

Sampling and Reconstruction of Visual Appearance

CSE 274 [Fall 2018], Lecture 7

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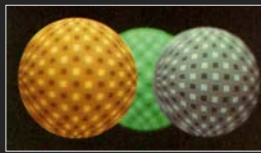


Basics of Denoising, Frequency Analysis

Monte Carlo Rendering (biggest application)

- Basic idea of denoising
- Frequency analysis one key concept
- Presentation of key papers at next class
- Relevant to other applications as well

Cook et al. [1984] results



depth of field



motion blur

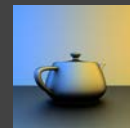


soft shadows
glossy reflection



Motivation

- Distribution effects (depth of field, motion blur, global illumination, soft shadows) are slow. Many dimensions sample



- Ray Tracing physically accurate but slow, not real-time
- Can we adaptively sample and filter for fast, real-time?

Sample result

Path traced scene

[Kalantari et al. 2015]



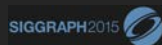
scene by Jo Ann Elliott

4 samples/pixel
(48.8 sec)

using only post-process filter!



PRADEE SEN



Adaptive sampling + reconstruction

- These algorithms use 2 kinds of noise reduction strategies, sometimes combined:

1. Adaptive sampling algorithms

- Use information from renderer to position new samples better to reduce noise

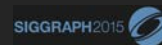
2. Reconstruction (filtering) algorithms

- Use information from renderer to remove MC noise directly

- Both methods have been explored in the past, but new algorithms make remarkable advances

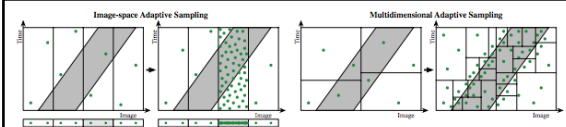


PRADEE SEN



Multi-Dimensional Adaptive Sampling

- Hachisuka, Jarosz, ... Zwicker, Jensen [MDAS 2008]
- Scenes with motion blur, depth of field, soft shadows
- Involves high-dimensional integral, converges slowly
- Exploit high-dimensional info to sample adaptively
- Sampling in 2D image plane or other dims inadequate
 - Need to consider full joint high-dimensional space



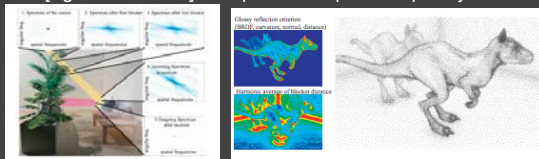
Multi-Dimensional Adaptive Sampling



Motion Blur and Depth of Field 32 samples per pixel

Resurgence (2008 -)

- Eurographics 2015 STAR report by Zwicker et al.
 - Papers below are key a-priori, frequency analysis methods
 - Many other approaches to be discussed in class
- [Durand et al. 2005] *Frequency analysis light transport*
 - Key theoretical ideas, but not initially very practical
- [Chai et al. 2000] Plenoptic Sampling (wedge spectrum)
- [Egan et al. 2009] First practical a-priori frequency method



Background: Fourier Analysis

Analysis in the frequency (not spatial) domain

- Sum of sine waves, with possibly different offsets (phase)
- Each wave different frequency, amplitude

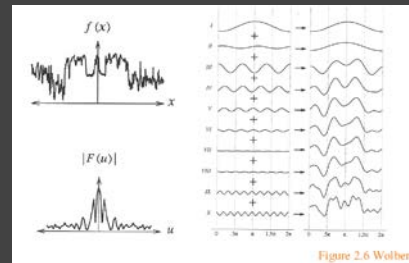


Figure 2.6 Wolberg

Fourier Transform

- Tool for converting from spatial to frequency domain

$$f(x) = \sum_{u=-\infty}^{+\infty} F(u)e^{2\pi iux}$$

$$e^{2\pi iux} = \cos(2\pi ux) + i \sin(2\pi ux)$$

- Or vice versa

$$i = \sqrt{-1}$$

- One of most important mathematical ideas
- Computational algorithm: Fast Fourier Transform
 - One of 10 great algorithms scientific computing
 - Makes Fourier processing possible (images etc.)
 - Not discussed here, but look up if interested

