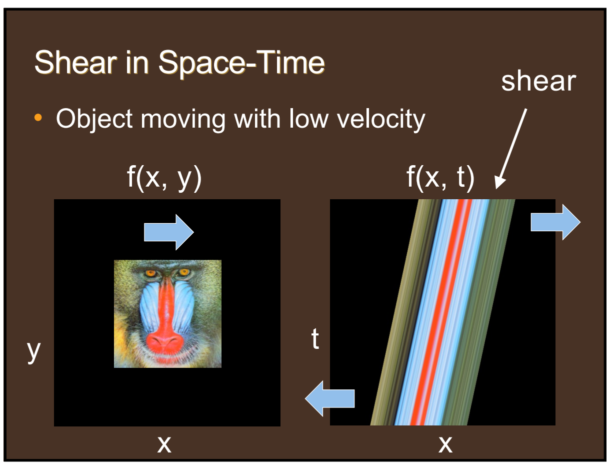

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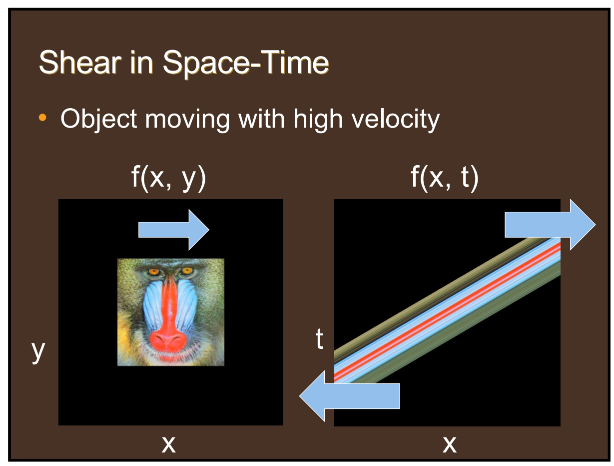

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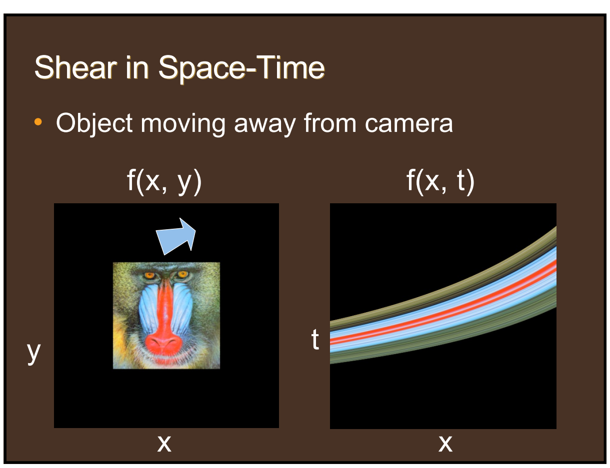

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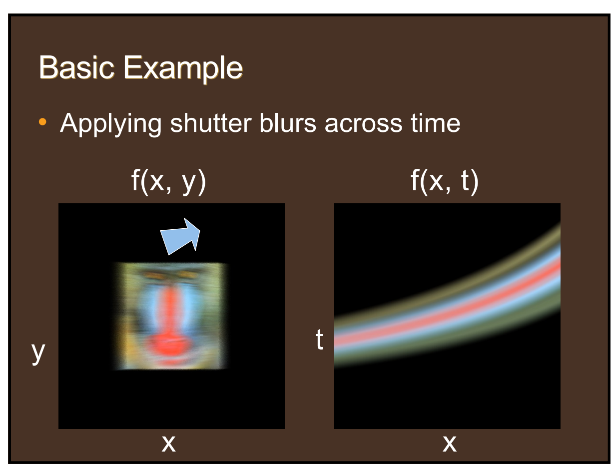

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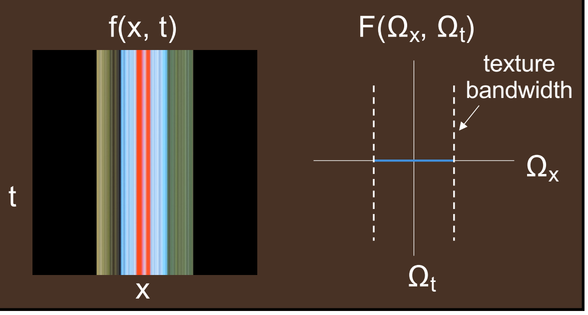
Basic Example – Fourier Domain

- Fourier spectrum, zero velocity



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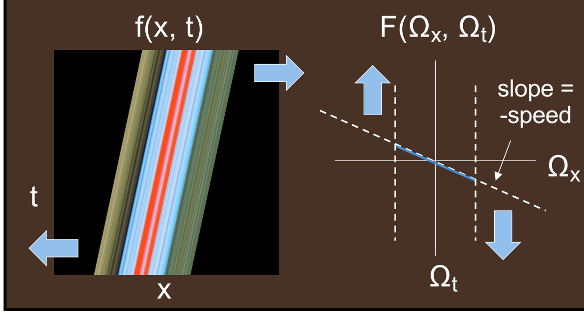
Basic Example – Fourier Domain

