Computer Graphics II: Rendering

CSE 168[Spr 25],Lecture 9: Importance Sampling
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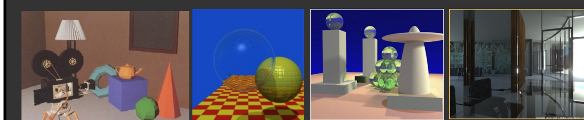
http://viscomp.ucsd.edu/classes/cse168/sp25



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To Do

- Start working on homework 3. Ask me if problems
- Also homework 4. This lecture covers material (Lecture is designed to follow assignment closely)
- Start thinking about final project

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Importance Sampling

- Talked about in Monte Carlo Path Tracing
- This assignment: implement at each bounce
- Use “good” pdf for sampling instead of uniform
- Extension to Multiple Importance Sampling (Veach 95)
  - Allows considering both lighting and BRDF sampling
  - Key development in production rendering (Academy Award)
  - Remains active topic of research (many papers in 2019+)

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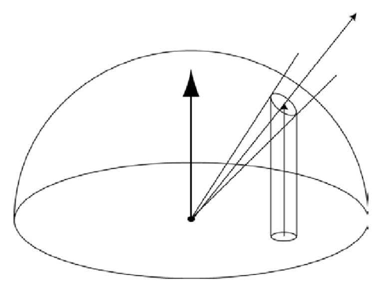
Sampling Projected Solid Angle

Generate cosine weighted distribution



CS348B Lecture 6 Pat Hanrahan, Spring 2004

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Cosine Importance Sampling

- Include cosine term in PDF (for indirect lighting)
- Previously, uniformly integrate over hemisphere $pdf(\omega_i) = \frac{1}{2\pi}$

$$\frac{1}{N}\sum_{k=1}^{N}\frac{L(\omega_{i,k})f(\omega_{i,k},\omega_o)(n\bullet\omega_{i,k})}{pdf(\omega_i)} = \frac{2\pi}{N}\sum_{k=1}^{N}L(\omega_{i,k})f(\omega_{i,k},\omega_o)(n\bullet\omega_{i,k})$$

- Now, consider a cosine PDF $pdf(\omega_i) = \frac{n\bullet\omega_i}{\pi}$

$$\frac{1}{N}\sum_{k=1}^{N}\frac{L(\omega_{i,k})f(\omega_{i,k},\omega_o)(n\bullet\omega_{i,k})}{pdf(\omega_i)} = \frac{\pi}{N}\frac{\sum_{k=1}^{N}L(\omega_{i,k})f(\omega_{i,k},\omega_o)(n\bullet\omega_{i,k})}{n\bullet\omega_{i,k}} = \frac{\pi}{N}\sum_{k=1}^{N}L(\omega_{i,k})f(\omega_{i,k},\omega_o)$$

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Cosine Sampling Upper Hemisphere

- Inversion method
  - In polar coords, density must be proportional to cos θ sin θ (remember d(solid angle) = sin θ $d\theta$ $d\phi$)
  - Integrate, invert ➔ $\cos^{-1}$($sqrt$(…))
- Recipe is (start with two random numbers ξ_1, ξ_2 in 0…1)
  - Generate ϕ in 0..2π $\phi = 2\pi\xi_2$
  - Generate z in 0..1 $z = sqrt(\xi_1)$ *// Note extra sqrt wrt uniform*
  - Let $\theta = \cos^{-1} z$ $\theta = acos(z) = acos(sqrt(\xi_1))$
  - $(x,y,z) = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$
- Rotate according to surface normal (z goes to normal)
  - Or create coordinate frame (as you did for uniform sampling)
- Modify indirect lighting estimator (remove $n\bullet\omega_i$) and replace 2π with π (indirect lighting, Russian Roulette)

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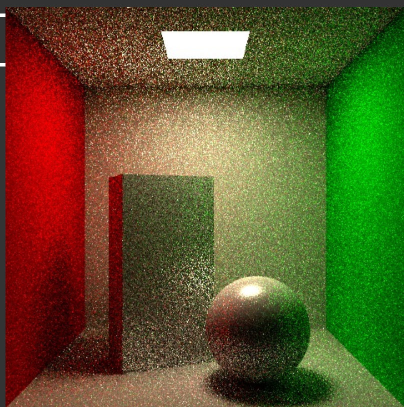

Uniform Hemisphere Sampling

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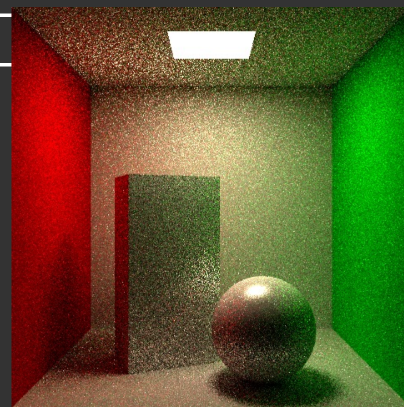

