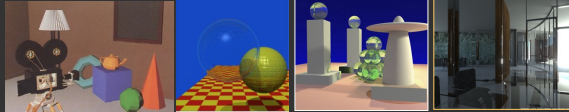


## Computer Graphics II: Rendering

CSE 168 [Spr 25], Lecture 3: Illumination and Reflection  
Ravi Ramamoorthi

<http://viscomp.ucsd.edu/classes/cse168/sp25>



1

## To Do

- Homework 1 (ray tracer) due Monday
- Next assignment direct lighting (on edX edge). Will cover that material next week

2

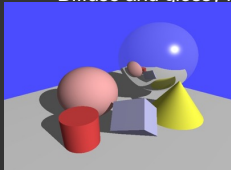
## Illumination Models

### Local Illumination

- Light directly from light sources to surface
- No shadows (cast shadows are a global effect)

### Global Illumination: multiple bounces (indirect light)

- Hard and soft shadows
- Reflections/refractions (already seen in ray tracing)
- Diffuse and glossy interreflections (radiosity, caustics)

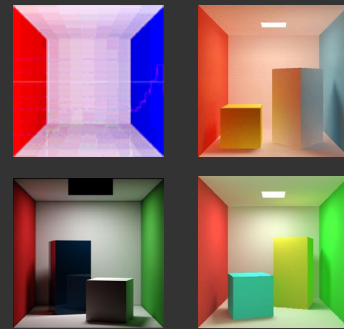


Some images courtesy Henrik Wann Jensen

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## Diffuse Interreflection

Diffuse interreflection, color bleeding [Cornell Box]



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## Radiosity



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## Caustics

Caustics: Focusing through specular surface



- Major research effort in 80s, 90s till today

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## Motivation: BRDFs, Radiometry

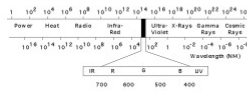
- Basics of Illumination, Reflection
- Formal radiometric analysis (not ad-hoc)
- Reflection Equation (Local Illumination)
- Discussion of BRDFs
- Rendering Equation (Global Illumination) on Thu
- Formal analysis important for correct implementation

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## Light

### Visible electromagnetic radiation

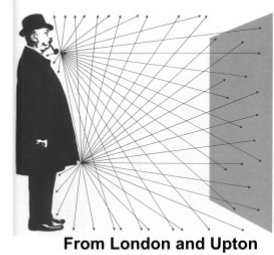
#### Power spectrum



#### Polarization

#### Photon (quantum effects)

#### Wave (interference, diffraction)



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## Radiometry

- Physical measurement of electromagnetic energy
- Measure spatial (and angular) properties of light
  - Radiance, Irradiance
  - Reflection functions: Bi-Directional Reflectance Distribution Function or BRDF
  - Reflection Equation
  - Simple BRDF models

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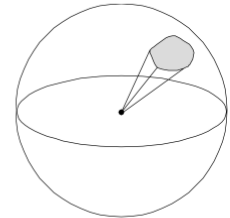
## Angles and Solid Angles

■ Angle  $\theta = \frac{l}{r}$

⇒ circle has  $2\pi$  radians

■ Solid angle  $\Omega = \frac{A}{R^2}$

⇒ sphere has  $4\pi$  steradians

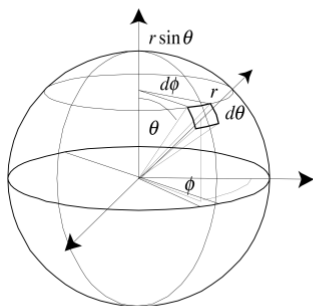


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## Differential Solid Angles



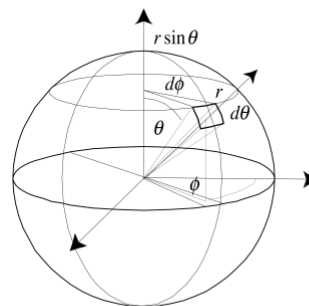
$$dA = (r d\theta)(r \sin \theta d\phi) \\ = r^2 \sin \theta d\theta d\phi$$

