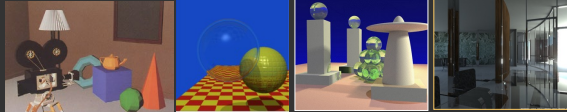


Computer Graphics II: Rendering

CSE 168[Spr 24], Lecture 9: Importance Sampling
Ravi Ramamoorthi

<http://viscomp.ucsd.edu/classes/cse168/sp24>



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To Do

- Start working on homework 3. Ask me if problems
- Also homework 4. This lecture covers material (Lecture is designed to follow assignment closely)
- Start thinking about final project

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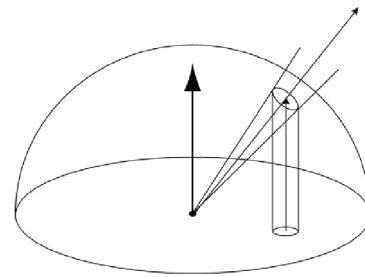
Importance Sampling

- Talked about in Monte Carlo Path Tracing
- This assignment: implement at each bounce
- Use "good" pdf for sampling instead of uniform
- Extension to Multiple Importance Sampling (Veach 95)
 - Allows considering both lighting and BRDF sampling
 - Key development in production rendering (Academy Award)
 - Remains active topic of research (many papers in 2019+)

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Sampling Projected Solid Angle

Generate cosine weighted distribution



CS348B Lecture 6

Pat Hanrahan, Spring 2004

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Cosine Importance Sampling

- Include cosine term in PDF (for indirect lighting)
- Previously, uniformly integrate over hemisphere $pdf(\omega_i) = \frac{1}{2\pi}$

$$\frac{1}{N} \sum_{k=1}^N \frac{L(\omega_{i,k}) f(\omega_{i,k}, \omega_o)(n \cdot \omega_{i,k})}{pdf(\omega_i)} = \frac{2\pi}{N} \sum_{k=1}^N L(\omega_{i,k}) f(\omega_{i,k}, \omega_o)(n \cdot \omega_{i,k})$$

- Now, consider a cosine PDF $pdf(\omega_i) = \frac{n \cdot \omega_i}{\pi}$

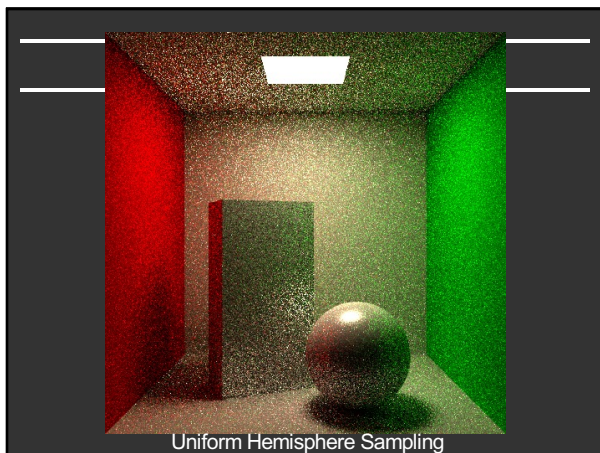
$$\frac{1}{N} \sum_{k=1}^N \frac{L(\omega_{i,k}) f(\omega_{i,k}, \omega_o)(n \cdot \omega_{i,k})}{pdf(\omega_i)} = \frac{\pi}{N} \sum_{k=1}^N \frac{L(\omega_{i,k}) f(\omega_{i,k}, \omega_o)(n \cdot \omega_{i,k})}{n \cdot \omega_{i,k}} = \frac{\pi}{N} \sum_{k=1}^N L(\omega_{i,k}) f(\omega_{i,k}, \omega_o)$$

