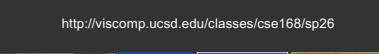


Computer Graphics II: Rendering

CSE 168 [Spr 26], Lecture 15: Volumetric Rendering

Ravi Ramamoorthi

<http://viscomp.ucsd.edu/classes/cse168/sp26>



1

To Do

- Start working on final projects (initial results and proposal due in a week). Ask me if problems
- Volumetric rendering (this lecture) may be one component of the final project (but hard, be careful)
- Increasingly accurate appearance requires volumetric scattering (even for skin, hair, fur)
- Continues to be an active area of research

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Volumetric Scattering

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Volumetric Scattering



A photograph of a street at night with heavy fog, illustrating volumetric scattering. Below the image is a row of six glasses containing liquids of increasing density, representing the scattering medium.

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Volumetric Scattering

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Volumetric Scattering

- Participating Media (medium participates: scattering)
 - Volumetric phenomena like clouds, smoke, fire
 - Subsurface scattering, translucency (wax, human skin)
 - These are not surfaces with well-defined BRDFs
 - Rather volumes where light can scatter
 - Medium is often known as a participating medium
- Surface Rendering: Radiance Constant along Ray
 - Only true in absence of participating media
 - No longer true for volumetric scattering
 - Often replace ray tracing with ray marching in medium
- Volumetric Properties
 - BRDF replaced by phase function
 - Must consider absorption and scattering in medium

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Homogeneous vs Heterogeneous

- Homogeneous: Properties constant everywhere
 - Example: Fog often represented as homogeneous
- Heterogeneous: Varies across space
 - Example: Smoke, fire etc.
 - Sometimes called inhomogeneous
- Homogeneous volumes often easier
 - Some computational shortcuts (transmittance etc.)
 - Some analytic formulae

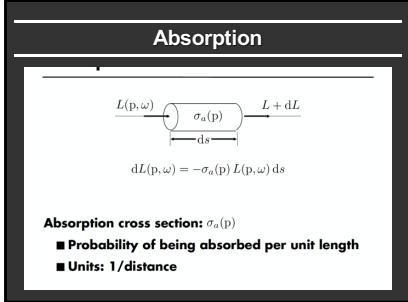
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Homogeneous vs Heterogeneous

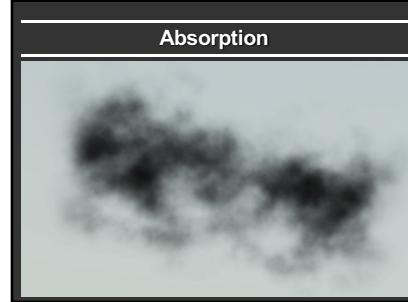
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Volumetric Interactions

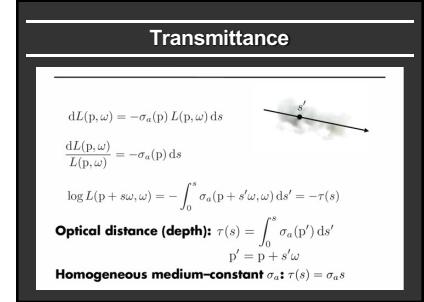
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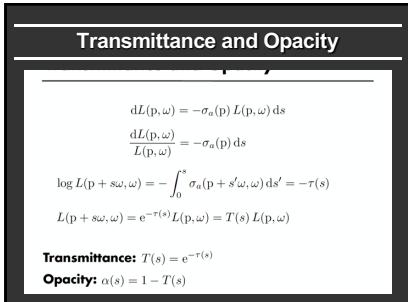
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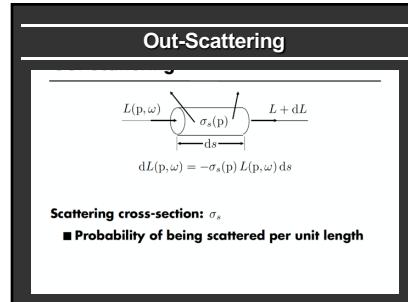
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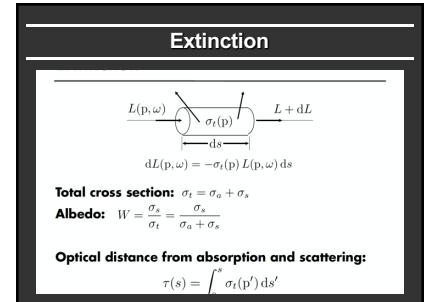
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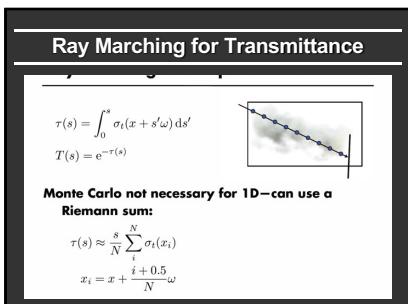
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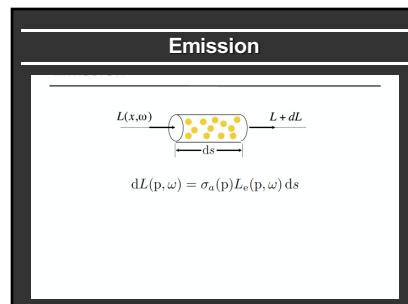
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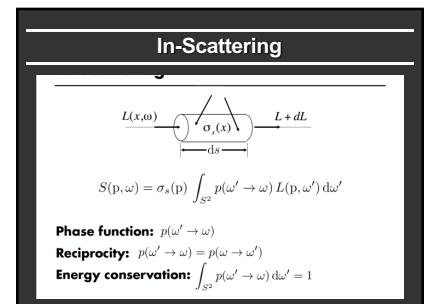
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Scattering Phase Functions

