


Computer Graphics II: Rendering

CSE 168[Spr 26], Lecture 15: Volumetric Rendering
Ravi Ramamoorthi

<http://viscomp.ucsd.edu/classes/cse168/sp26>



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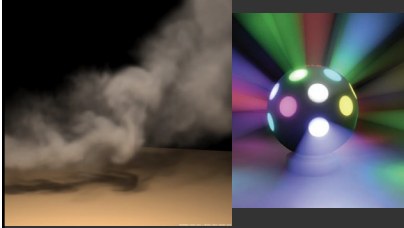
To Do

- Start working on final projects (initial results and proposal due in a week). Ask me if problems
- Volumetric rendering (this lecture) may be one component of the final project (but hard, be careful)
- Increasingly accurate appearance requires volumetric scattering (even for skin, hair, fur)
- Continues to be an active area of research

Many slides courtesy Pat Hanrahan/Matt Pharr (Stanford CS 348b) and Steve Rottenberg, Henrik Wann Jensen (UCSD CSE 168)


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Volumetric Scattering



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Volumetric Scattering



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Volumetric Scattering

Computer Graphics, Volume 21, Number 4, July 1987


THE SIMULATION OF PARTICIPATING MEDIUM TRANSPORT
IN THE RENDERING OF A PARTICIPATING MEDIUM

WILLI M. SLICKER
ANDREW A. CHARTERIS

Program of Computer Graphics and School of Mechanical and
Aerospace Engineering
Cornell University
Ithaca, New York 14853

ABSTRACT

The most useful tool for simulating radiative transport in the presence of a participating medium is the Monte Carlo method. This paper describes the Monte Carlo method for simulating the transport of radiation in a participating medium. The Monte Carlo method is used to simulate the transport of radiation in a participating medium. The Monte Carlo method is used to simulate the transport of radiation in a participating medium. The Monte Carlo method is used to simulate the transport of radiation in a participating medium.



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Volumetric Scattering

- Participating Media (medium participates; scattering)
 - Volumetric phenomena like clouds, smoke, fire
 - Subsurface scattering, translucency (wax, human skin)
 - These are not surfaces with well-defined BRDFs
 - Rather volumes where light can scatter
 - Medium is often known as a participating medium
- Surface Rendering: Radiance Constant along Ray
 - Only true in absence of participating media
 - No longer true for volumetric scattering
 - Often replace ray tracing with ray marching in medium
- Volumetric Properties
 - BRDF replaced by phase function
 - Must consider absorption and scattering in medium

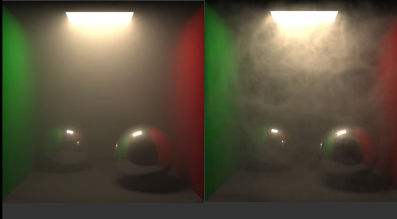
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Homogeneous vs Heterogeneous

- Homogeneous: Properties constant everywhere
 - Example: Fog often represented as homogeneous
- Heterogeneous: Varies across space
 - Example: Smoke, fire etc.
 - Sometimes called inhomogeneous
- Homogeneous volumes often easier
 - Some computational shortcuts (transmittance etc.)
 - Some analytic formulae

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Homogeneous vs Heterogeneous



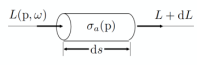
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Volumetric Interactions

- 4 different processes affect radiance of a beam
 - Absorption
 - Out-Scattering
 - Emission
 - In-Scattering

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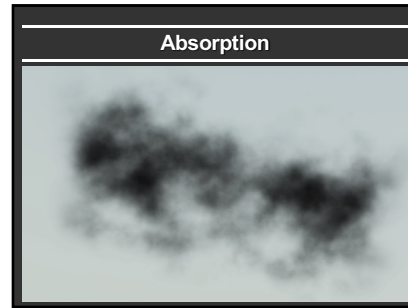
Absorption



$$dL(p, \omega) = -\sigma_a(p) L(p, \omega) ds$$

Absorption cross section: $\sigma_a(p)$
 ■ Probability of being absorbed per unit length
 ■ Units: 1/distance

