Computer Graphics II: Rendering

CSE 168[Spr 25],Lecture 10: Materials and BRDFs
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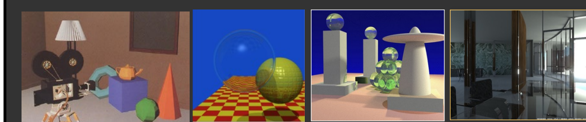
http://viscomp.ucsd.edu/classes/cse168/sp25



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To Do

- Start working on homework 3. Ask me if problems
- Also homework 4. Have covered material
- Start thinking about final project

Some slides courtesy Steve Rotenberg and Pat Hanrahan

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Materials and BRDFs

- Key part of renderer: different materials/BRDFs
- Abstract BRDF/Material interface (for MIS)
  - Evaluate (for given incident, outgoing direction)
  - Sample (given outgoing, importance sample incident)
  - PDF (for MIS, evaluate sampling PDF arbitrary direction)
  - Also for value of sample, need to compute eval/PDF (sometimes can simplify this, new value function=eval/PDF)
- Any physical or non-physical BRDF must fit above
  - Evaluation is usually easy (BRDF formula)
  - Can encompass analytic formulae, table measurements
  - Sampling can be hard and is crucial (see my 2004 paper for general importance sampling, special cases for some)
  - PDF function can be non-trivial, make sure math correct

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Diffuse Surfaces

- Simplest Case: Lambertian Reflectance
- BRDF is simply a constant: $f = \frac{\rho}{\pi}$
- Note energy conservation, divide albedo by π
- Note cosine incident term in final evaluation $\tilde{f} = \frac{\rho}{\pi}\cos\theta_i$
- Evaluate BRDF is straightforward
- Sample? Sample hemisphere (or cosine-weight)
- PDF is $\frac{1}{2\pi}$ or (if cosine-weight) $\frac{\cos\theta_i}{\pi}$
- Value/weight with cosine sampling is simply ρL_i

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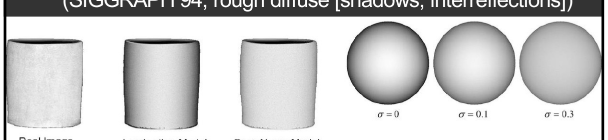
Oren-Nayar Model

- Generalization of Lambert's Reflectance Model (SIGGRAPH 94, rough diffuse [shadows, interreflections])



Importance sampling can be complicated (but exact sampling is not required)

Simplest: Lambertian sampling/PDF
But Eval uses Oren-Nayar; Eval/PDF
(will cancel leading Lambertian term only)

$$L_r = \frac{\rho}{\pi} \cdot \cos\theta_i \cdot (A + (B \cdot \max[0, \cos(\phi_i - \phi_r)] \cdot \sin\alpha \cdot \tan\beta)) \cdot E_0$$

where

$$A = 1 - 0.5\frac{\sigma^2}{\sigma^2 + 0.33},$$

$$B = 0.45\frac{\sigma^2}{\sigma^2 + 0.09},$$

$$\alpha = \max(\theta_i, \theta_r),$$

$$\beta = \min(\theta_i, \theta_r),$$

From Wikipedia

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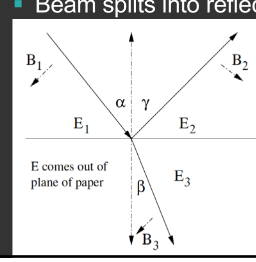
Fresnel Surfaces

- Idealized Fresnel surfaces are perfectly smooth boundary between dielectric (air,glass,water) and another dielectric, or a dielectric and a metal
- Beam splits into reflected/refracted (Snell's law)



$$\sin\alpha = n\sin\beta$$

$$\alpha = \gamma$$

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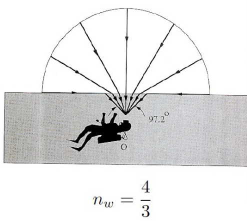
Optical Manhole