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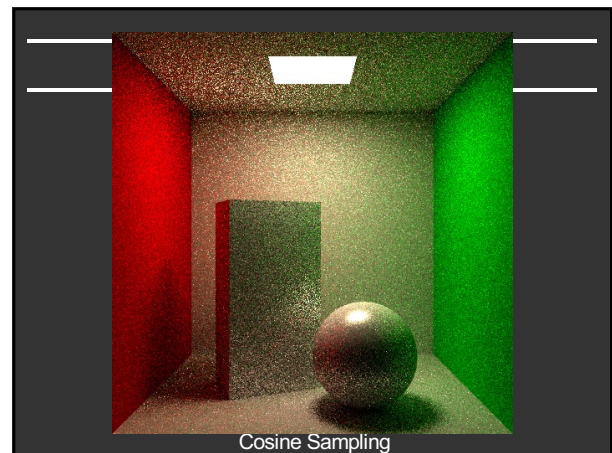
Cosine Sampling Upper Hemisphere

- Inversion method
 - In polar coords, density must be proportional to $\cos \theta \sin \theta$ (remember $d(\text{solid angle}) = \sin \theta d\theta d\phi$)
 - Integrate, invert $\rightarrow \cos^{-1}(\text{sqrt}(\dots))$
- Recipe is (start with two random numbers ξ_1, ξ_2 in $0 \dots 1$)
 - Generate ϕ in $0 \dots 2\pi$ $\phi = 2\pi\xi_2$
 - Generate z in $0 \dots 1$ $z = \text{sqrt}(\xi_1)$ // Note extra sqrt wrt uniform
 - Let $\theta = \cos^{-1} z$ $\theta = \text{acos}(z) = \text{acos}(\text{sqrt}(\xi_1))$
 - $(x, y, z) = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$
- Rotate according to surface normal (z goes to normal)
 - Or create coordinate frame (as you did for uniform sampling)
- Modify indirect lighting estimator (remove $n \cdot \omega_i$) and replace 2π with π (indirect lighting, Russian Roulette)

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Specular BRDFs

- Cosine importance sampling works well for near-Lambertian BRDFs (modest improvement)
- But more sophisticated sampling for specular BRDFs
- Will talk about general BRDFs next lecture
- For now, for assignment: Modified Phong, GGX
- Sampling BRDFs in general is non-trivial
 - Can simply normalize to get PDF, but sampling non-trivial
 - For now, sample a simpler BRDF, then divide by PDF
 - (This procedure is always guaranteed to work)

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BRDF Importance Sampling

- Phong BRDF: $f_r \sim \cos^s \beta$ where β is angle between outgoing ray and ideal mirror direction
- Constant scale = $k_s(s+2)/(2\pi)$
- Can't sample this times $\cos \theta_i$
 - Can only sample BRDF itself, then multiply by $\cos \theta_i$
 - That's OK – still better than random sampling

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BRDF Importance Sampling

- Recipe for sampling specular term:
 - Generate z in $0..1$
 - Let $y = \cos^{-1}(z^{1/(s+1)})$
 - Generate ϕ_i in $0..2\pi$
 - This gives direction w.r.t. ideal mirror direction
- Convert to (x,y,z) , then rotate such that z points along mirror dir.

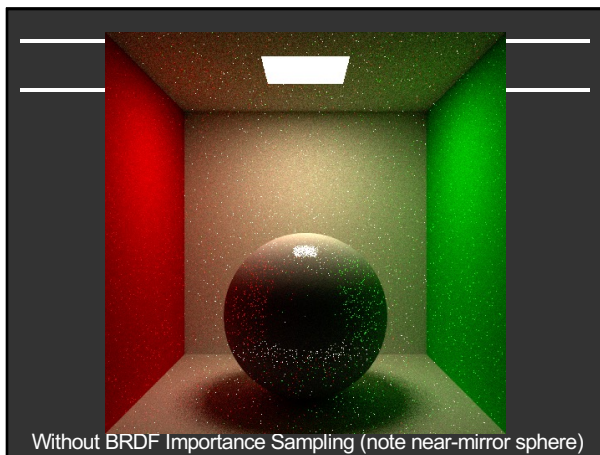
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Formal Modified Phong Sampling