Fourier Transform

- Simple case, function sum of sines, cosines

$$f(x) = \sum_{u=-\infty}^{+\infty} F(u)e^{2\pi iux}$$

$$F(u) = \int_0^1 f(x)e^{-2\pi iux} dx$$

- Continuous infinite case

$$\text{Forward Transform: } F(u) = \int_{-\infty}^{+\infty} f(x)e^{-2\pi iux} dx$$

$$\text{Inverse Transform: } f(x) = \int_{-\infty}^{+\infty} F(u)e^{2\pi iux} du$$

Fourier Transform

- Simple case, function sum of sines, cosines

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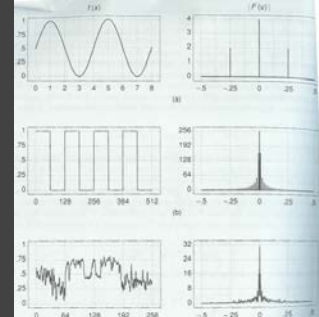
- Discrete case

$$F(u) = \sum_{x=0}^{x=N-1} f(x) [\cos(2\pi ux / N) - i \sin(2\pi ux / N)], \quad 0 \leq u \leq N-1$$

$$f(x) = \frac{1}{N} \sum_{u=0}^{u=N-1} F(u) [\cos(2\pi ux / N) + i \sin(2\pi ux / N)], \quad 0 \leq x \leq N-1$$

Fourier Transform: Examples 1

Single sine curve
(+constant DC term)



$$f(x) = \sum_{u=-\infty}^{+\infty} F(u)e^{2\pi iux}$$

$$F(u) = \int_0^1 f(x)e^{-2\pi iux} dx$$

Fourier Transform Examples 2

Forward Transform: $F(u) = \int_{-\infty}^{+\infty} f(x)e^{-2\pi iux} dx$

Inverse Transform: $f(x) = \int_{-\infty}^{+\infty} F(u)e^{2\pi iux} du$

- Common examples

$f(x)$	$F(u)$
$\delta(x - x_0)$	$e^{-2\pi iux_0}$
1	$\delta(u)$
e^{-ax^2}	$\sqrt{\pi/a} e^{-\pi^2 u^2 / a}$

Fourier Transform Properties

Forward Transform: $F(u) = \int_{-\infty}^{+\infty} f(x)e^{-2\pi iux} dx$

Inverse Transform: $f(x) = \int_{-\infty}^{+\infty} F(u)e^{2\pi iux} du$

- Common properties

- Linearity: $F(af(x) + bg(x)) = aF(f(x)) + bF(g(x))$

- Derivatives: [integrate by parts] $F(f'(x)) = \int_{-\infty}^{+\infty} f'(x)e^{-2\pi iux} dx = 2\pi iuF(u)$

- 2D Fourier Transform

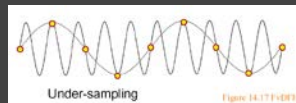
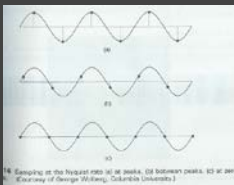
Forward Transform: $F(u,v) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(x,y)e^{-2\pi iux} e^{-2\pi ivy} dx dy$

- Convolution (next)

Inverse Transform: $f(x,y) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} F(u,v)e^{2\pi iux} e^{2\pi ivy} du dv$

Sampling Theorem, Bandlimiting

- A signal can be reconstructed from its samples, if the original signal has no frequencies above half the sampling frequency – Shannon
- The minimum sampling rate for a bandlimited function is called the Nyquist rate



Sampling Theorem, Bandlimiting

- A signal can be reconstructed from its samples, if the original signal has no frequencies above half the sampling frequency – Shannon
- The minimum sampling rate for a bandlimited function is called the Nyquist rate
- A signal is bandlimited if the highest frequency is bounded. This frequency is called the bandwidth
- In general, when we transform, we want to filter to bandlimit before sampling, to avoid aliasing

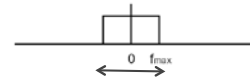
Antialiasing

- Sample at higher rate
 - Not always possible
 - Real world: lines have infinitely high frequencies, can't sample at high enough resolution
- Prefilter to bandlimit signal
 - Low-pass filtering (blurring)
 - Trade blurriness for aliasing