Total internal reflection



$n_w = \frac{4}{3}$

From Livingston and Lynch

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Fresnel Surfaces

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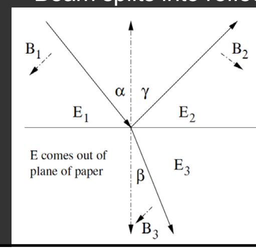

$$\sin\alpha = n \sin\beta$$

$$\alpha = \gamma$$

$$r_\perp = \frac{\cos\alpha - n\cos\beta}{\cos\alpha + n\cos\beta}$$

$$r_\parallel = \frac{n\cos\alpha - \cos\beta}{n\cos\alpha + \cos\beta}$$

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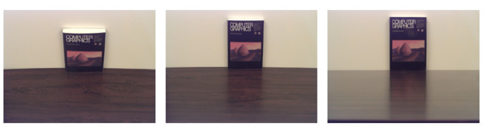
Experiment

Reflections from a shiny floor



From Lafortune, Foo, Torrance, Greenberg, SIGGRAPH 97

Reflection is greater at glancing angles

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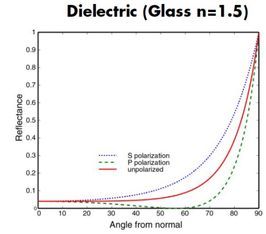
Fresnel Reflectance

Dielectric (Glass n=1.5)



F(0)=0.04

Schlick Approximation $F(\theta) = F(0) + (1 - F(0))(1 - \cos\theta)^5$

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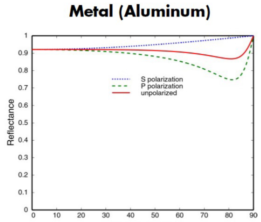
Fresnel Reflectance

Metal (Aluminum)



Gold F(0)=0.82
Silver F(0)=0.95

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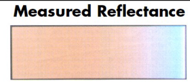
Reflection from Metals

Reflectance of Copper as a function of wavelength and angle of incidence

Measured Reflectance



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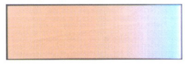
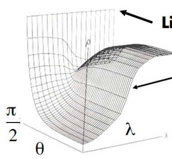
Cook-Torrance Reflection

Reflectance of Copper as a function of wavelength and angle of incidence

Measured Reflectance





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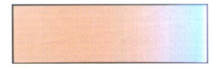
Cook-Torrance Reflection Model

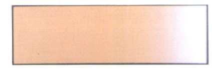
Interpolate between color measured at normal incidence and the light color

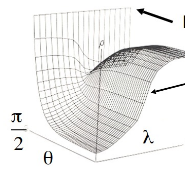
Use Fresnel formula to interpolate

Measured Reflectance



Approximated Reflectance





Cook-Torrance approximation

$$R(\theta) = R(0) + (R(\pi/2) - R(0)) \max(0, \frac{F(\theta) - F(0)}{F(\pi/2) - F(0)})$$

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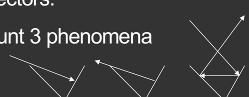
(Cook-)Torrance-Sparrow

- Assume the surface is made up grooves at the microscopic level. (General *Microfacet Theory*)

- Assume the faces of these grooves (called microfacets) are perfect reflectors.
- Take into account 3 phenomena



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(Cook-)Torrance-Sparrow

$$f = \frac{F(\theta_i)G(\omega_i,\omega_r)D(\theta_h)}{4\cos(\theta_i)\cos(\theta_r)}$$

Fresnel term: allows for wavelength dependency

Geometric Attenuation: reduces the output based on the amount of shadowing or masking that occurs.

Distribution: distribution function determines what percentage of microfacets are oriented to reflect in the viewer direction.