Cosine Sampling

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Specular BRDFs

- Cosine importance sampling works well for near-Lambertian BRDFs (modest improvement)
- But more sophisticated sampling for specular BRDFs
- Will talk about general BRDFs next lecture
- For now, for assignment: Modified Phong, GGX
- Sampling BRDFs in general is non-trivial
  - Can simply normalize to get PDF, but sampling non-trivial
  - For now, sample a simpler BRDF, then divide by PDF
  - (This procedure is always guaranteed to work)

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BRDF Importance Sampling

- Phong BRDF: $f_r \sim \cos^s\beta$ where β is angle between outgoing ray and ideal mirror direction
- Constant scale = $k_s(s+2)/(2\pi)$
- Can't sample this times cos θ_i
  - Can only sample BRDF itself, then multiply by cos θ_i
  - That's OK – still better than random sampling

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BRDF Importance Sampling

- Recipe for sampling specular term:
  - Generate *z* in 0..1
  - Let $\gamma = \cos^{-1}(z^{1/(s+1)})$
  - Generate ϕ_γ in 0..2π
  - This gives direction w.r.t. ideal mirror direction
- Convert to (x,y,z), then rotate such that *z* points along mirror dir.

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Formal Modified Phong Sampling

- Multiply by cosine, transport function (note colors)
- Modified Phong is an approximation $t = \frac{\bar{k}_s}{\bar{k}_d + \bar{k}_s}$

$$(1-t)\frac{n \bullet \omega_i}{\pi} + t\frac{s+1}{2\pi}(r \bullet \omega_i)^s$$

- Generate 3 random numbers: ξ_0, ξ_1, ξ_2 in 0…1
- Use ξ_0 to decide diffuse (>t) or specular (≤t)
- Generate ϕ in 0..2π $\phi = 2\pi\xi_2$
- If diffuse $\theta = \text{acos}(\text{sqrt}(\xi_1))$ [coord. frame normal n]
- If specular $\theta = \text{acos}(\xi_1^{1/(s+1)})$ [coord. frame refl r]
- Compute BRDF / PDF (if below visible, BRDF = 0)

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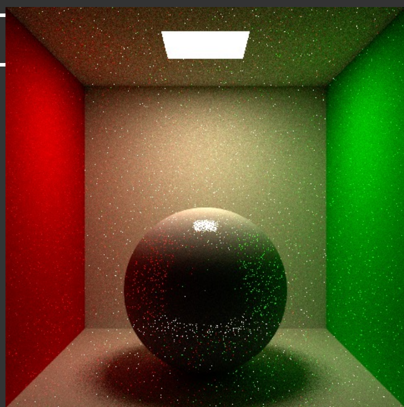

Without BRDF Importance Sampling (note near-mirror sphere)

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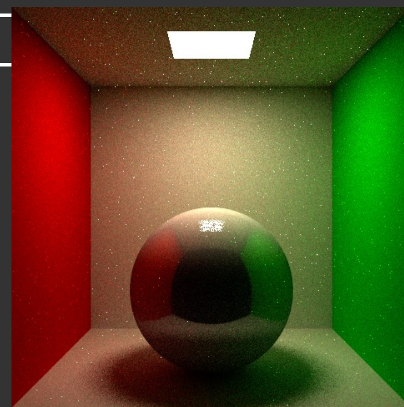

With BRDF Importance Sampling (note near-mirror sphere)

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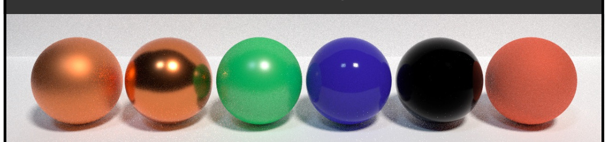
GGX Microfacet Model

- Physically-Based Reflectance Model
- Widely used in practice
- Will discuss BRDFs in more detail next time
- Brief review here, see assignment for details



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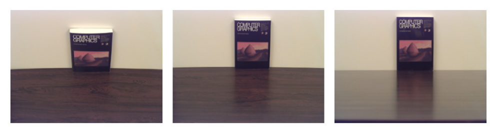
Experiment

Reflections from a shiny floor



From Lafortune, Foo, Torrance, Greenberg, SIGGRAPH 97

Reflection is greater at glancing angles

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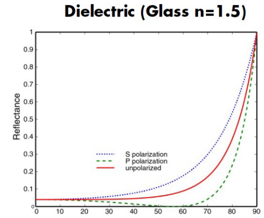
Fresnel Reflectance