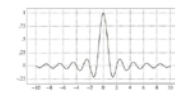
Ideal bandlimiting filter

- Formal derivation is exercise

Frequency domain



Spatial domain



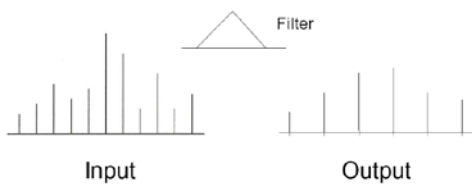
if full width $f_{\max} = 1$

$$\text{Sinc}(x) = \frac{\sin \pi x}{\pi x}$$

Figure 4.5 Wolberg

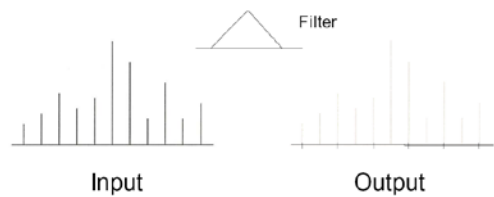
Convolution 1

- Spatial domain: output pixel is weighted sum of pixels in neighborhood of input image
 - Pattern of weights is the "filter"



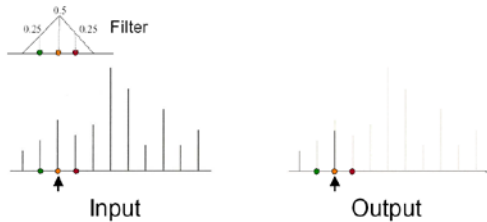
Convolution 2

- Example 1:



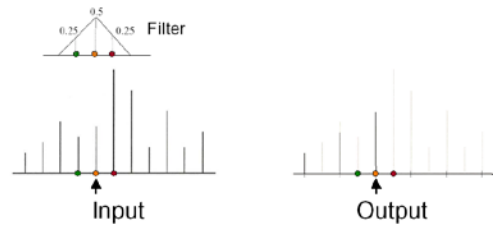
Convolution 3

- Example 1:



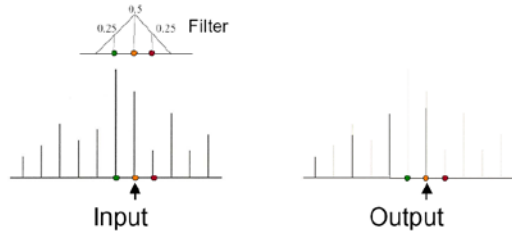
Convolution 4

- Example 1:



Convolution 5

• Example 1:



Convolution in Frequency Domain

Forward Transform: $F(u) = \int_{-\infty}^{\infty} f(x)e^{-2\pi iux} dx$

Inverse Transform: $f(x) = \int_{-\infty}^{\infty} F(u)e^{2\pi iux} du$

- Convolution (f is signal ; g is filter [or vice versa])

$$h(y) = \int_{-\infty}^{\infty} f(x)g(y-x)dx = \int_{-\infty}^{\infty} g(x)f(y-x)dx$$

$$h = f * g \text{ or } f \otimes g$$

- Fourier analysis (frequency domain multiplication) $H(u) = F(u)G(u)$

A Frequency Analysis of Light Transport

F. Durand, MIT CSAIL

N. Holzschuch, C. Soler, ARTIS/GRAVIR-IMAG INRIA

E. Chan, MIT CSAIL

F. Sillion, ARTIS/GRAVIR-IMAG INRIA

Illumination effects

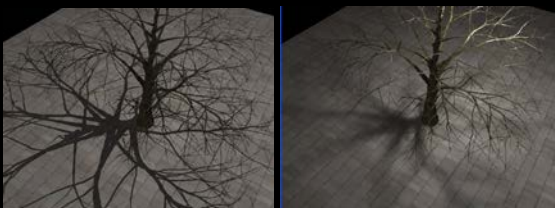
- Blurry reflections:



From [Ramamoorthi and Hanrahan 2001]

Illumination effects

- Shadow boundaries:



Point light source

Area light source

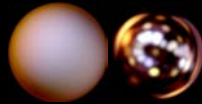
© U. Assarsson 2005.