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Transmittance

$$dL(p, \omega) = -\sigma_a(p) L(p, \omega) ds$$

$$\frac{dL(p, \omega)}{L(p, \omega)} = -\sigma_a(p) ds$$

$$\log L(p + s\omega, \omega) = -\int_0^s \sigma_a(p + s'\omega, \omega) ds' = -\tau(s)$$

Optical distance (depth): $\tau(s) = \int_0^s \sigma_a(p') ds'$
 $p' = p + s'\omega$
Homogeneous medium-constant σ_a : $\tau(s) = \sigma_a s$



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Transmittance and Opacity

$$dL(p, \omega) = -\sigma_a(p) L(p, \omega) ds$$

$$\frac{dL(p, \omega)}{L(p, \omega)} = -\sigma_a(p) ds$$

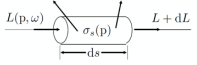
$$\log L(p + s\omega, \omega) = -\int_0^s \sigma_a(p + s'\omega, \omega) ds' = -\tau(s)$$

$$L(p + s\omega, \omega) = e^{-\tau(s)} L(p, \omega) = T(s) L(p, \omega)$$

Transmittance: $T(s) = e^{-\tau(s)}$
Opacity: $\alpha(s) = 1 - T(s)$

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Out-Scattering

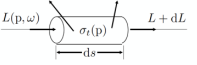


$$dL(p, \omega) = -\sigma_s(p) L(p, \omega) ds$$

Scattering cross-section: σ_s
 ■ Probability of being scattered per unit length

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Extinction



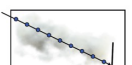
$$dL(p, \omega) = -\sigma_t(p) L(p, \omega) ds$$

Total cross section: $\sigma_t = \sigma_a + \sigma_s$
Albedo: $W = \frac{\sigma_s}{\sigma_t} = \frac{\sigma_s}{\sigma_a + \sigma_s}$
Optical distance from absorption and scattering:
 $\tau(s) = \int_0^s \sigma_t(p') ds'$

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Ray Marching for Transmittance

$$\tau(s) = \int_0^s \sigma_t(x + s'\omega) ds'$$

$$T(s) = e^{-\tau(s)}$$


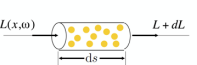
Monte Carlo not necessary for 1D—can use a Riemann sum:

$$\tau(s) \approx \sum_{i=1}^N \sigma_t(x_i)$$

$$x_i = x + \frac{i + 0.5}{N} \omega$$

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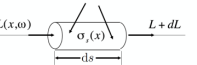
Emission



$$dL(p, \omega) = \sigma_e(p) L_e(p, \omega) ds$$

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In-Scattering



$$S(p, \omega) = \sigma_s(p) \int_{S^2} p(\omega' \rightarrow \omega) L(p, \omega') d\omega'$$

Phase function: $p(\omega' \rightarrow \omega)$
Reciprocity: $p(\omega' \rightarrow \omega) = p(\omega \rightarrow \omega')$
Energy conservation: $\int_{S^2} p(\omega' \rightarrow \omega) d\omega' = 1$

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Scattering Phase Functions

- Light interacts with volume, scatters in some spherical distribution
- Similar to light scattering off a surface
- Phase function analogous to a surface BRDF
- Depends only on cosine of incident-outgoing
- Like BRDFs, volumetric phase functions must be reciprocal and conserve energy
- Similar to BRDFs, we want to do importance sampling and evaluation of phase functions

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Phase Functions

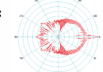
Phase angle $\cos \theta = \omega \cdot \omega'$

Phase functions

■ **Isotropic:** $p(\cos \theta) = \frac{1}{4\pi}$

■ **Rayleigh:** $p(\cos \theta) = \frac{3}{4}(1 + \cos^2 \theta)$ with $\sigma_s \propto \frac{1}{\lambda^4}$