Dielectric (Glass n=1.5)



F(0)=0.04

Schlick Approximation $F(\theta) = F(0) + (1 - F(0))(1 - \cos\theta)^5$

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(Cook-)Torrance-Sparrow

- Assume the surface is made up grooves at the microscopic level. (General *Microfacet Theory*)

- Assume the faces of these grooves (called microfacets) are perfect reflectors.
- Take into account 3 phenomena

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(Cook-)Torrance-Sparrow

Fresnel term: allows for wavelength dependency

Geometric Attenuation: reduces the output based on the amount of shadowing or masking that occurs.

$$f = \frac{F(\theta_i)G(\omega_i,\omega_r)D(\theta_h)}{4\cos(\theta_i)\cos(\theta_r)}$$

Distribution: distribution function determines what percentage of microfacets are oriented to reflect in the viewer direction.

How much of the macroscopic surface is visible to the light source

How much of the macroscopic surface is visible to the viewer

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GGX Microfacet Model

- Specular term (see assignment for G, F)

$$f(\omega_i,\omega_o) = \frac{k_d}{\pi} + f_{GGX}(\omega_i,\omega_o)$$

$$f_{GGX}(\omega_i,\omega_o) = \frac{F(\omega_i,\mathbf{h};k_s)G(\omega_i,\omega_o)D(\mathbf{h})}{4(\omega_i \bullet \mathbf{n})(\omega_o \bullet \mathbf{n})}$$

$$D(\mathbf{h}) = \frac{\alpha^2}{\pi \cos^4\theta_h(\alpha^2 + \tan^2\theta_h)^2}$$

- Importance Sampling PDF (includes cosine term)
  - Neglects F and G terms, must do BRDF / PDF

$$pdf(\omega_i \mid \omega_o) = (1-t)\frac{n \bullet \omega_i}{\pi} + t\frac{D(\mathbf{h})(\mathbf{n} \bullet \mathbf{h})}{4(\omega_i \bullet \mathbf{h})}$$

  - Note that t is clamped at a min of 0.25 to give some specular samples even for low k_s (because of Fresnel)

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Importance Sampling GGX

- High-Level idea similar to modified Phong
- Generate 3 random numbers: ξ_0, ξ_1, ξ_2 in 0…1
- Use ξ_0 to decide diffuse (>t) or specular (≤t)
- Generate ϕ in 0..2π $\phi=2\pi\xi_2$ (*if specular, this is* ϕ_h)
- If diffuse θ= acos(sqrt(ξ_1)) [coord. frame normal n]
- ***If specular*** $\theta_h = \arctan\left(\frac{\alpha\sqrt{\xi_1}}{\sqrt{1-\xi_1}}\right)$ [coord. frame halfvector h]
  - *Must compute incident direction from outgoing, half-vector*
  - *Rotate **h** about normal, reflect outgoing about half-vector*
- Compute BRDF / PDF (if below visible, BRDF = 0)

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Multiple Importance Sampling

- Veach 95 classic scene (4 lights, 4 glossiness)
- BRDF importance sampling only (no NEE, so no explicit direct lighting or light sampling pass)
  - Mostly noisy but sharper reflections handled well
- Compare with light sampling (NEE)
  - Mostly better but noisy for sharp reflections
- Can we combine BRDF, Light(NEE) sampling?
  - MIS (Veach95) provides a way, bounds
  - Very robust, works well shiny/rough etc.
  - Key development in production rendering
  - Remains topic of interest (many papers in 2019)

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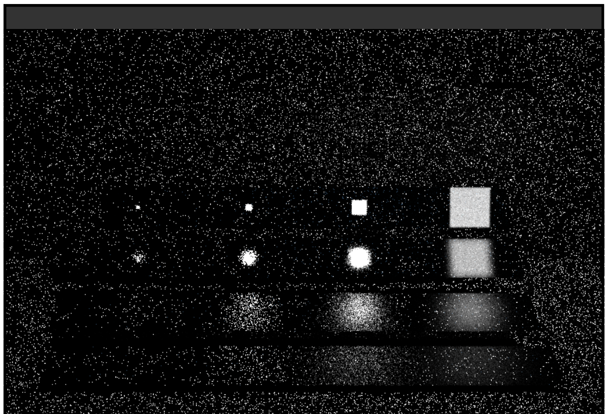

BRDF Importance Sampling (note top right images)