Problem statement

- How does light interaction in a scene explain the frequency content?
- Theoretical framework:
 - Understand the frequency spectrum of the radiance function
 - From the equations of light transport

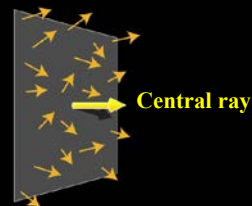
Unified framework:

- Spatial frequency
(e.g. shadows, textures)
- Angular frequency
(e.g. blurry highlight)



Local light field

- 4D light field, around a *central ray*



Local light field

- 4D light field, around a *central ray*
- We study its spectrum during transport



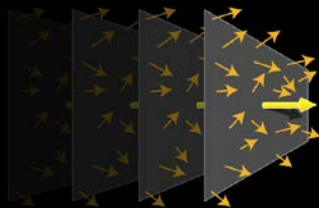
Local light field

- 4D light field, around a *central ray*
- We study its spectrum during transport



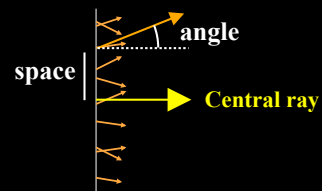
Local light field

- 4D light field, around a *central ray*
- We study its spectrum during transport



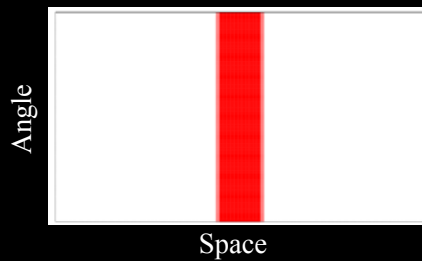
Local light field parameterization

- Space and angle



Local light field representation

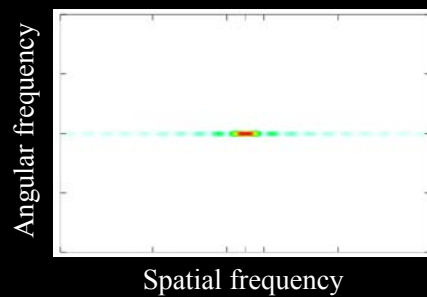
- Density plot:



Local light field Fourier spectrum

- We are interested in the Fourier spectrum of the local light field
- Also represented as a density plot

Local light field Fourier spectrum

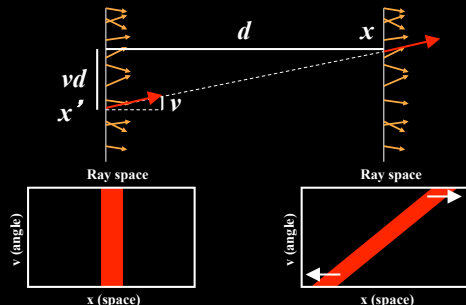


Fourier analysis 101

- Spectrum corresponds to blurriness:
 - Sharpest feature has size $\sim 1/F_{\max}$
- Convolution theorem:
 - Multiplication of functions: spectrum is convolved
 - Convolution of functions: spectrum is multiplied
- Classical spectra:
 - Box becomes sinc
 - Dirac becomes constant

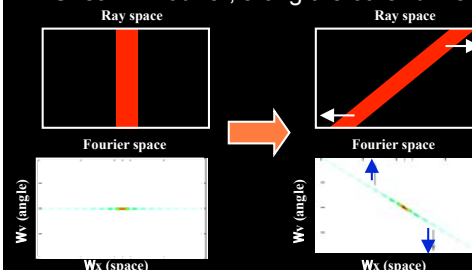
Transport

- Shear: $x' = x - v d$



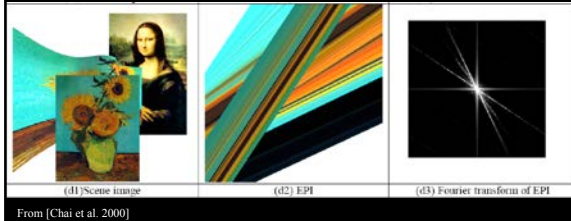
Transport in Fourier space

- Shear in primal: $x' = x - v d$
- Shear in Fourier, along the other dimension



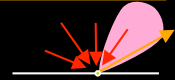
Transport becomes Shear

- This is consistent with light field spectra [Chai et al. 00, Isaksen et al. 00]



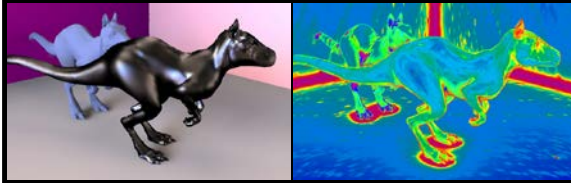
BRDF integration

- Ray-space: **convolution**
 - Outgoing light: convolution of incoming light and BRDF
 - For rotationally-invariant BRDFs
- Fourier domain: **multiplication**
 - Outgoing spectrum: multiplication of incoming spectrum and BRDF spectrum