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## Differential Solid Angles



$$dA = (r d\theta)(r \sin \theta d\phi) \\ = r^2 \sin \theta d\theta d\phi$$

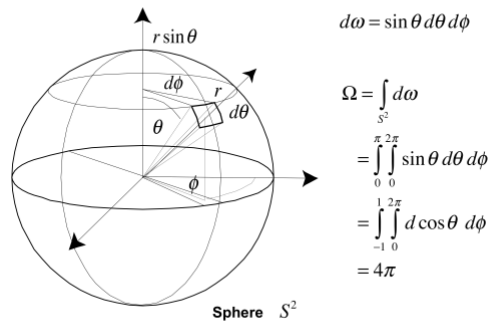
$$d\omega = \frac{dA}{r^2} = \sin \theta d\theta d\phi$$

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## Differential Solid Angles



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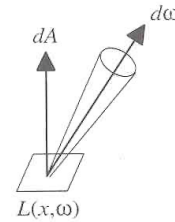
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## Radiance

- Power per unit projected area perpendicular to the ray per unit solid angle in the direction of the ray

- Symbol:  $L(x, \omega)$  ( $\text{W}/\text{m}^2 \text{sr}$ )

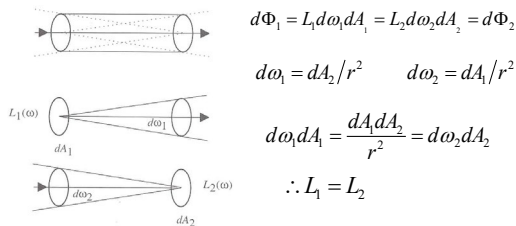
- Flux given by  $d\Phi = L(x, \omega) \cos \theta \, d\omega \, dA$



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## Radiance properties

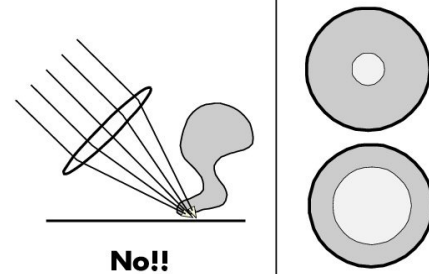
- Radiance constant as propagates along ray
  - Derived from conservation of flux
  - Fundamental in Light Transport.



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## Quiz

Does radiance increase under a magnifying glass?



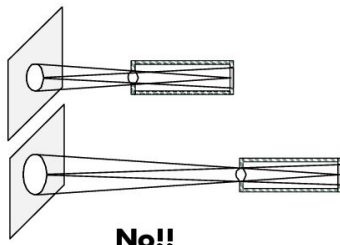
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## Quiz

Does the brightness that a wall appears to the eye depend on the distance of the viewer to the wall?



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## Radiance properties

- Sensor response proportional to radiance (constant of proportionality is throughput)

- Far away surface: See more, but subtends smaller angle
- Wall equally bright across viewing distances

### Consequences

- Radiance associated with rays in a ray tracer
- Other radiometric quants derived from radiance
- This course primarily about computing radiance*

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## Irradiance, Radiosity

- Irradiance  $E$  is radiant power per unit area
- Integrate incoming radiance over hemisphere
  - Projected solid angle ( $\cos \theta d\omega$ )
  - Uniform illumination: Irradiance =  $\pi$  [CW 24,25]
  - Units:  $\text{W/m}^2$
- Radiant Exitance (radiosity)
  - Power per unit area leaving surface (like irradiance)

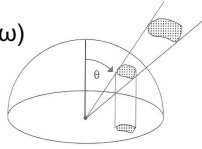


Figure 2.8: Projection of differential area.

