

Sampling and Reconstruction of Visual Appearance

CSE 274 [Fall 2018], Lecture 2

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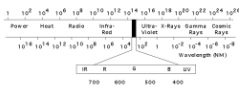
Motivation: BRDFs, Radiometry

- Basics of Illumination, Reflection
- Formal radiometric analysis (not ad-hoc)
- Reflection Equation
- Monte Carlo Rendering next week
- Appreciate formal analysis in a graduate course, even if not absolutely essential in practice

Light

Visible electromagnetic radiation

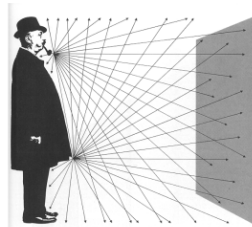
Power spectrum



Polarization

Photon (quantum effects)

Wave (interference, diffraction)



From London and Upton

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Radiometry

- Physical measurement of electromagnetic energy
- Measure spatial (and angular) properties of light
 - Radiance, Irradiance
 - Reflection functions: Bi-Directional Reflectance Distribution Function or BRDF
 - Reflection Equation
 - Simple BRDF models

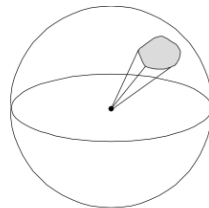
Angles and Solid Angles

■ Angle $\theta = \frac{l}{r}$

⇒ circle has 2π radians

■ Solid angle $\Omega = \frac{A}{R^2}$

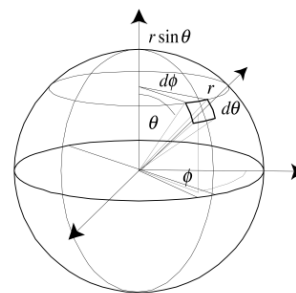
⇒ sphere has 4π steradians



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Differential Solid Angles

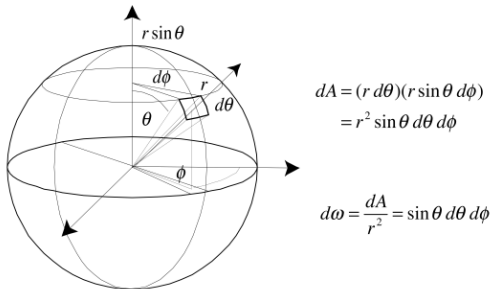


$$dA = (r d\theta)(r \sin \theta d\phi) = r^2 \sin \theta d\theta d\phi$$

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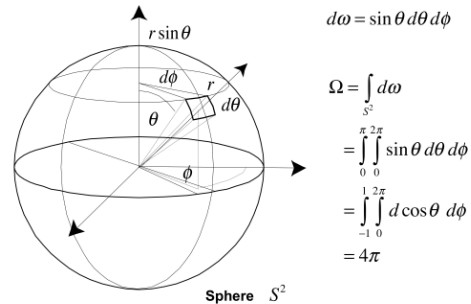
Differential Solid Angles



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Differential Solid Angles



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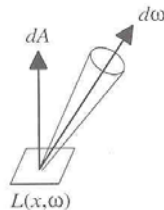
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Radiance

- Power per unit projected area perpendicular to the ray per unit solid angle in the direction of the ray

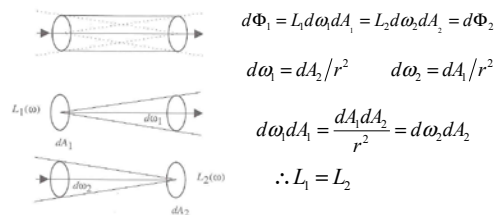
- Symbol: $L(x, \omega)$ ($\text{W/m}^2 \text{sr}$)

- Flux given by $d\Phi = L(x, \omega) \cos \theta d\omega dA$



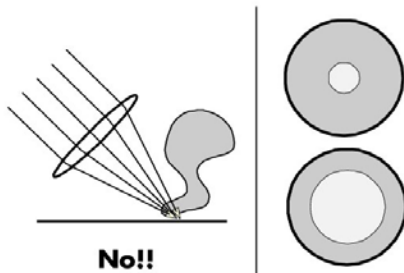
Radiance properties

- Radiance constant as propagates along ray
 - Derived from conservation of flux
 - Fundamental in Light Transport.