Adaptive shading sampling

- Per-pixel prediction of max. frequency (bandwidth)
 - Based on curvature, BRDF, distance to occluder, etc.
 - No spectrum computed, just estimate max frequency



Adaptive shading sampling

- Per-pixel prediction of max. frequency (bandwidth)
 - Based on curvature, BRDF, distance to occluder, etc.
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Uniform sampling



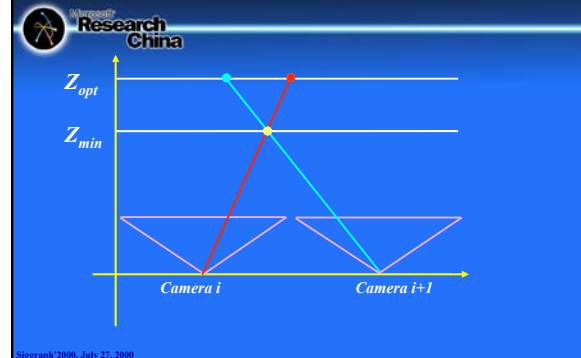
Adaptive sampling



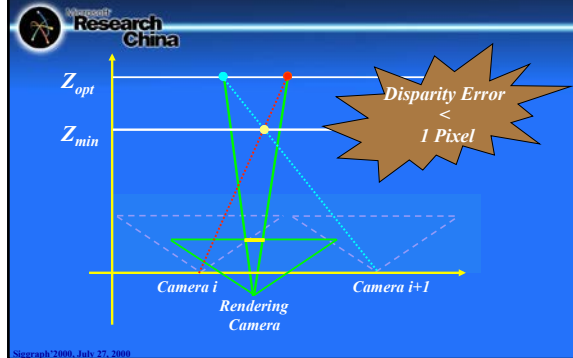
Plenoptic Sampling

- Plenoptic Sampling. *Chai, Tong, Chan, Shum 00*
- Signal-processing on light field
- Minimal sampling rate for antialiased rendering
- Relates to depth range, Fourier analysis
- Fourier spectra derived for 2D light fields for simplicity. Same ideas extend to 4D
- Key paper in many newer methods on sheared and axis-aligned filtering for adaptive sampling