- Low velocity, small shear in both domains



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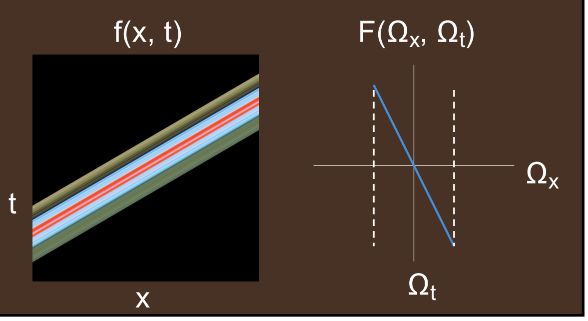
Basic Example – Fourier Domain

- Large shear



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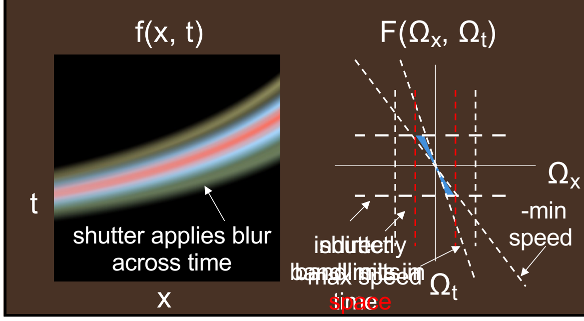
Basic Example – Fourier Domain

- Non-linear motion, wedge shaped spectra



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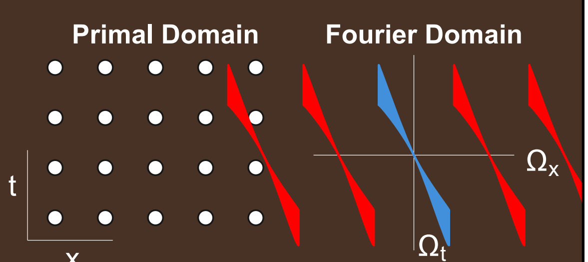
Sampling in Fourier Domain

- Sampling produces replicas in Fourier domain
- Sparse sampling produces dense replicas



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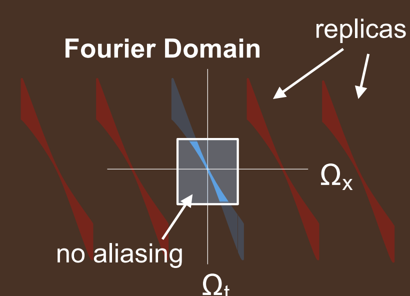
Standard Reconstruction Filtering

- Standard filter, dense sampling (slow)



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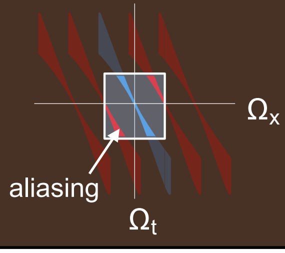
Standard Reconstruction Filter

- Standard filter, sparse sampling (fast)

Fourier Domain



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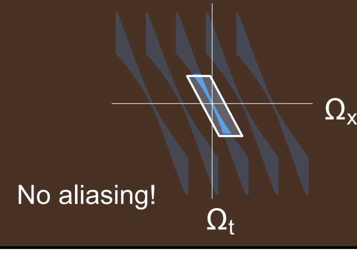
Sheared Reconstruction Filter

- Our sheared filter, sparse sampling (fast)

Fourier Domain



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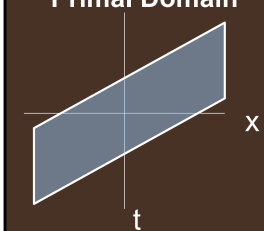
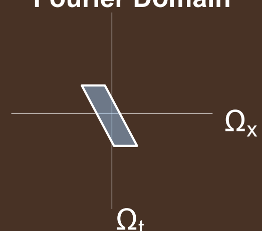
Sheared Reconstruction Filter

- Compact shape in Fourier = wide in primal

Primal Domain



Fourier Domain



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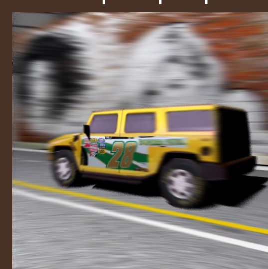
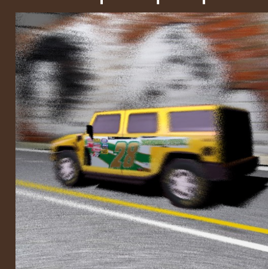
Car Scene

Our Method,
4 samples per pixel



Stratified Sampling
4 samples per pixel



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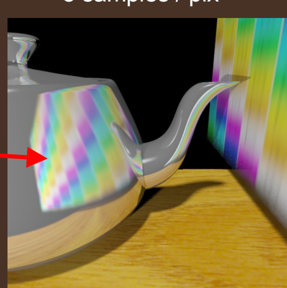
Teapot Scene

Our Method
8 samples / pix

motion blurred reflection



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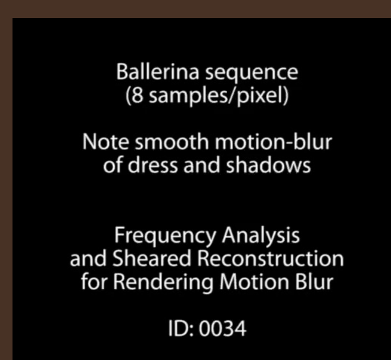
Ballerina Video



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