How much of the macroscopic surface is visible to the light source

How much of the macroscopic surface is visible to the viewer

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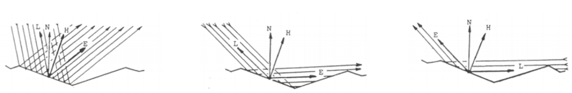
Geometric Attenuation

- *Geometric attenuation* refers to the decrease in light reflection due to both shadowing and masking

$$G = \min\left(1, \frac{2(\mathbf{n}\cdot\mathbf{h})(\mathbf{n}\cdot\mathbf{v})}{(\mathbf{v}\cdot\mathbf{h})}, \frac{2(\mathbf{n}\cdot\mathbf{h})(\mathbf{n}\cdot\mathbf{L})}{(\mathbf{v}\cdot\mathbf{h})}\right)$$



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Microfacet Distribution Function

- There have been various functions proposed that describe the distribution of microfacets around the average surface normal
- Gaussian: $D = ce^{-(\alpha/m)^2}$
- Beckmann: $D = \frac{1}{m^2\cos^4\alpha} e^{-\left(\frac{\tan^2\alpha}{m^2}\right)}$

where

α=acos(**n**·**h**)
m=root mean square slope of microfacets
c=an arbitrary constant (?)

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Torrance-Sparrow Model

K. E. Torrance, E. M. Sparrow,
Theory of the off-specular reflection from roughened surfaces,
JOSA 1967

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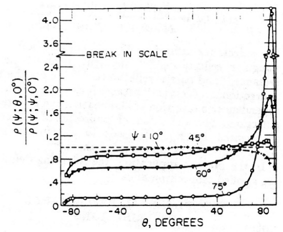
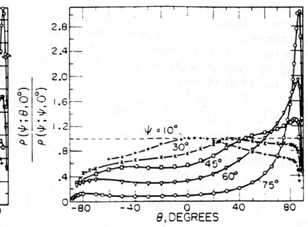
Experiment: "Off-Specular" Peak

Peak of reflection is not at the angle of reflection



Magnesium Oxide
Dielectric



Aluminum
Metal

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Torrance-Sparrow Theory

$$f_r(\omega_i \to \omega_r) = \frac{F(\theta_i')S(\theta_i)S(\theta_r)D(\alpha)}{4\cos\theta_i \cos\theta_r}$$

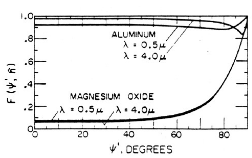

FIG. 6. Fresnel reflectance.

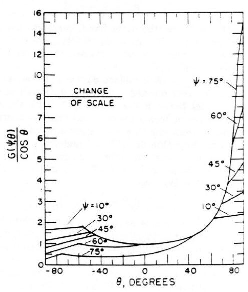

FIG. 7. The factor $G(\psi,\theta)/\cos\theta$ in the plane of incidence for various incidence angles ψ.

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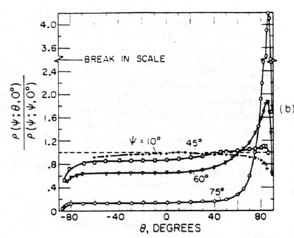
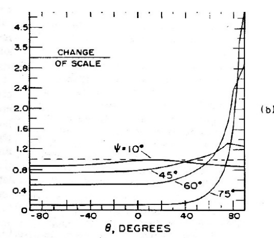
Torrance-Sparrow Model Prediction





Magnesium Oxide

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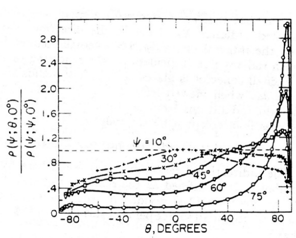
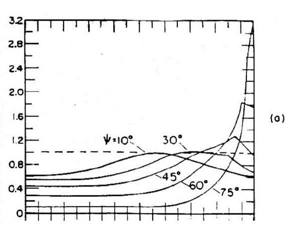
Torrance-Sparrow Model Prediction





Aluminum

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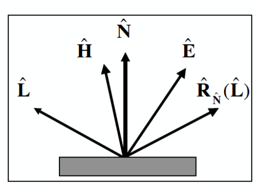
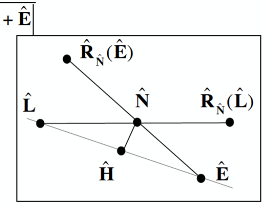
Reflection Geometry

$$\hat{\mathbf{H}} = \frac{\hat{\mathbf{L}} + \hat{\mathbf{E}}}{|\hat{\mathbf{L}} + \hat{\mathbf{E}}|}$$





