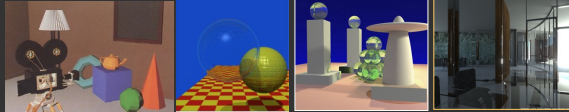


## Computer Graphics II: Rendering

CSE 168 [Spr 25], Lecture 6: Direct Lighting Details  
Ravi Ramamoorthi

<http://viscomp.ucsd.edu/classes/cse168/sp25>



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## To Do

- Homework 2 (Direct Lighting) due Apr 24
- Assignment is on edX edge
- START EARLY
- This lecture goes through details of direct lighting, visibility, Monte Carlo for the assignment
- Ask re any questions

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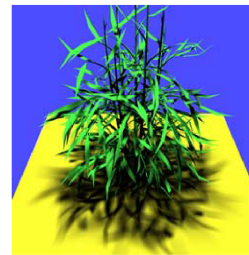
## Area Lights

- Physically correct (physically no perfect point light)
- Can talk in terms of radiance at surface
- Enable soft shadows, partial visibility/occlusion
- Shoot multiple rays to light source, average
- Need proper rendering equation for accurate result
  - Only direct lighting here, so reflection equation suffices

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## Example: Soft Shadows

$$E(x) = \int_{H^2} L_i(x, \omega) \cos \theta d\omega$$



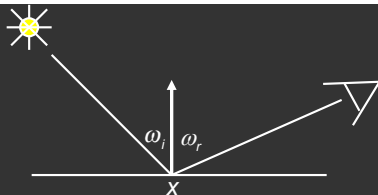
### Challenges

- Visibility and blockers
- Varying light distribution
- Complex source geometry

Source: Agrawala, Ramamoorthi, Heirich, Moll, 2000

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## Reflection Equation

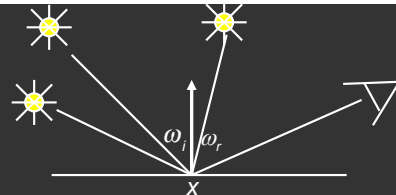


$$L_r(x, \omega_r) = L_e(x, \omega_r) + L_i(x, \omega_i) f(x, \omega_i, \omega_r) (\omega_i \cdot n)$$

Reflected Light (Output Image)	Emission	Incident Light (from light source)	BRDF	Cosine of Incident angle
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## Reflection Equation



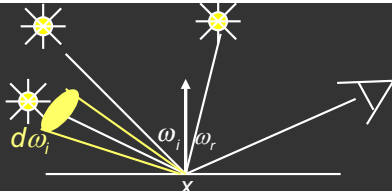
Sum over all light sources

$$L_r(x, \omega_r) = L_e(x, \omega_r) + \sum L_i(x, \omega_i) f(x, \omega_i, \omega_r) (\omega_i \cdot n)$$

Reflected Light (Output Image)	Emission	Incident Light (from light source)	BRDF	Cosine of Incident angle
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### Reflection Equation



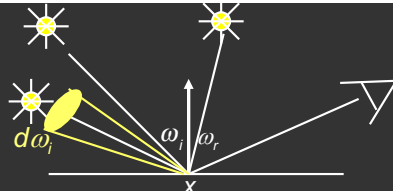
Replace sum with integral

$$L_r(x, \omega_r) = L_e(x, \omega_r) + \int_{\Omega} L_i(x, \omega_i) f(x, \omega_i, \omega_r) \cos \theta_i d\omega_i$$

Reflected Light (Output Image)	Emission	Incident Light (from light source)	BRDF	Cosine of Incident angle
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### Reflection Equation



As written in assignment (for one pixel)

$$L_r(\omega_o) = L_e(\omega_o) + \int_{\Omega} L_i(\omega_i) f(\omega_i, \omega_o) \max(n \cdot \omega_i, 0) d\omega_i$$

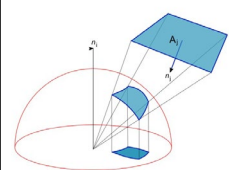
Reflected Light (Outgoing Dirn. Drop spat. Coord. )	Emission	Incident Light (from area light)	BRDF	Cosine of Incident angle (max not explicit hw)
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### Uniform Polygonal Area Source