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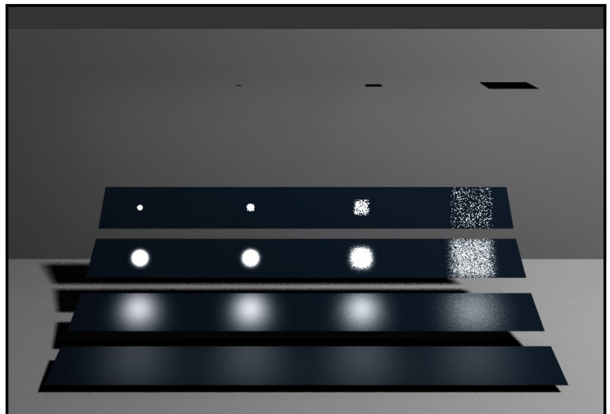

Light Sampling (Next Event Estimation for Direct Lighting)

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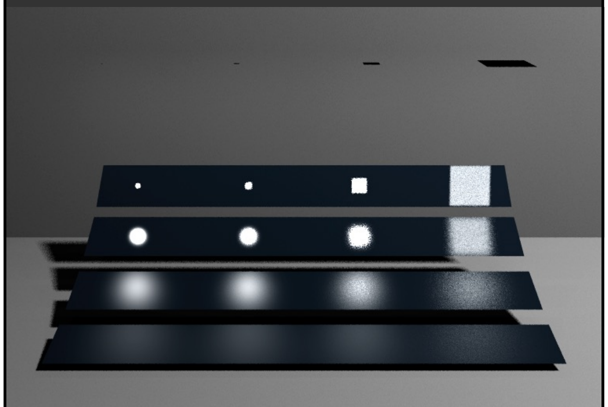

Multiple Importance Sampling (NEE+BRDF Sampling)

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Multiple Importance Sampling

- MIS relies on NEE almost everywhere, but relies on BRDF importance sampling when needed
- Multi-sample: sample both distributions at each intersection *(for direct lighting, needs code change)*
- General case: N sampling techniques (inner summation is unbiased estimator each technique)

$$\int_X f(x)dx \approx \sum_{i=1}^{N} \frac{1}{N_i} \sum_{j=1}^{N_i} w_i(x_{ij}) \frac{f(x_{ij})}{pdf_i(x_{ij})}$$

- Weights must sum to 1, unbiased

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Multiple Importance Sampling

- General case: N sampling techniques (inner summation is unbiased estimator each technique)

$$\int_X f(x)dx \approx \sum_{i=1}^{N} \frac{1}{N_i} \sum_{j=1}^{N_i} w_i(x_{ij}) \frac{f(x_{ij})}{pdf_i(x_{ij})}$$

- Weights must sum to 1, unbiased
  - Interesting theory (ongoing, papers in 2019)
  - Veach and Guibas 95 proposed balance, power heuristics (provably “good” under certain assumptions)
  - We use power heuristic with β = 2
  - Subtle point: PDF must be able to be evaluated anywhere (not just own samples)

$$w_i(\omega) = \frac{pdf_i^{\beta}(\omega)}{\sum_{k=1}^{N} pdf_k^{\beta}(\omega)}$$

- Natural abstract interface for sampling and MIS
  - Eval(), Sample(), PDF() [sometimes Value() = Eval/PDF]

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Lighting/BRDF Sampling

- For now, 1 sample on light (NEE), 1 from BRDF
  - We already know BRDF PDF
  - Light PDF implicitly on light, convert to angle $d\omega = dA\frac{\cos\theta}{R^2}$

$$pdf_{light}(\omega) = \frac{R^2}{(\mathbf{n}_{light} \bullet \omega)A}$$

  - For multiple lights, simple normalization (see homework)
  - Combine NEE and BRDF sampling (power heuristic)

$$w_i(\omega) = \frac{pdf_i^{\beta}(\omega)}{\sum_{k=1}^{N} pdf_k^{\beta}(\omega)}$$

  - *All of this for direct lighting only, indirect unchanged (BRDF)*
    - *Modify code to do BRDF sampling for direct lighting with MIS*

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MIS Implementation

- Can be tricky, see assignment
- First disable NEE, BRDF sampling for direct
  - Separate NEE function, toggle light/BRDF sampling
- Now implement pdf(nee)
  - Beware divide by zero, see assignment for specifics
- Implement weight function
  - Visualize weighted lighting, weighted BRDF
  - Then combine them with MIS, enable both techniques
- See assignment carefully
  - MIS for *direct lighting only* (Veach scene no indirect)
  - *Don’t try to modify your indirect BRDF with MIS*
  - Note gamma correction for this assignment

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MIS weights

Weighted contribution BRDF Importance

Weighted contribution of NEE

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