$\cos\theta_i = \hat{\mathbf{L}} \bullet \hat{\mathbf{N}}$

$\cos\theta_r = \hat{\mathbf{E}} \bullet \hat{\mathbf{N}}$

$\cos\theta_s = \hat{\mathbf{E}} \bullet \mathbf{R}_{\hat{N}}(\hat{\mathbf{L}}) = \mathbf{R}_{\hat{N}}(\hat{\mathbf{E}}) \bullet \hat{\mathbf{L}}$

$\cos\theta_g = \hat{\mathbf{E}} \bullet \hat{\mathbf{L}}$

$\cos\theta_{s'} = \hat{\mathbf{H}} \bullet \hat{\mathbf{N}}$

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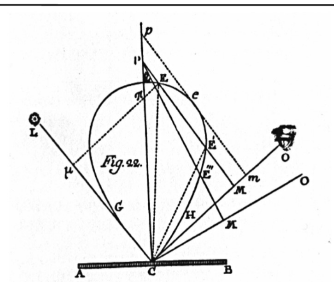
Microfacet BRDFs ("Little Faces")



P. Bouguer, Treatise on Optics, 1760

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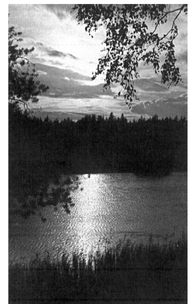
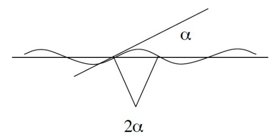
Reflection of the Sun from Waves





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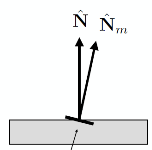
Microfacet Distributions



Normalize projected area

$$\int_{H^2} dA(\omega_m)\, cos\theta_m\, d\omega_m = dA$$

Probability distribution

$$\int_{H^2} D(\omega_m)\, cos\theta_m\, d\omega_m = 1$$

Area distribution $dA(\omega_m)$

Microfacet distribution $D(\omega_m) = dA(\omega_m)/dA$

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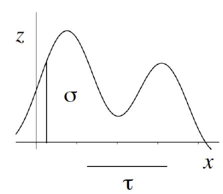
Beckmann Distribution

Gaussian distribution of heights

$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{z^2}{2\sigma^2}}$$



Beckmann distribution of normals (mirrors)

$$D(\omega_m) = \frac{e^{\frac{-\tan^2\theta_m}{\alpha^2}}}{\pi\alpha^2\cos^4\theta_m} \qquad \alpha = \sqrt{2}\frac{\sigma}{\tau}$$

mean slope

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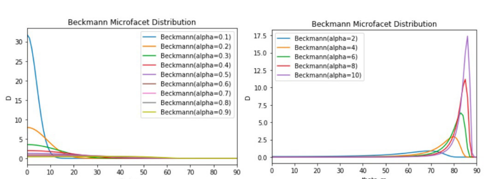
Beckmann Distribution



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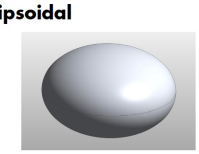
Trowbridge-Reitz (GGX) Distribution

Elipsoidal



$$z = \alpha(1 - x^2 - y^2)^{(1/2)}$$

GGX distribution of normals

$$D(\omega_m) = \frac{1}{\pi\alpha^2\cos^4\theta_m(1 + \frac{\tan^2\theta_m}{\alpha^2})^2}$$

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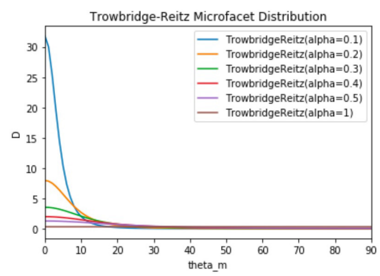
Trowbridge-Reitz (GGX) Distribution