- Integrate projection of polygon P onto sphere
- Assume uniform radiance incident from light  $L_i$
- Assume no emission (and no occlusion for now)
- Nusselt's analog (wikipedia image) project to (unit hemi)sphere, then circle (unit disk at equator) for cosine term [but only valid for Lambertian, or modulate by BRDF]

$$L_d(\omega_o) = \int_{\Omega_p} L_i f(\omega_i, \omega_o) (n \cdot \omega_i) d\omega_i$$

$$= \frac{k_a}{\pi} L_i \int_{\Omega_p} (n \cdot \omega_i) d\omega_i \quad [\text{Lambertian}]$$


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### Analytic Irradiance

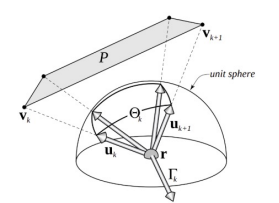
- Analytic irradiance for a polygon [Baum89, Arvo94-95]
- Useful to get precise answer, check Monte Carlo
- Define "irradiance vector" [Arvo 94]

$$\Phi(r) = \frac{1}{2} \sum_{i=1}^n \Theta_i(r) \Gamma_i(r)$$

where  $\Theta_1, \Theta_2, \dots, \Theta_n$  are the angles subtended by the  $n$  edges of the polygon as seen from  $r$

$\Theta_i = \angle \mathbf{v}_i \mathbf{r} \mathbf{v}_{i+1}$ , where  $\mathbf{v}_i$  are vertices of polygon, in cyclic order ( $n+1 = 1$ )

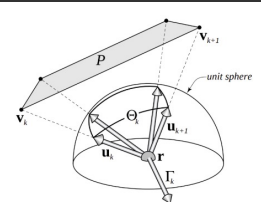
$\Gamma_1, \Gamma_2, \dots, \Gamma_n$  are the unit normals of the polygonal cone with cross section  $P$  and apex at  $r$



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### Analytic Irradiance

- Analytic irradiance for a polygon [Baum89, Arvo94-95]
- Useful to get precise answer, check Monte Carlo
- Define "irradiance vector" [Arvo 94]

$$\Phi(r) = \frac{1}{2} \sum_{i=1}^n \Theta_i(r) \Gamma_i(r)$$


$$\Theta_i(r) = \cos^{-1} \left( \frac{(\mathbf{v}_i - \mathbf{r}) \cdot (\mathbf{v}_{i+1} - \mathbf{r})}{\|\mathbf{v}_i - \mathbf{r}\| \|\mathbf{v}_{i+1} - \mathbf{r}\|} \right)$$

$$\Gamma_i(r) = \frac{(\mathbf{v}_i - \mathbf{r}) \times (\mathbf{v}_{i+1} - \mathbf{r})}{\|(\mathbf{v}_i - \mathbf{r}) \times (\mathbf{v}_{i+1} - \mathbf{r})\|}$$

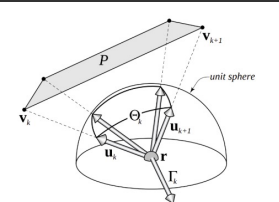
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### Analytic Irradiance

- Analytic irradiance for a polygon [Baum89, Arvo94-95]
- Useful to get precise answer, check Monte Carlo
- Define "irradiance vector" [Arvo 94]

$$\Phi(r) = \frac{1}{2} \sum_{i=1}^n \Theta_i(r) \Gamma_i(r)$$

$E = L_i (\Phi(r) \cdot \mathbf{n}(r))$  [Irradiance]

$$L_o(\omega_o) = f E = \frac{k_a}{\pi} E = \frac{k_a}{\pi} L_i (\Phi(r) \cdot \mathbf{n}(r))$$


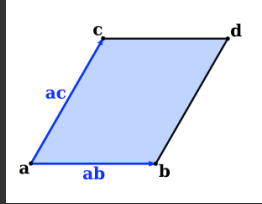
$$\Theta_i(r) = \cos^{-1} \left( \frac{(\mathbf{v}_i - \mathbf{r}) \cdot (\mathbf{v}_{i+1} - \mathbf{r})}{\|\mathbf{v}_i - \mathbf{r}\| \|\mathbf{v}_{i+1} - \mathbf{r}\|} \right)$$

$$\Gamma_i(r) = \frac{(\mathbf{v}_i - \mathbf{r}) \times (\mathbf{v}_{i+1} - \mathbf{r})}{\|(\mathbf{v}_i - \mathbf{r}) \times (\mathbf{v}_{i+1} - \mathbf{r})\|}$$