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## Directional Power Arriving at a Surface

$$d^2\Phi_i(x, \omega) = L_i(x, \omega) \cos \theta dA d\omega$$

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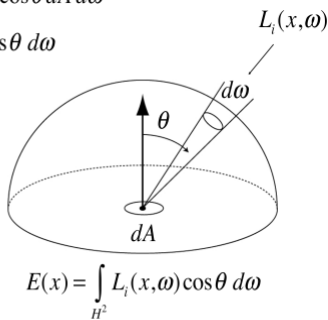
## Irradiance from the Environment

$$d^2\Phi_i(x, \omega) = L_i(x, \omega) \cos \theta dA d\omega$$

$$dE(x, \omega) = L_i(x, \omega) \cos \theta d\omega$$



Light meter



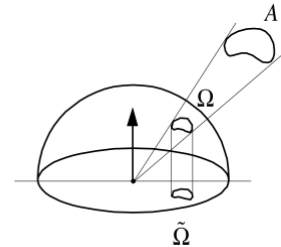
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## Uniform Area Source

$$\begin{aligned} E(x) &= \int_{H^2} L \cos \theta d\omega \\ &= L \int_{\Omega} \cos \theta d\omega \\ &= L \tilde{\Omega} \end{aligned}$$

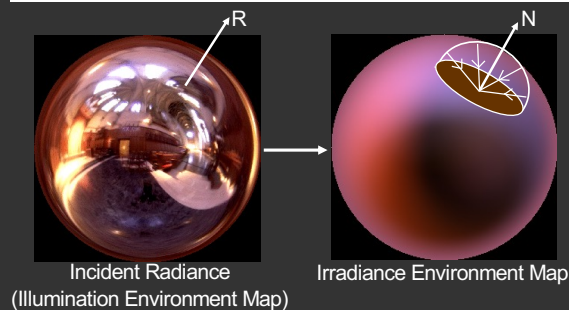


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## Irradiance Environment Maps



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## Radiometry

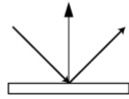
- Physical measurement of electromagnetic energy
- Measure spatial (and angular) properties of light
  - Radiance, Irradiance
  - Reflection functions: Bi-Directional Reflectance Distribution Function or BRDF
  - Reflection Equation
  - Simple BRDF models

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## Types of Reflection Functions

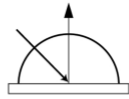
### Ideal Specular

- Reflection Law
- Mirror



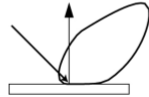
### Ideal Diffuse

- Lambert's Law
- Matte



### Specular

- Glossy
- Directional diffuse



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## Materials



Plastic

Metal

Matte

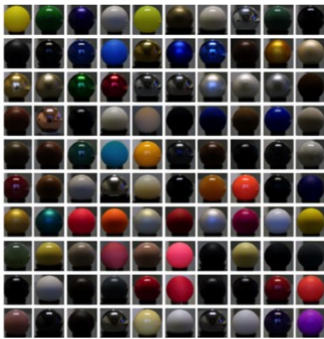
From Apodaca and Gritz, *Advanced RenderMan*

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## Spheres [Matusik et al.]



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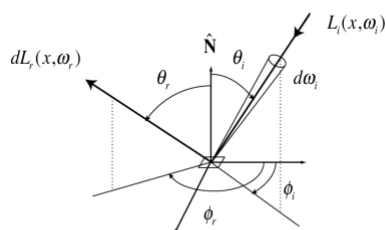
## Building up the BRDF

- Bi-Directional Reflectance Distribution Function [Nicodemus 77]
- Function based on incident, view direction
- Relates incoming light energy to outgoing
- Unifying framework for many materials

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## The BRDF

### Bidirectional Reflectance-Distribution Function



$$f_r(\omega_i \rightarrow \omega_r) \equiv \frac{dL_r(\omega_i \rightarrow \omega_r)}{dE_i} \left[ \frac{1}{sr} \right]$$