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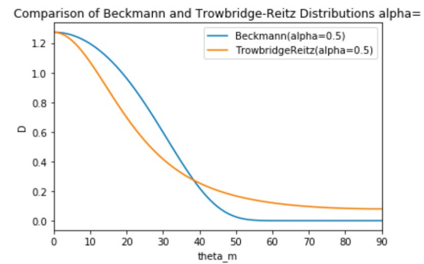
Comparison



Trowbridge-Reitz has a longer tail

Trowbridge-Reitz matches experimental data better

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Shadowing Reduces Reflected Energy

Without self-shadowing

With self-shadowing

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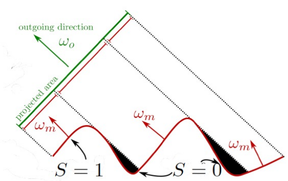
Visible Projected Area

From Heitz 2014



$$\int S(\omega_o) \max(0, \omega_m \cdot \omega_o) \, D(\omega_m) \, d\omega_m = \cos\theta_o$$

The sum of the *visible* areas of the rough surface as viewed from the outgoing direction should equal the projected area of the underlying mean surface

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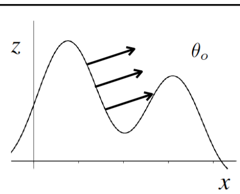
Smith Self-Shadowing Function

Assume probability of shadowing is independent of the normal



$$S(\theta_o) = \frac{1}{1 + \Lambda(\theta_o)}$$

$$\Lambda(\theta_o) = \frac{\mathrm{erf}(a) - 1}{2} + \frac{1}{2a\sqrt{\pi}} \exp(-a^2) \qquad a = \frac{1}{\alpha \tan\theta_o}$$

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} \, dt$$

From Smith, 1967

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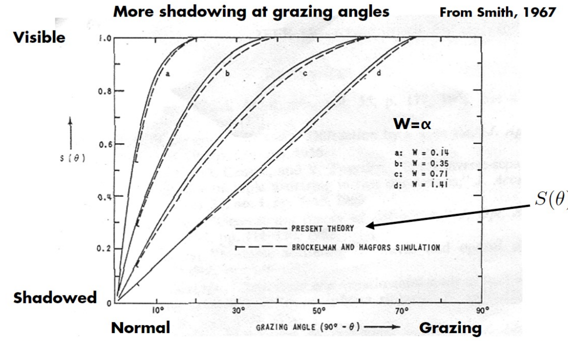
Smith Self-Shadowing Function

More shadowing at grazing angles **From Smith, 1967**



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BRDF Sampling

- Have dealt with BRDF evaluation, need importance sampling and PDF functions for MIS
- In 2004, no good importance sampling schemes for most BRDFs, including common Torrance-Sparrow
- From Lawrence et al. 04, factor BRDF into data-driven terms that can each be importance sampled
- Now some form of light/BRDF sampling common in production (standard in RenderMan 16, 2011-)

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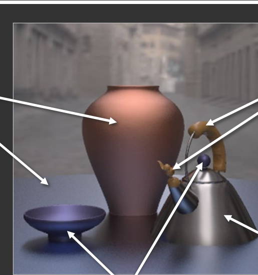
Motivation



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Key Idea

- Project 4D BRDF into sum of products of
  2D function dependent on ω_o and
  2D function dependent on ω_i :

$$f_r(\omega_o,\omega_i)(n\cdot\omega_i) = \sum_{j=1}^{J} F_j(\omega_o)\; G_j(\omega_p)$$

ω_p depends **only** on the incoming direction and some re-parameterization of the hemisphere.

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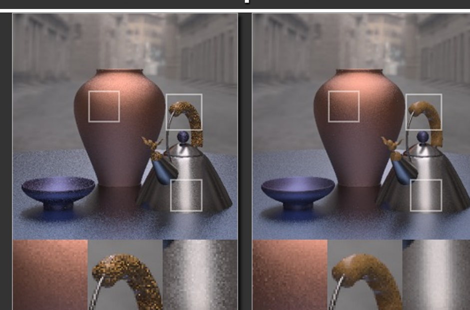
300 Samples/Pixel



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