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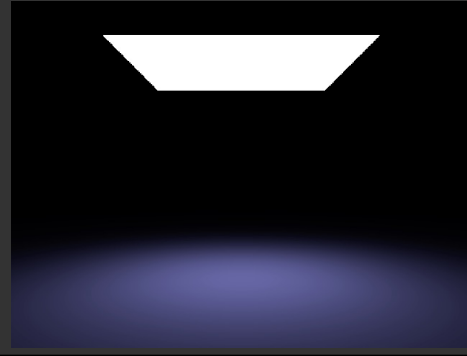
## Implementing Analytic Irradiance

- New parallelogram light quadLight <a> <ab> <ac> <radiance>
- quadLight 0 0 0 1 0 0 0 1 1 1 1
- Primary rays like ray tracer
- Use analytic formula for shading
- Add contribs. from all lights
- No emission, visibility (yet)



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## Example (No Noise)



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## General Direct Lighting

- Include (partial) visibility by tracing multiple rays

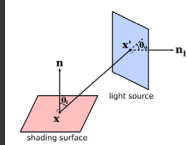
$$L_d(\omega_o) = L_i \int_{\Omega_p} f(\omega_i, \omega_o) (n \cdot \omega_i) V(\omega_i) d\omega_i$$

- Include non-Lambertian reflectance (Phong)

$$f(\omega_i, \omega_o) = \frac{k_d}{\pi} + k_s \frac{(s+2)}{2\pi} \max(r \cdot \omega_i, 0)^s$$

- Integrate over area light, change area measure

$$d\omega_i = \frac{n_i \cdot \omega_i}{R^2} dA$$



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## General Direct Lighting

- Include (partial) visibility by tracing multiple rays

$$L_d(\omega_o) = L_i \int_{\Omega_p} f(\omega_i, \omega_o) (n \cdot \omega_i) V(\omega_i) d\omega_i$$

- Include non-Lambertian reflectance (Phong)

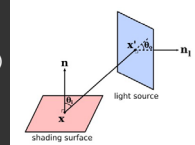
$$f(\omega_i, \omega_o) = \frac{k_d}{\pi} + k_s \frac{(s+2)}{2\pi} \max(r \cdot \omega_i, 0)^s$$

- Integrate over area light, change area measure

$$d\omega_i = \frac{n_i \cdot \omega_i}{R^2} dA$$

$$L_d(\mathbf{x}, \omega_o) = L_i \int_P f(\mathbf{x}, \omega_i, \omega_o) G(\mathbf{x}, \mathbf{x}') V(\mathbf{x}, \mathbf{x}') dA(\mathbf{x}')$$

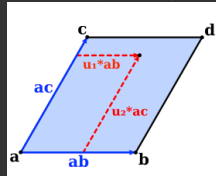
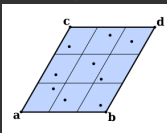
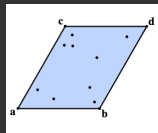
$$G(\mathbf{x}, \mathbf{x}') = \frac{(n \cdot \omega_i)(n_i \cdot \omega_i)}{R^2} = \frac{\cos \theta_i \cos \theta_o}{|\mathbf{x} - \mathbf{x}'|^2}$$



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## Monte Carlo Solution

- Sample area light (two random numbers give  $\mathbf{x}'_k$ )
- Random or stratified (compare)