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## BRDF

- Reflected Radiance proportional Irradiance
- Constant proportionality: BRDF
- Ratio of outgoing light (radiance) to incoming light (irradiance)
  - Bidirectional Reflection Distribution Function
  - (4 Vars) units 1/sr

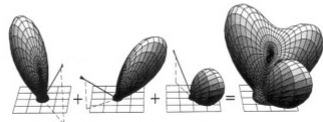
$$f(\omega_i, \omega_r) = \frac{L_r(\omega_r)}{L_i(\omega_i) \cos \theta_i d\omega_i}$$

$$L_r(\omega_r) = L_i(\omega_i) f(\omega_i, \omega_r) \cos \theta_i d\omega_i$$

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## Properties of BRDF's

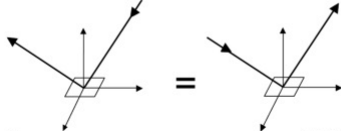
### 1. Linearity



From Sillion, Arvo, Westin, Greenberg

### 2. Reciprocity principle

$$f_r(\omega_r \rightarrow \omega_i) = f_r(\omega_i \rightarrow \omega_r)$$



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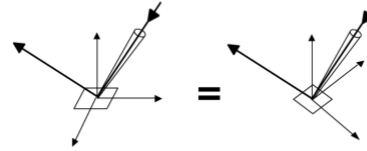
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## Properties of BRDF's

### 3. Isotropic vs. anisotropic

$$f_r(\theta_i, \phi_i; \theta_r, \phi_r) = f_r(\theta_i, \theta_r, \phi_r - \phi_i)$$



### Reciprocity and isotropy

$$f_r(\theta_i, \theta_r, \phi_r - \phi_i) = f_r(\theta_r, \theta_i, \phi_i - \phi_r) = f_r(\theta_i, \theta_r, |\phi_r - \phi_i|)$$

### 4. Energy conservation

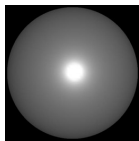
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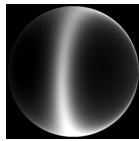
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## Isotropic vs Anisotropic

- Isotropic: Most materials (you can rotate about normal without changing reflections)
- Anisotropic: brushed metal etc. preferred tangential direction



Isotropic

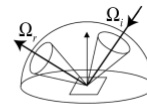


Anisotropic

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## Energy Conservation

$$\begin{aligned} \frac{d\Phi_r}{d\Phi_i} &= \frac{\int_{\Omega_r} L_r(\omega_r) \cos \theta_r d\omega_r}{\int_{\Omega_i} L_i(\omega_i) \cos \theta_i d\omega_i} \\ &= \frac{\int_{\Omega_r} \int_{\Omega_i} f_r(\omega_i \rightarrow \omega_r) L_i(\omega_i) \cos \theta_i d\omega_i \cos \theta_r d\omega_r}{\int_{\Omega_i} L_i(\omega_i) \cos \theta_i d\omega_i} \\ &\leq 1 \end{aligned}$$

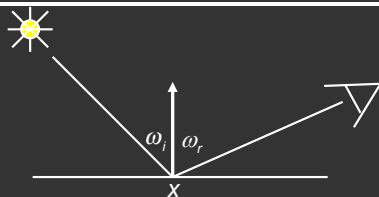


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## Reflection Equation

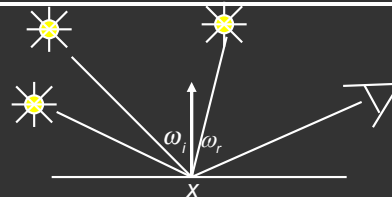


$$L_r(x, \omega_r) = L_e(x, \omega_r) + L_i(x, \omega_i) f(x, \omega_i, \omega_r) (\omega_i \cdot n)$$

Reflected Light (Output Image)   Emission   Incident Light (from light source)   BRDF   Cosine of Incident angle

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## Reflection Equation



Sum over all light sources

$$L_r(x, \omega_r) = L_e(x, \omega_r) + \sum L_i(x, \omega_i) f(x, \omega_i, \omega_r) (\omega_i \cdot n)$$