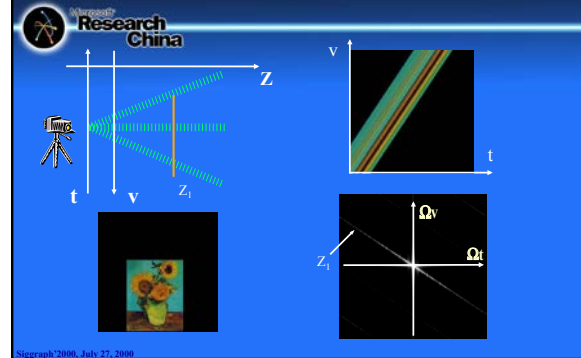
A Geometrical Intuition



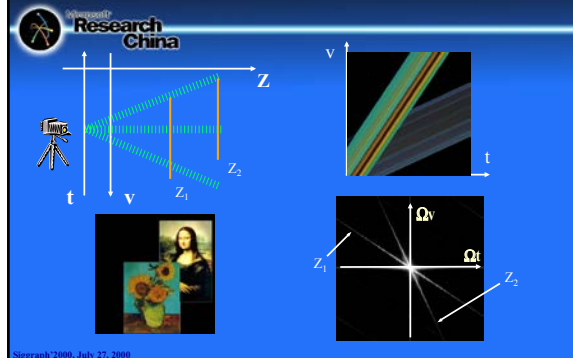
A Geometrical Intuition



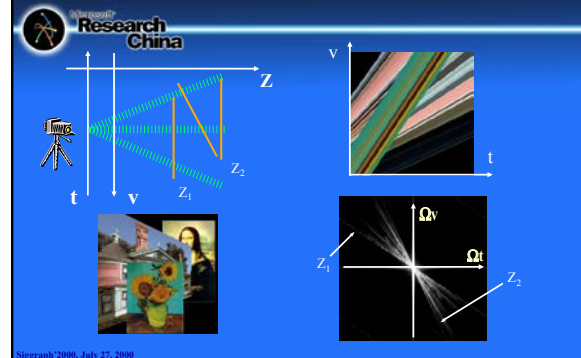
A Constant Plane

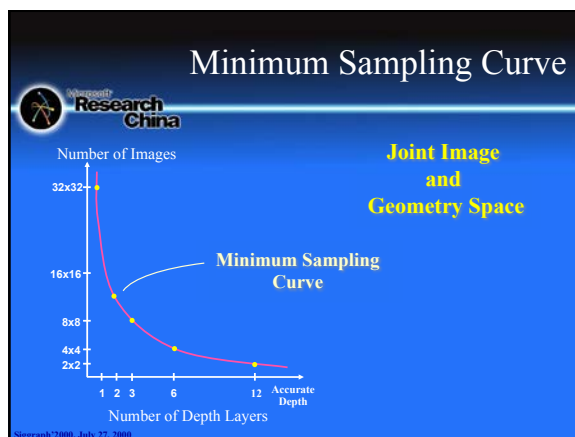
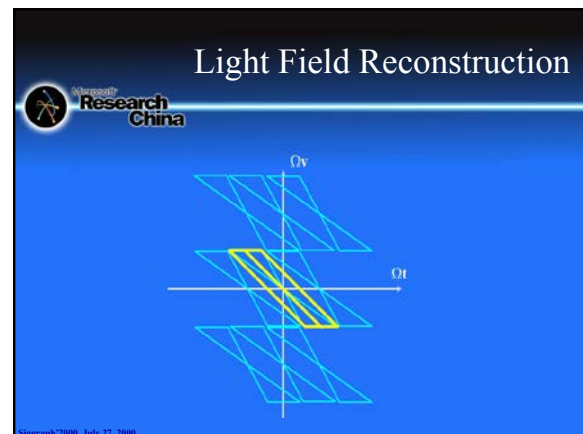
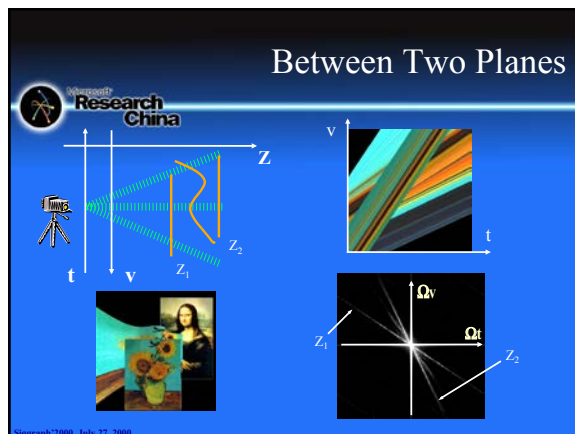


Two Constant Planes



Between Two Planes





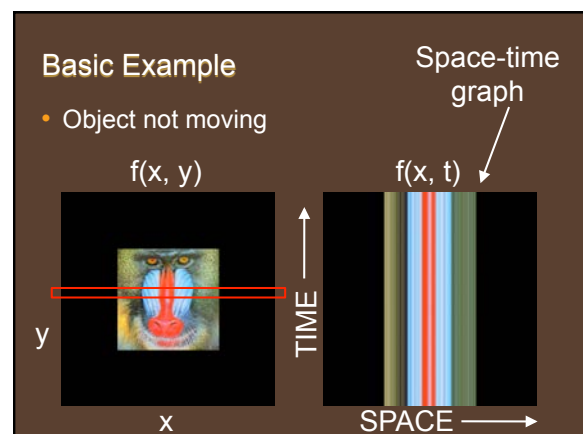
Frequency Analysis and Sheared Reconstruction for Rendering Motion Blur

Kevin Egan	Columbia University
Yu-Ting Tseng	Columbia University
Nicolas Holzschuch	INRIA -- LJK
Frédéric Durand	MIT CSAIL
Ravi Ramamoorthi	University of California, Berkeley