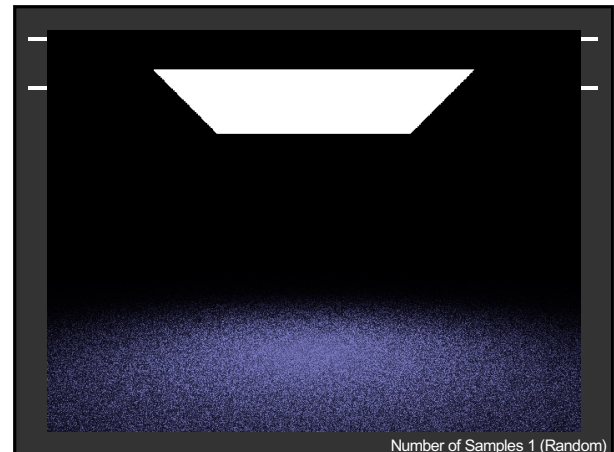


- Monte Carlo evaluation of direct lighting (N samples)

$$L_d(\mathbf{x}, \omega_o) = L_i \int_P f(\mathbf{x}, \omega_i, \omega_o) G(\mathbf{x}, \mathbf{x}') V(\mathbf{x}, \mathbf{x}') dA(\mathbf{x}')$$

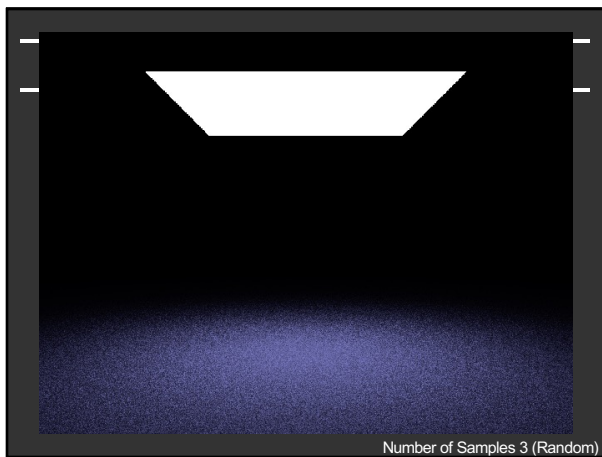
$$L_d(\mathbf{x}, \omega_o) \approx L_i \frac{A}{N} \sum_{k=1}^N f(\mathbf{x}, \omega_i(k), \omega_o) G(\mathbf{x}, \mathbf{x}'_k) V(\mathbf{x}, \mathbf{x}'_k)$$

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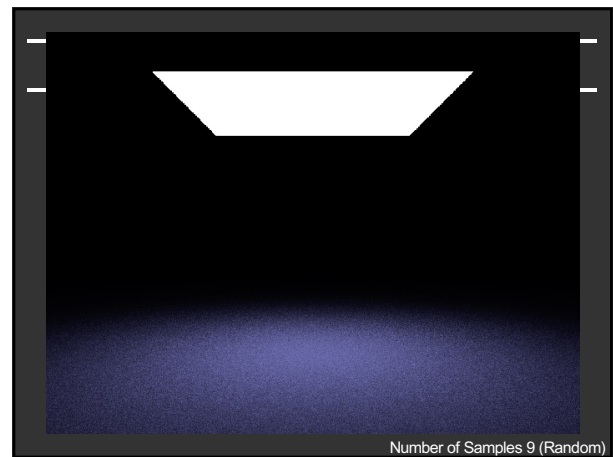


Number of Samples 1 (Random)