Quiz

Does radiance increase under a magnifying glass?

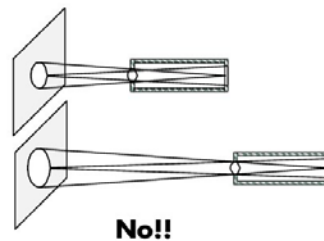


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Quiz

Does the brightness that a wall appears to the eye depend on the distance of the viewer to the wall?



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Radiance properties

- Sensor response proportional to radiance (constant of proportionality is throughput)
 - Far away surface: See more, but subtends smaller angle
 - Wall equally bright across viewing distances
- Consequences
 - Radiance associated with rays in a ray tracer
 - Other radiometric quants derived from radiance

Irradiance, Radiosity

- Irradiance E is radiant power per unit area
- Integrate incoming radiance over hemisphere
 - Projected solid angle ($\cos \theta d\omega$)
 - Uniform illumination: Irradiance = π [CW 24,25]
 - Units: W/m^2
- Radiant Exitance (radiosity)
 - Power per unit area leaving surface (like irradiance)

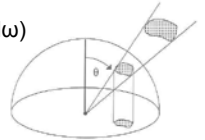


Figure 2.8: Projection of differential area.

Directional Power Arriving at a Surface

$$d^2 \Phi_i(x, \omega) = L_i(x, \omega) \cos \theta dA d\omega$$

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Irradiance from the Environment

$$d^2 \Phi_i(x, \omega) = L_i(x, \omega) \cos \theta dA d\omega$$

$$dE(x, \omega) = L_i(x, \omega) \cos \theta d\omega$$



Light meter

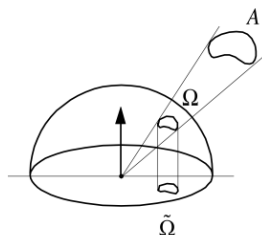
$$E(x) = \int_{H^2} L_i(x, \omega) \cos \theta d\omega$$

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Uniform Area Source

$$\begin{aligned} E(x) &= \int_{H^2} L \cos \theta d\omega \\ &= L \int_{\Omega} \cos \theta d\omega \\ &= L \tilde{\Omega} \end{aligned}$$



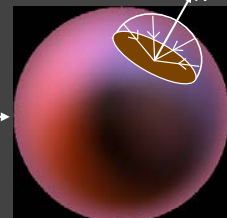
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Irradiance Environment Maps



Incident Radiance
(Illumination Environment Map)



Irradiance Environment Map

Radiometry

- Physical measurement of electromagnetic energy
- Measure spatial (and angular) properties of light
 - Radiance, Irradiance
 - Reflection functions: Bi-Directional Reflectance Distribution Function or BRDF
 - Reflection Equation
 - Simple BRDF models

Types of Reflection Functions

Ideal Specular

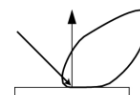
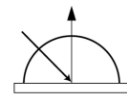
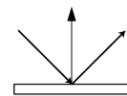
- Reflection Law
- Mirror

Ideal Diffuse

- Lambert's Law
- Matte

Specular

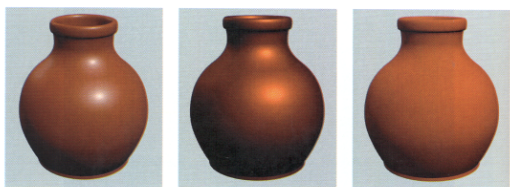
- Glossy
- Directional diffuse



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Materials



Plastic

Metal

Matte

From Apodaca and Gritz, *Advanced RenderMan*

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Spheres [Matusik et al.]