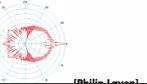
- Light interacts with volume, scatters in some spherical distribution
- Similar to light scattering off a surface
- Phase function analogous to a surface BRDF
- Depends only on cosine of incident-outgoing
- Like BRDFs, volumetric phase functions must be reciprocal and conserve energy
- Similar to BRDFs, we will want to do importance sampling and evaluation of phase functions

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Phase Functions

Phase angle $\cos \theta = \omega \cdot \omega'$

Phase functions

- Isotropic:** $p(\cos \theta) = \frac{1}{4\pi}$
- Rayleigh:** $p(\cos \theta) = \frac{3}{4}(1 + \cos^2 \theta)$ with $\sigma_s \propto \frac{1}{\lambda^4}$
- Mie:** 

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Rayleigh Scattering

- Rayleigh scattering describes the scattering of light by particles much smaller than the wavelength

$$p(\cos \theta) = \frac{3}{16\pi} (1 + \cos^2 \theta)$$

$$\sigma_s = \frac{2\pi^5}{3} \frac{d^6}{\lambda^4} \frac{(n^2 - 1)^2}{(n^2 + 2)}$$

Where λ is the wavelength of light, d is the diameter of the particle, and n is the index of refraction of the particle.

- The strong dependence on wavelength (λ^{-4}) causes greater scattering towards the blue end of the spectrum
- The blue color of the sky is caused by Rayleigh scattering of sunlight by air molecules

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Rayleigh Scattering: Blue Sky, Red Sunset



From Greenler: Rainbows, Halos, and Glories
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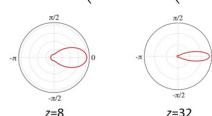
Mie Scattering

- Scatter electromagnetic waves by spherical particles
- Size of particles same scale as wavelength of light
- Water droplets in atmosphere, fat droplets in milk
- After Gustave Mie, Ludvig Lorenz

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Empirical Mie Approximation

- The following empirical function is often used to approximate the shape of Mie scattering

$$p(\cos \theta) = \frac{1}{4\pi} \left(\frac{1}{2} + \frac{(z+1)}{2} \left(\frac{1 + \cos \theta}{2} \right)^z \right)$$


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Henyey-Greenstein Function

- The Henyey-Greenstein phase function is an empirical function originally designed to model the scattering in galactic dust clouds

$$p(\cos \theta) = \frac{1 - g^2}{4\pi(1 + g^2 - 2g\cos \theta)^{1.5}}$$

$g=0.45$

- It uses an anisotropy parameter g that ranges between -1 (full backscatter) and 1 (full forward scatter), and is isotropic for $g=0$

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Direct Illumination in a Volume

$$S_d(p', \omega) = \sigma_s(p') \int_{S^2} p(\omega' \rightarrow \omega) L_d(p', \omega') d\omega'$$

Can treat like direct illumination at a surface

- Sample from phase function's distribution
- Sample from light source distributions
- Weight using multiple importance sampling



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Direct Illumination in a Volume

$$S_d(p', \omega) = \sigma_s(p') \int_{S^2} p(\omega' \rightarrow \omega) L_d(p', \omega') d\omega'$$

Estimator: $\sigma_s(p') \frac{1}{N} \sum_i^N \frac{p(\omega_i \rightarrow \omega) L_d(p', \omega_i)}{p(\omega_i)}$

Computing direct lighting, L_d can be expensive

- Not just a shadow ray - need to compute transmittance



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Transmittance for Shadow Rays

Besides Monte Carlo, precomputed transmittance can be faster for point, distant lights

3D grid [Kajiya and von Herzen 1984]

Deep Shadow Maps [Lukovic & Veach 2009] Adaptive Volumetric Shadow Maps [Salvi et al. 2010]

Two line graphs show transmittance (y-axis, 0.2 to 1.0) versus depth (x-axis, 0.2 to 0.8). The first graph shows a step function with a sharp drop at depth 0.4. The second graph shows a smooth curve with a similar drop, labeled 'Deep Shadow Maps' and 'Adaptive Volumetric Shadow Maps'.