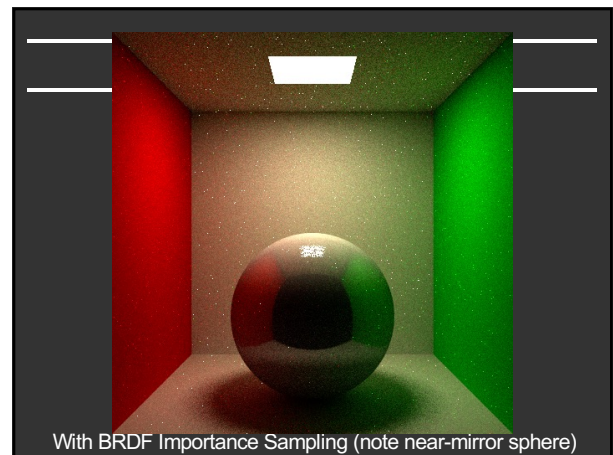
- Multiply by cosine, transport function (note colors)
- Modified Phong is an approximation $t = \frac{\bar{k}_s}{k_d + k_s}$

$$(1-t) \frac{n \cdot \omega_i}{\pi} + t \frac{s+1}{2\pi} (r \cdot \omega_i)^s$$
- Generate 3 random numbers: ξ_0, ξ_1, ξ_2 in $0..1$
- Use ξ_0 to decide diffuse ($>t$) or specular ($\leq t$)
- Generate ϕ in $0..2\pi$ $\phi = 2\pi\xi_2$
- If diffuse $\theta = \arccos(\sqrt{\xi_1})$ [coord. frame normal n]
- If specular $\theta = \arccos(\xi_1^{1/(s+1)})$ [coord. frame refl r]
- Compute BRDF / PDF (if below visible, BRDF = 0)

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GGX Microfacet Model

- Physically-Based Reflectance Model
- Widely used in practice
- Will discuss BRDFs in more detail next time
- Brief review here, see assignment for details

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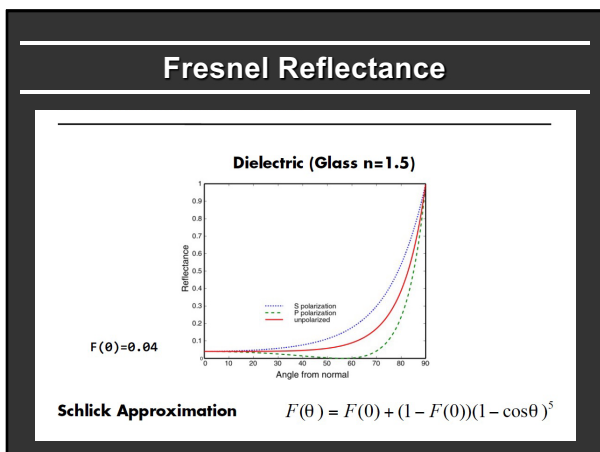
Experiment

Reflections from a shiny floor

From Lafortune, Foo, Torrance, Greenberg, SIGGRAPH 97

Reflection is greater at glancing angles

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(Cook-)Torrance-Sparrow

- Assume the surface is made up of grooves at the microscopic level. (*General Microfacet Theory*)
- Assume the faces of these grooves (called microfacets) are perfect reflectors.
- Take into account 3 phenomena

Shadowing Masking Interreflection

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(Cook-)Torrance-Sparrow

Fresnel term:
allows for
wavelength
dependency

Geometric Attenuation:
reduces the output based on the
amount of shadowing or masking
that occurs.

$$f = \frac{F(\theta_i)G(\omega_i, \omega_r)D(\theta_h)}{4\cos(\theta_i)\cos(\theta_r)}$$

How much of the
macroscopic
surface is visible
to the light source

How much of
the macroscopic
surface is visible
to the viewer

Distribution:
distribution function
determines what
percentage of
microfacets are
oriented to reflect
in the viewer
direction.