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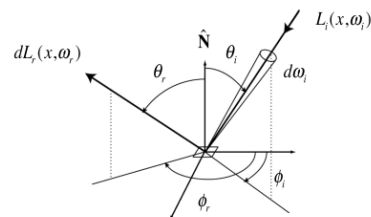
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Building up the BRDF

- Bi-Directional Reflectance Distribution Function [Nicodemus 77]
- Function based on incident, view direction
- Relates incoming light energy to outgoing
- Unifying framework for many materials

The BRDF

Bidirectional Reflectance-Distribution Function



$$f_r(\omega_i \rightarrow \omega_r) \equiv \frac{dL_r(x, \omega_r)}{dE_i} \left[\frac{1}{sr} \right]$$

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BRDF

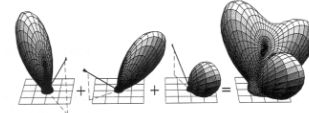
- Reflected Radiance proportional Irradiance
- Constant proportionality: BRDF
- Ratio of outgoing light (radiance) to incoming light (irradiance)
 - Bidirectional Reflection Distribution Function
 - (4 Vars) units 1/sr

$$f(\omega_i, \omega_r) = \frac{L_r(\omega_r)}{L_i(\omega_i) \cos \theta_i d\omega_i}$$

$$L_r(\omega_r) = L_i(\omega_i) f(\omega_i, \omega_r) \cos \theta_i d\omega_i$$

Properties of BRDF's

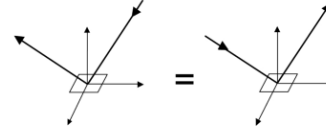
1. Linearity



From Sillion, Arvo, Westin, Greenberg

2. Reciprocity principle

$$f_r(\omega_r \rightarrow \omega_i) = f_r(\omega_i \rightarrow \omega_r)$$



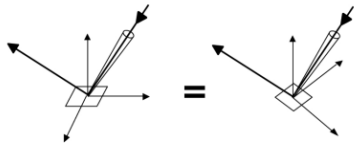
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Properties of BRDF's

3. Isotropic vs. anisotropic

$$f_r(\theta_i, \phi_i; \theta_r, \phi_r) = f_r(\theta_i, \theta_r, \phi_r - \phi_i)$$



Reciprocity and isotropy

$$f_r(\theta_i, \theta_r, \phi_r - \phi_i) = f_r(\theta_r, \theta_i, \phi_i - \phi_r) = f_r(\theta_i, \theta_r, |\phi_r - \phi_i|)$$

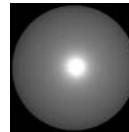
4. Energy conservation

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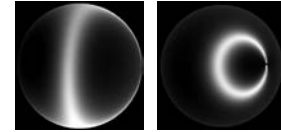
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Isotropic vs Anisotropic

- Isotropic: Most materials (you can rotate about normal without changing reflections)
- Anisotropic: brushed metal etc. preferred tangential direction



Isotropic



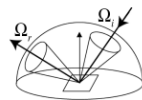
Anisotropic

Energy Conservation

$$\frac{d\Phi_r}{d\Phi_i} = \frac{\int_{\Omega_r} L_r(\omega_r) \cos \theta_r d\omega_r}{\int_{\Omega_i} L_i(\omega_i) \cos \theta_i d\omega_i}$$

$$= \frac{\int_{\Omega_r} \int_{\Omega_i} f_r(\omega_i \rightarrow \omega_r) L_i(\omega_i) \cos \theta_i d\omega_i \cos \theta_r d\omega_r}{\int_{\Omega_i} L_i(\omega_i) \cos \theta_i d\omega_i}$$

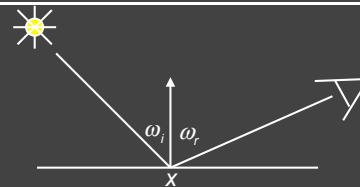
$$\leq 1$$



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Reflection Equation



$$L_r(x, \omega_r) = L_e(x, \omega_r) + L_i(x, \omega_i) f(x, \omega_i, \omega_r) (\omega_i \cdot n)$$

Reflected Light (Output Image) Emission Incident Light (from light source) BRDF Cosine of Incident angle