SIGGRAPH 2009 NEW ORLEANS

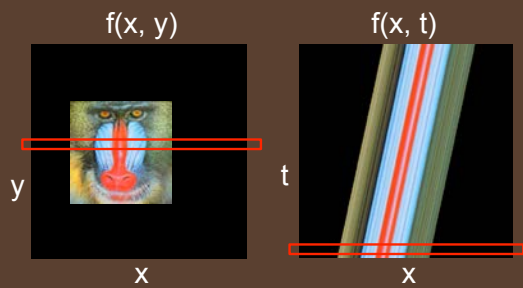
Observation

- Motion blur is expensive
- Motion blur *removes* spatial complexity



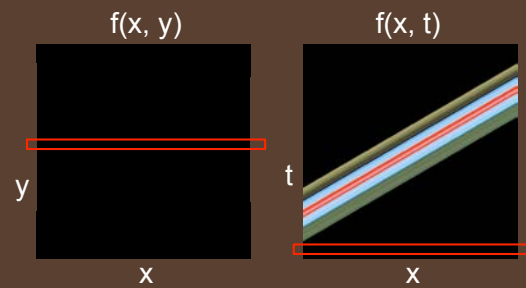
Basic Example

- Low velocity, $t \in [0.0, 1.0]$



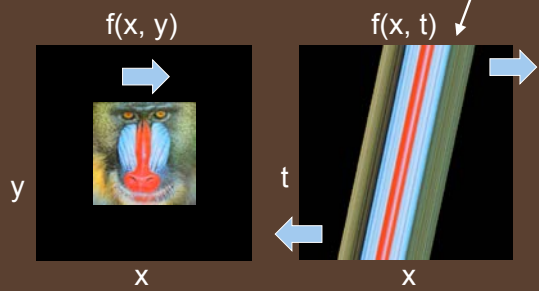
Basic Example

- High velocity, $t \in [0.0, 1.0]$



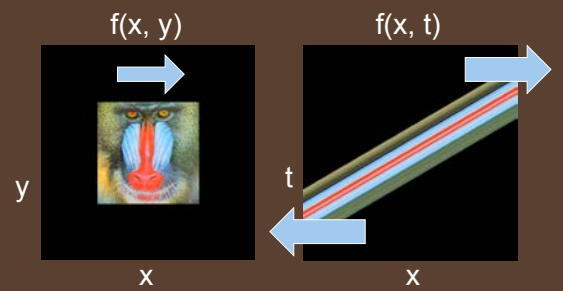
Shear in Space-Time

- Object moving with low velocity



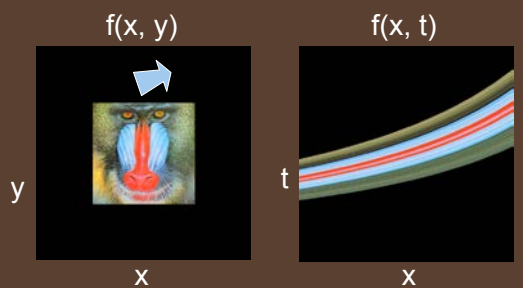
Shear in Space-Time

- Object moving with high velocity



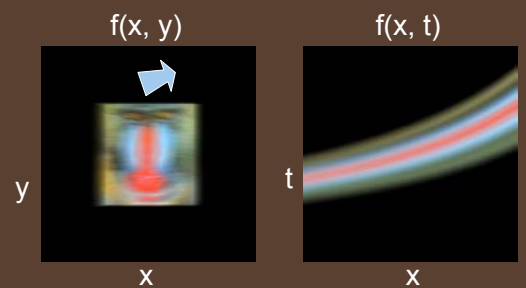
Shear in Space-Time

- Object moving away from camera



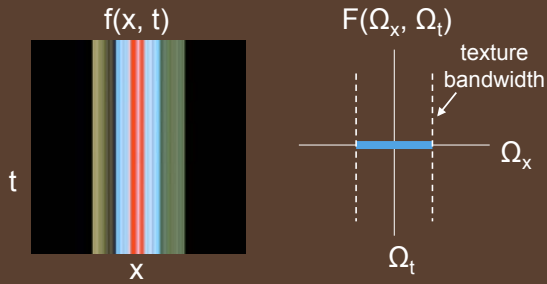
Basic Example

- Applying shutter blurs across time



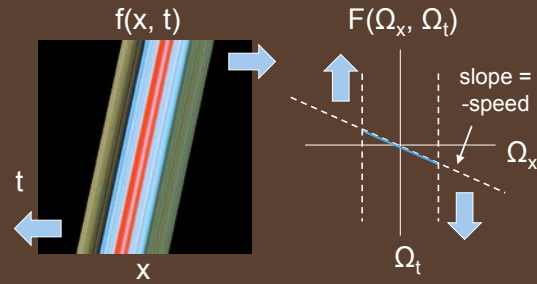
Basic Example – Fourier Domain

- Fourier spectrum, zero velocity



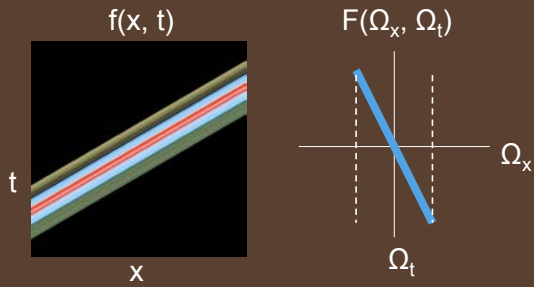
Basic Example – Fourier Domain

- Low velocity, small shear in both domains



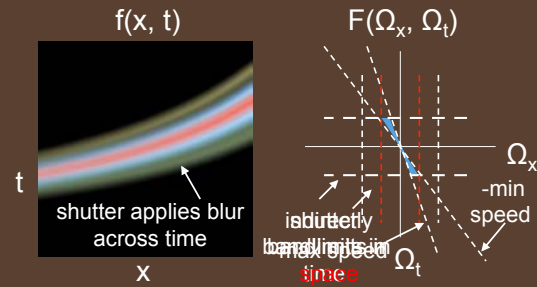
Basic Example – Fourier Domain

- Large shear



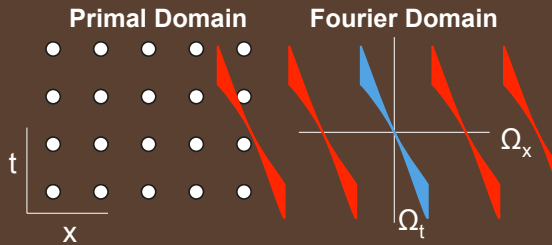