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GGX Microfacet Model

- Specular term (see assignment for G, F)

$$f(\omega_i, \omega_o) = \frac{k_s}{\pi} + f_{GGX}(\omega_i, \omega_o)$$

$$f_{GGX}(\omega_i, \omega_o) = \frac{F(\omega_i, \mathbf{h}, k_s)G(\omega_i, \omega_o)D(\mathbf{h})}{4(\omega_i \cdot \mathbf{n})(\omega_o \cdot \mathbf{n})}$$

$$D(\mathbf{h}) = \frac{\alpha^2}{\pi \cos^4 \theta_h (\alpha^2 + \tan^2 \theta_h)^2}$$
- Importance Sampling PDF (includes cosine term)
 - Neglects F and G terms, must do BRDF / PDF

$$pdf(\omega_i | \omega_o) = (1-t) \frac{\mathbf{n} \cdot \omega_i}{\pi} + t \frac{D(\mathbf{h})(\mathbf{n} \cdot \mathbf{h})}{4(\omega_i \cdot \mathbf{h})}$$
 - Note that t is clamped at a min of 0.25 to give some specular samples even for low k_s (because of Fresnel)

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Importance Sampling GGX

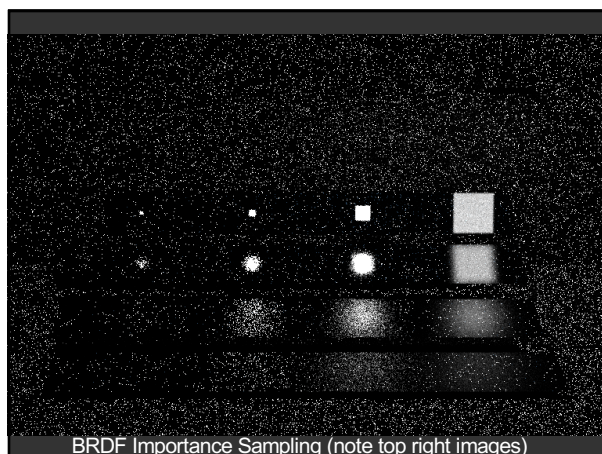
- High-Level idea similar to modified Phong
- Generate 3 random numbers: ξ_0, ξ_1, ξ_2 in $0 \dots 1$
- Use ξ_0 to decide diffuse ($>t$) or specular ($\leq t$)
- Generate ϕ in $0 \dots 2\pi$ $\phi = 2\pi\xi_2$ (if specular, this is ϕ_h)
- If diffuse $\theta = \arccos(\sqrt{\xi_1})$ [coord. frame normal $\mathbf{n}$]
- If specular** $\theta_h = \arctan\left(\frac{\alpha\sqrt{\xi_1}}{\sqrt{1-\xi_1}}\right)$ [coord. frame halfvector $\mathbf{h}$]
 - Must compute incident direction from outgoing, half-vector
 - Rotate $\mathbf{h}$ about normal, reflect outgoing about half-vector
- Compute BRDF / PDF (if below visible, BRDF = 0)

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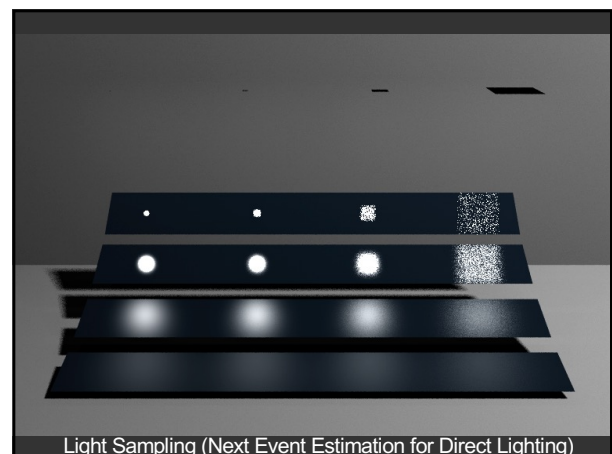
Multiple Importance Sampling

- Veach 95 classic scene (4 lights, 4 glossiness)
- BRDF importance sampling only (no NEE, so no explicit direct lighting or light sampling pass)
 - Mostly noisy but sharper reflections handled well
- Compare with light sampling (NEE)
 - Mostly better but noisy for sharp reflections
- Can we combine BRDF, Light(NEE) sampling?
 - MIS (Veach95) provides a way, bounds
 - Very robust, works well shiny/rough etc.
 - Key development in production rendering
 - Remains topic of interest (many papers in 2019)