Reflection Equation

Sum over all light sources

$$L_r(x, \omega_r) = L_e(x, \omega_r) + \sum L_i(x, \omega_i) f(x, \omega_i, \omega_r) (\omega_i \cdot n)$$

Reflected Light (Output Image)	Emission	Incident Light (from light source)	BRDF	Cosine of Incident angle
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Reflection Equation

Replace sum with integral

$$L_r(x, \omega_r) = L_e(x, \omega_r) + \int_{\Omega} L_i(x, \omega_i) f(x, \omega_i, \omega_r) \cos \theta_i d\omega_i$$

Reflected Light (Output Image)	Emission	Incident Light (from light source)	BRDF	Cosine of Incident angle
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Radiometry

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Brdf Viewer plots

Diffuse Torrance-Sparrow Anisotropic

by written by Szymon Rusinkiewicz

Ideal Diffuse Reflection

Assume light is equally likely to be reflected in any output direction (independent of input direction).

$$L_{r,d}(\omega_r) = \int f_{r,d} L_i(\omega_i) \cos \theta_i d\omega_i$$

$$= f_{r,d} \int L_i(\omega_i) \cos \theta_i d\omega_i$$

$$= f_{r,d} E$$

$$M = \int L_r(\omega_r) \cos \theta_r d\omega_r = L_r \int \cos \theta_r d\omega_r = \pi L_r$$

$$\rho_d = \frac{M}{E} = \frac{\pi L_r}{E} = \frac{\pi f_{r,d} E}{E} = \pi f_{r,d} \Rightarrow f_{r,d} = \frac{\rho_d}{\pi}$$

Lambert's Cosine Law $M = \rho_d E = \rho_d E_i \cos \theta_i$

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Phong Model

$$(\hat{E} \cdot \mathbf{R}(\hat{L}))^p = (\hat{L} \cdot \mathbf{R}(\hat{E}))^p$$

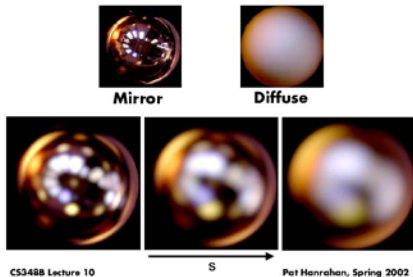
Reciprocity: $(\hat{E} \cdot \mathbf{R}(\hat{L}))^p = (\hat{L} \cdot \mathbf{R}(\hat{E}))^p$

Distributed light source!

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Specular Term (Phong)

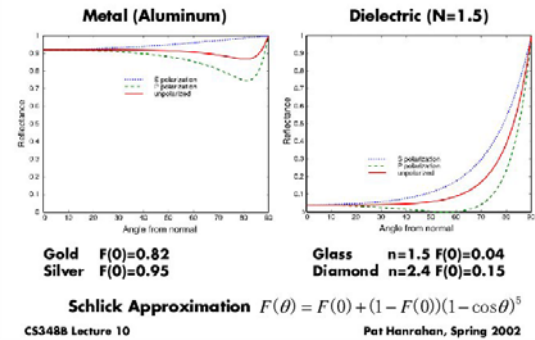
Phong Model



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Fresnel Reflectance



Experiment

Reflections from a shiny floor



From Lafortune, Foo, Torrance, Greenberg, SIGGRAPH 97

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Torrance-Sparrow

Fresnel term:
allows for wavelength
dependency

Geometric Attenuation:
reduces the output based on the
amount of shadowing or masking
that occurs

$$f = \frac{F(\theta_i)G(\omega_i, \omega_r)D(\theta_h)}{4\cos(\theta_i)\cos(\theta_r)}$$

How much of the
macroscopic surface
is visible to the light
source

How much of the
macroscopic surface
is visible
to the viewer

Distribution:
distribution function
determines what
percentage of
microfacets are
oriented to reflect in
the viewer direction.

Other BRDF models

- Empirical: Measure and build a 4D table
- Anisotropic models for hair, brushed steel
- Cartoon shaders, funky BRDFs
- Capturing spatial variation
- Very active area of research