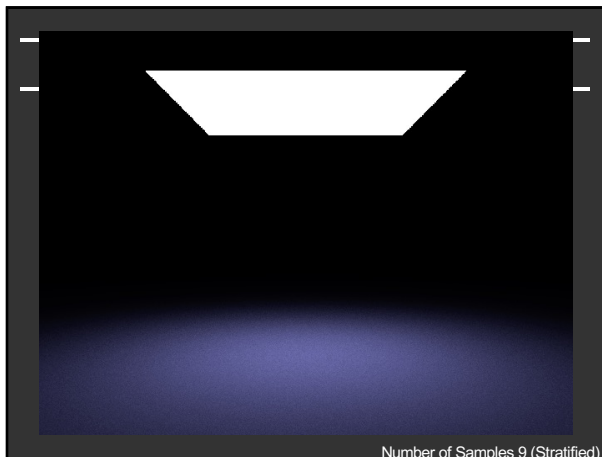
18



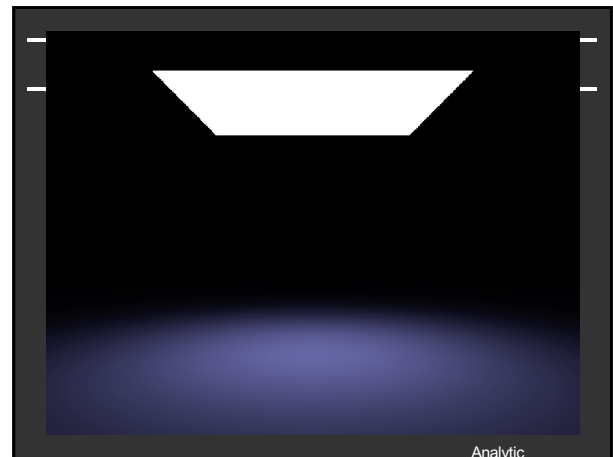
19



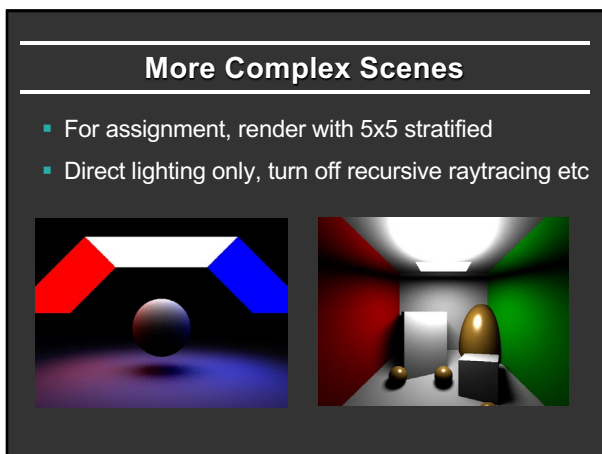
20



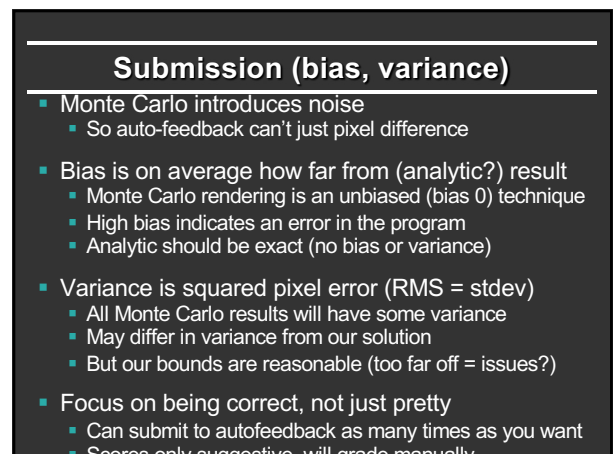
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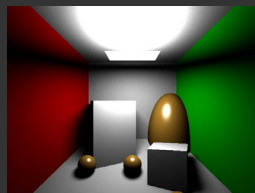
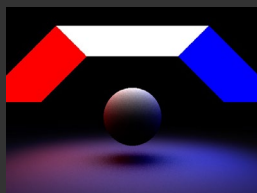
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### More Complex Scenes

- For assignment, render with 5x5 stratified
- Direct lighting only, turn off recursive raytracing etc



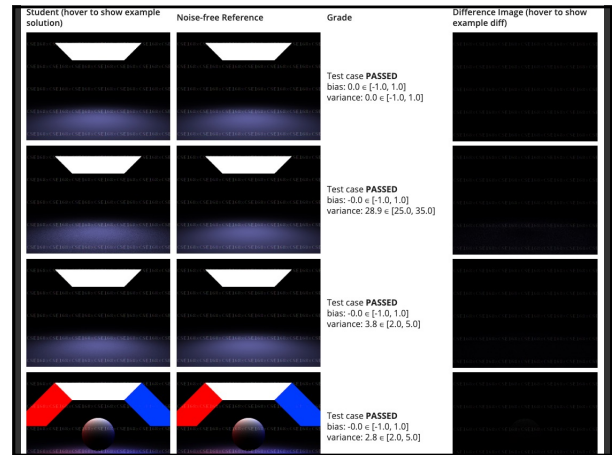
### Submission (bias, variance)