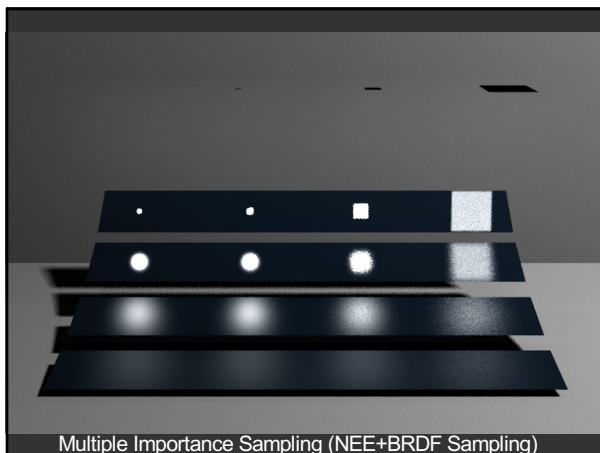
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Multiple Importance Sampling

- MIS relies on NEE almost everywhere, but relies on BRDF importance sampling when needed
- Multi-sample: sample both distributions at each intersection
- General case: N sampling techniques (inner summation is unbiased estimator each technique)

$$\int_x f(x) dx \approx \sum_{i=1}^N \frac{1}{N_i} \sum_{j=1}^{N_i} w_i(x_{ij}) \frac{f(x_{ij})}{pdf_i(x_{ij})}$$

- Weights must sum to 1, unbiased

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Multiple Importance Sampling

- General case: N sampling techniques (inner summation is unbiased estimator each technique)

$$\int_x f(x) dx \approx \sum_{i=1}^N \frac{1}{N_i} \sum_{j=1}^{N_i} w_i(x_{ij}) \frac{f(x_{ij})}{pdf_i(x_{ij})}$$

- Weights must sum to 1, unbiased
 - Interesting theory (ongoing, papers in 2019)
 - Veach and Guibas 95 proposed balance, power heuristics (provably "good" under certain assumptions)
 - We use power heuristic with $\beta = 2$
 - Subtle point: PDF must be able to be evaluated anywhere (not just own samples)
- Natural abstract interface for sampling and MIS
 - Eval(), Sample(), PDF() [sometimes Value() = Eval/PDF]

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Lighting/BRDF Sampling

- For now, 1 sample on light (NEE), 1 from BRDF

- We already know BRDF PDF
- Light PDF implicitly on light, convert to angle $d\omega = dA \frac{\cos \theta}{R^2}$

$$pdf_{light}(\omega) = \frac{R^2}{(\mathbf{n}_{light} \cdot \omega)A}$$

- For multiple lights, simple normalization (see homework)
- Combine NEE and BRDF sampling (power heuristic)

$$w_i(\omega) = \frac{pdf_i^{\beta}(\omega)}{\sum_{k=1}^N pdf_k^{\beta}(\omega)}$$

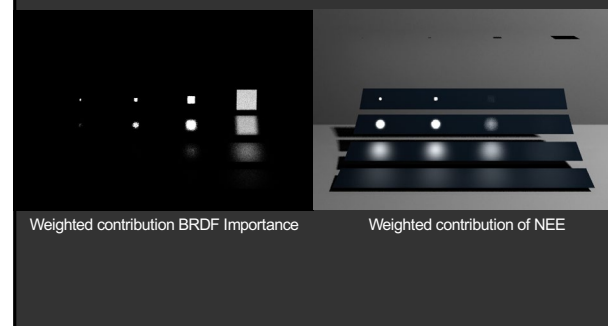
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MIS Implementation

- Can be tricky, see assignment
- First disable NEE, BRDF sampling for direct
 - Separate NEE function, toggle light/BRDF sampling
- Now implement pdf(nee)
 - Beware divide by zero, see assignment for specifics
- Implement weight function
 - Visualize weighted lighting, weighted BRDF
 - Then combine them with MIS, enable both techniques
- See assignment carefully
 - MIS for direct lighting only (Veach scene no indirect)
 - Note gamma correction for this assignment

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MIS weights



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