■ **Mie:**



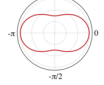
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Rayleigh Scattering

- Rayleigh scattering describes the scattering of light by particles much smaller than the wavelength

$$p(\cos \theta) = \frac{3}{16\pi}(1 + \cos^2 \theta)$$

$$\sigma_s = \frac{2\pi^5 d^6}{3 \lambda^4} \frac{(n^2 - 1)^2}{(n^2 + 2)^2}$$



- Where λ is the wavelength of light, d is the diameter of the particle, and n is the index of refraction of the particle
- The strong dependence on wavelength (λ^4) causes greater scattering towards the blue end of the spectrum
- The blue color of the sky is caused by Rayleigh scattering of sunlight by air molecules

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Rayleigh Scattering: Blue Sky, Red Sunset



From Greenler: Rainbows, Halos, and Glories

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Mie Scattering

- Scatter electromagnetic waves by spherical particles
- Size of particles same scale as wavelength of light
- Water droplets in atmosphere, fat droplets in milk
- After Gustave Mie, Ludvig Lorenz

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Empirical Mie Approximation

- The following empirical function is often used to approximate the shape of Mie scattering

$$p(\cos \theta) = \frac{1}{4\pi} \left(\frac{1}{2} + \frac{(z+1)}{2} \left(\frac{1 + \cos \theta}{2} \right)^z \right)$$



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Heney-Greenstein Function

- The Heney-Greenstein phase function is an empirical function originally designed to model the scattering in galactic dust clouds

$$p(\cos \theta) = \frac{1 - g^2}{4\pi(1 + g^2 - 2g \cos \theta)^{1.5}}$$



- It uses an anisotropy parameter g that ranges between -1 (full backscatter) and 1 (full forward scatter), and is isotropic for $g=0$

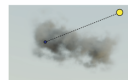
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Direct Illumination in a Volume

$$S_d(p', \omega) = \sigma_s(p') \int_{S^2} p(\omega' \rightarrow \omega) L_d(p', \omega') d\omega'$$

Can treat like direct illumination at a surface

- Sample from phase function's distribution
- Sample from light source distributions
- Weight using multiple importance sampling



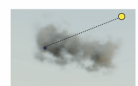
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Direct Illumination in a Volume

$$S_d(p', \omega) = \sigma_s(p') \int_{S^2} p(\omega' \rightarrow \omega) L_d(p', \omega') d\omega'$$

$$\text{Estimator: } \sigma_s(p') \frac{1}{N} \sum_{i=1}^N \frac{p(\omega_i \rightarrow \omega) L_d(p', \omega_i)}{p(\omega_i)}$$

- Computing direct lighting, L_d , can be expensive
- Not just a shadow ray—need to compute transmittance



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Transmittance for Shadow Rays

Besides Monte Carlo, precomputed transmittance can be faster for point, distant lights

3D grid [Kajiya and van der Zant 1984]

Deep Shadow Maps [Lokovic & Vach 2000]
Adaptive Volumetric Shadow Maps [Salvi et al. 2010]

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Single-Scattering

Minneart: Color and Light In The Open Air

pbrt: Spot-Lit Ball In The Fog

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The Volume Rendering Equation

Integro-differential equation:

$$\frac{\partial L(p, \omega)}{\partial s} = -\sigma_t L(p, \omega) + S(p, \omega)$$

Integro-integral equation:

$$L(p, \omega) = \int_0^\infty T(p') S(p', \omega) ds'$$

Attenuation: absorption and scattering
 $e^{-\int_0^s \sigma_t(p'') ds''}$

Source: in-scattering (and emission)
 $\sigma_s(p') \int_{S^2} p(\omega' \rightarrow \omega) L(p', \omega') d\omega'$

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Volumetric Path Tracing

Integro-integral equation:

$$L(p, \omega) = \int_0^\infty T(p') S(p', \omega) ds'$$

Monte Carlo integration: sample $s' \sim p(s)$

Estimator: $\frac{T(p') S(p', \omega)}{p(s')}$

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Evaluating the Estimator: S

Include indirect illumination in the source term:

$$S(x, \omega) = \sigma_s(x) \int_{S^2} p(\omega' \rightarrow \omega) L(x, \omega') d\omega'$$