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Single-Scattering

Minneart: Color and Light In The Open Air

pbrt: Spot-Lit Ball In The Fog

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The Volume Rendering Equation

Integro-differential equation:

$$\frac{\partial L(p, \omega)}{\partial s} = -\sigma_t L(p, \omega) + S(p, \omega)$$

Integro-integral equation:

$$L(p, \omega) = \int_0^{\infty} T(p') S(p', \omega) ds'$$

Attenuation: absorption and scattering

$$e^{-\int_0^{s'} \sigma_t(p'') ds''}$$

Source: in-scattering (and emission)

$$\sigma_s(p') \int_{S^2} p(\omega' \rightarrow \omega) L(p', \omega') d\omega'$$

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Volumetric Path Tracing

Integro-integral equation:

$$L(p, \omega) = \int_0^{\infty} T(p') S(p', \omega) ds'$$

Monte Carlo integration: sample $s' \sim p(s)$

Estimator: $\frac{T(p') S(p', \omega)}{p(s')}$

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Evaluating the Estimator: S

Include indirect illumination in the source term:

$$S(x, \omega) = \sigma_s(x) \int_{S^2} p(\omega' \rightarrow \omega) L(x, \omega') d\omega'$$

$$L(x, \omega') = L_d(x, \omega') + L_i(x, \omega')$$

Compute direct lighting as before

Sample incident direction from the phase function's distribution, trace a ray recursively...

$L_i(x, \omega') \approx \frac{p(\omega'' \rightarrow \omega') L(x, \omega'')}{p(\omega'')}$

Uniform spherical directions: $p(\omega'') = \frac{1}{4\pi}$

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Linear Sampling of T

We want samples along a finite ray $[0, t_{\max}]$.

Uniform probability along the ray:

$$p(t) = \frac{1}{t_{\max}}$$

Sampling recipe:

$$\xi = \int_0^t p(t) dt \quad t = \xi t_{\max}$$

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Exact Sampling of Uniform T

We want samples along a finite ray $[0, t_{\max}]$, $p(t) \propto e^{-\sigma t}$

Normalize to find PDF:

$$\int_0^{t_{\max}} e^{-\sigma t} dt = -\frac{1}{\sigma}(e^{-\sigma t_{\max}} - 1) = c \quad p(t) = ce^{-\sigma t}$$

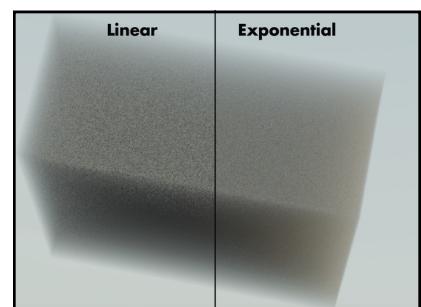
Invert to find t for a random sample:

$$\xi = \int_0^t p(t) dt \quad t = -\frac{1}{\sigma} \log(1 - \xi(1 - e^{-\sigma t_{\max}}))$$

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Volumetric Path Tracing

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Estimator: $\frac{T(p')S(p', \omega)}{p(s')}$

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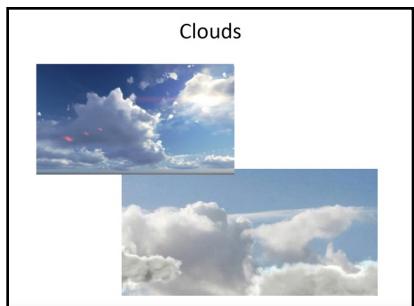
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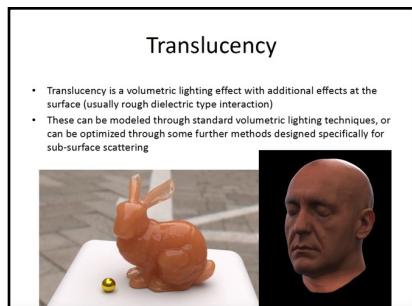
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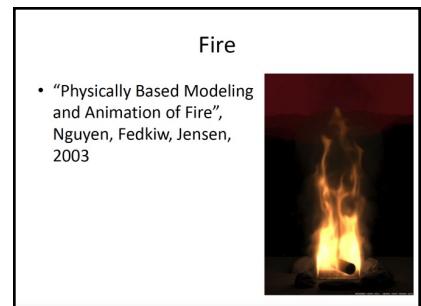
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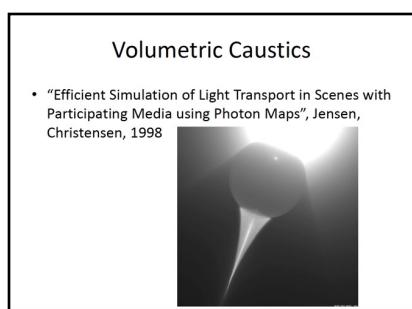
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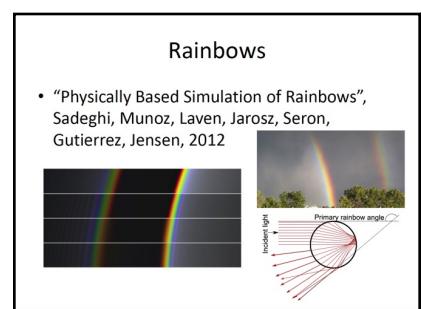
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Atmospheric Phenomena



Corona



Ice Crystal Halo



Glory