Reflected Light (Output Image)   Emission   Incident Light (from light source)   BRDF   Cosine of Incident angle

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## Reflection Equation

Replace sum with integral

$$L_r(x, \omega_r) = L_e(x, \omega_r) + \int_{\Omega} L_i(x, \omega_i) f(x, \omega_i, \omega_r) \cos \theta_i d\omega_i$$

Reflected Light (Output Image)	Emission	Incident Light (from light source)	BRDF	Cosine of Incident angle
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## Environment Maps

- Light as a function of direction, from entire environment
- Captured by photographing a chrome steel or mirror sphere
- Accurate only for one point, but distant lighting same at other scene locations (typically use only one env. map)

Blinn and Newell 1976, Miller and Hoffman, 1984  
Later, Greene 86, Cabral et al. 87

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## Environment Maps

- Environment maps widely used as lighting representation
- Many modern methods deal with offline and real-time rendering with environment maps
- Image-based complex lighting + complex BRDFs

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## Radiometry

- Physical measurement of electromagnetic energy
- Measure spatial (and angular) properties of light
  - Radiance, Irradiance
  - Reflection functions: Bi-Directional Reflectance Distribution Function or BRDF
  - Reflection Equation
  - Simple BRDF models

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## Brdf Viewer plots

Diffuse

Torrance-Sparrow

Anisotropic

bv written by Szymon Rusinkiewicz

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## Ideal Diffuse Reflection

**Assume light is equally likely to be reflected in any output direction (independent of input direction).**

$$L_{r,d}(\omega_r) = \int f_{r,d} L_i(\omega_i) \cos \theta_i d\omega_i$$

$$= f_{r,d} \int L_i(\omega_i) \cos \theta_i d\omega_i$$

$$= f_{r,d} E$$

$$M = \int L_r(\omega_r) \cos \theta_r d\omega_r = L_r \int \cos \theta_r d\omega_r = \pi L_r$$

$$\rho_d = \frac{M}{E} = \frac{\pi L_r}{E} = \frac{\pi f_{r,d} E}{E} = \pi f_{r,d} \Rightarrow f_{r,d} = \frac{\rho_d}{\pi}$$

**Lambert's Cosine Law**  $M = \rho_d E = \rho_d E_s \cos \theta_s$

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### Phong Model

Reciprocity:  $(\hat{E} \cdot \mathbf{R}(\hat{L}))^s = (\hat{L} \cdot \mathbf{R}(\hat{E}))^s$

Distributed light source!

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### Specular Term (Phong)

#### Phong Model

Mirror Diffuse

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### Fresnel Reflectance

**Metal (Aluminum)**

Gold  $F(0)=0.82$   
Silver  $F(0)=0.95$

**Dielectric (N=1.5)**

Glass  $n=1.5$   $F(0)=0.04$   
Diamond  $n=2.4$   $F(0)=0.15$

Schlick Approximation  $F(\theta) = F(0) + (1 - F(0))(1 - \cos\theta)^5$

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### Experiment

#### Reflections from a shiny floor

From Lafortune, Foo, Torrance, Greenberg, SIGGRAPH 97

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### Torrance-Sparrow

Fresnel term: allows for wavelength dependency

Geometric Attenuation: reduces the output based on the amount of shadowing or masking that occurs.

Distribution: distribution function determines what percentage of microfacets are oriented to reflect in the viewer direction.

$$f = \frac{F(\theta_i)G(\omega_i, \omega_r)D(\theta_h)}{4\cos(\theta_i)\cos(\theta_r)}$$

How much of the macroscopic surface is visible to the light source

How much of the macroscopic surface is visible to the viewer

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### Other BRDF models

- Empirical: Measure and build a 4D table
- Anisotropic models for hair, brushed steel
- Cartoon shaders, funky BRDFs
- Capturing spatial variation
- Very active area of research

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