- Monte Carlo introduces noise
  - So auto-feedback can't just pixel difference
- Bias is on average how far from (analytic?) result
  - Monte Carlo rendering is an unbiased (bias 0) technique
  - High bias indicates an error in the program
  - Analytic should be exact (no bias or variance)
- Variance is squared pixel error ( $RMS = stdev$ )
  - All Monte Carlo results will have some variance
  - May differ in variance from our solution
  - But our bounds are reasonable (too far off = issues?)
- Focus on being correct, not just pretty
  - Can submit to autofeedback as many times as you want
  - Scores only suggestive, will grade manually

## Submission (bias, variance)

- Monte Carlo introduces noise
  - So auto-feedback can't just pixel difference
- Bias is on average how far from result (close to 0)
  - Monte Carlo rendering is an unbiased technique
  - High bias indicates an error in the program
- Variance is squared pixel error (RMS = stdev)
  - May differ in variance from our solution
  - But our bounds are reasonable (too far off = issues?)

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## Extra: Reducing Variance

- Increase bias slightly by evaluating everything except visibility (at center [of stratum], or analytic)

$$L_d(\mathbf{x}, \omega_o) = L_i \frac{A}{N} \sum_{k=1}^N f(\mathbf{x}, \omega_i(k), \omega_o) G(\mathbf{x}, \mathbf{x}_k) V(\mathbf{x}, \mathbf{x}_k)$$

$$L_d(\mathbf{x}, \omega_o) \approx L_i \frac{A}{N} \sum_{k=1}^N f(\mathbf{x}, \omega_i(\mathbf{c}_k), \omega_o) G(\mathbf{x}, \mathbf{c}_k) V(\mathbf{x}, \mathbf{x}_k)$$

$\mathbf{c}_k$  is center of stratum, no noise (not Monte Carlo)

- Noise now only from visibility, none otherwise
  - No noise if no occlusions/shadows
  - Exact if use analytic formula (to evaluate unshadowed irradiance from each strata) instead of stratum center

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## Extra: Point Lights

- Point lights not physical (but widely used)
- How to include in Monte Carlo Rendering?
- Units aren't even same (can't use radiance)
  - Power per solid angle, not power per unit area per solid angle
- Think of as delta functions (area goes to 0, total radiance integrated over light  $I = L_i A$  remains same)
  - No cosine term for light, inverse square falloff, no integral

$$L_d(\omega_o) = L_i \int_{\Omega_p} f(\omega_i, \omega_o) (n \cdot \omega_i) V(\omega_i) d\omega_i$$

$$L_d(\omega_o) = \frac{I}{R^2} f(\omega_i, \omega_o) (n \cdot \omega_i) V(\omega_i)$$

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## Extra: Directional Lights

- First, assume small angular extent (even sun subtends an angle of 0.5 degrees)  $L_d(\omega_o) = L_i \int_{\Omega_p} f(\omega_i, \omega_o) (n \cdot \omega_i) V(\omega_i) d\omega_i$

$$L_d(\omega_o) \approx L_i \frac{\Delta\omega}{N} \sum_{k=1}^N f(\omega_i(k), \omega_o) (n \cdot \omega_i) V(\omega_i(k))$$

- If pure directional light (delta function), then fix product  $E = L_i \Delta\omega$  (same units as irradiance)

$$L_d(\omega_o) = E f(\omega_i, \omega_o) (n \cdot \omega_i) V(\omega_i)$$

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## Combining Point, Directional, Area

- Small angle, area expressed in radiance  $L_i$ 
  - Use physical units  $\text{Wm}^{-2} \text{sr}^{-1}$
- Point is in intensity  $\text{Wsr}^{-1}$ 
  - This is not in same units (e.g. 0...1) as area lights
  - Need to normalize by multiplying by an area unit
  - Point light same intensity as area light: multiply by area
- Directional is in irradiance units  $\text{Wm}^{-2}$ 
  - This is not in same units (e.g. 0...1) as area/point lights
  - Need to normalize by multiplying by solid angle
  - Directional light same intensity as area light: multiply by average solid angle subtended by area light
  - Directional same intensity as point: divide by average squared distance to point light, mult by 2 for avg cosine

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