Basic Example – Fourier Domain

- Non-linear motion, wedge shaped spectra



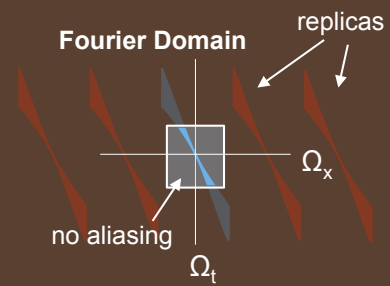
Sampling in Fourier Domain

- Sampling produces **replicas** in Fourier domain
- Sparse sampling produces dense replicas



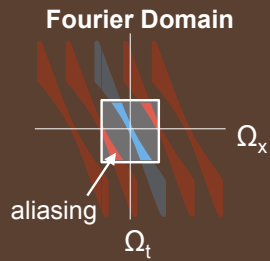
Standard Reconstruction Filtering

- Standard filter, dense sampling (slow)



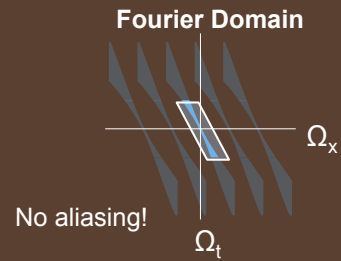
Standard Reconstruction Filter

- Standard filter, sparse sampling (fast)



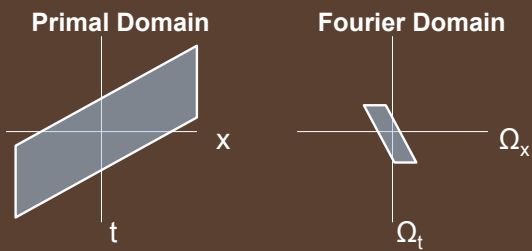
Sheared Reconstruction Filter

- Our sheared filter, sparse sampling (fast)



Sheared Reconstruction Filter

- Compact shape in Fourier = wide in primal

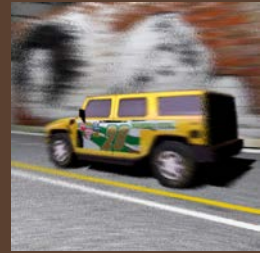


Car Scene

Our Method,
4 samples per pixel



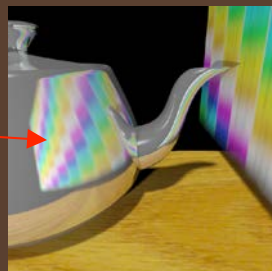
Stratified Sampling
4 samples per pixel



Teapot Scene

Our Method
8 samples / pix

motion blurred
reflection



Ballerina Video

Ballerina sequence
(8 samples/pixel)

Note smooth motion-blur
of dress and shadows

Frequency Analysis
and Sheared Reconstruction
for Rendering Motion Blur

ID: 0034