\downarrow

$$L(x, \omega) = L_d(x, \omega) + L_i(x, \omega)$$

- Compute direct lighting as before
- Sample incident direction from the phase function's distribution, trace a ray recursively...

$$L_i(x, \omega) \approx \frac{p(\omega'' \rightarrow \omega) L(x, \omega'')}{p(\omega'')} \quad \text{Uniform spherical directions: } p(\omega'') = \frac{1}{4\pi}$$

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Linear Sampling of T

We want samples along a finite ray $[0, t_{\max}]$.

- Uniform probability along the ray:
 $p(t) = \frac{1}{t_{\max}}$
- Sampling recipe:
 $\xi = \int_0^t p(t) dt \quad t = \xi t_{\max}$

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Exact Sampling of Uniform T

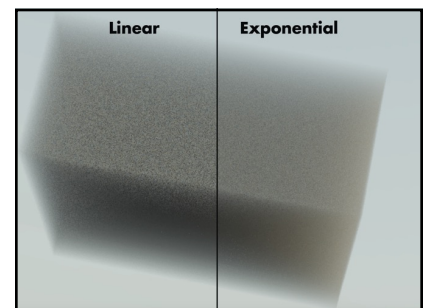
We want samples along a finite ray $[0, t_{\max}]$, $p(t) \propto e^{-\sigma t}$

- Normalize to find PDF:
 $\int_0^{t_{\max}} e^{-\sigma t} dt = -\frac{1}{\sigma} (e^{-\sigma t_{\max}} - 1) = c \quad p(t) = c e^{-\sigma t}$
- Invert to find t for a random sample:
 $\xi = \int_0^t p(t) dt$
 $t = -\frac{1}{\sigma} \log(1 - \xi(1 - e^{-\sigma t_{\max}}))$

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Volumetric Path Tracing

Integro-integral equation:

$$L(p, \omega) = \int_0^\infty T(p') S(p', \omega) ds'$$

Monte Carlo integration: sample $s' \sim p(s)$

Estimator: $\frac{T(p') S(p', \omega)}{p(s')}$

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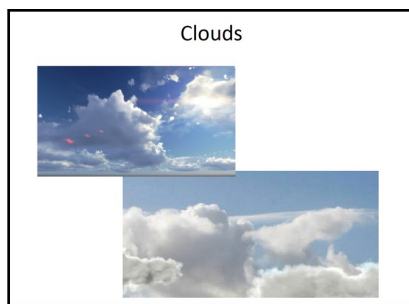
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Translucency

- Translucency is a volumetric lighting effect with additional effects at the surface (usually rough dielectric type interaction)
- These can be modeled through standard volumetric lighting techniques, or can be optimized through some further methods designed specifically for sub-surface scattering

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Fire

- "Physically Based Modeling and Animation of Fire", Nguyen, Fedkiw, Jensen, 2003

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Sky Rendering

- "A Practical Analytical Model for Daylight", Preetham, Shirley, Smits, 1999
- "A Physically Based Night Sky Model", Jensen, Durand, Stark, Premoze, Dorsey, Shirley, 2001
- "Precomputed Atmospheric Scattering", Bruneton, Neyret, 2008
- "An Analytic Model for Full Spectral Sky-Dome Radiance", Hosek, Wilkie, 2012

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Volumetric Caustics

- "Efficient Simulation of Light Transport in Scenes with Participating Media using Photon Maps", Jensen, Christensen, 1998

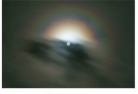
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Rainbows

- "Physically Based Simulation of Rainbows", Sadeghi, Munoz, Laven, Jarosz, Seron, Gutierrez, Jensen, 2012

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Atmospheric Phenomena



Corona



Ice Crystal Halo



Glory