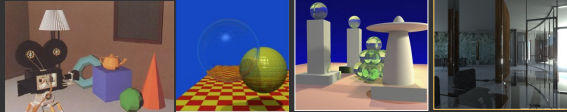


Computer Graphics II: Rendering

CSE 168[Spr 24], Lecture 15: Volumetric Rendering
Ravi Ramamoorthi

<http://viscomp.ucsd.edu/classes/cse168/sp24>



1

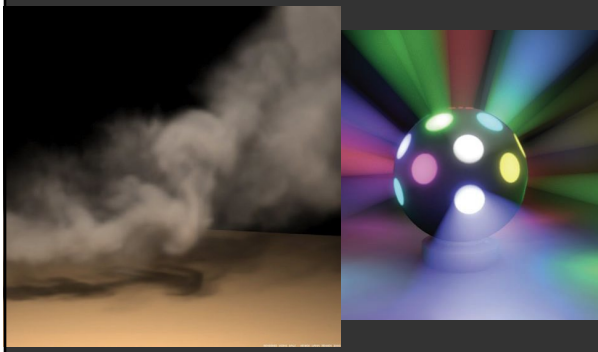
To Do

- Start working on final projects (initial results and proposal due in a week). Ask me if problems
- Volumetric rendering (this lecture) may be one component of the final project (but hard, be careful)
- Increasingly accurate appearance requires volumetric scattering (even for skin, hair, fur)
- Continues to be an active area of research

Many slides courtesy Pat Hanrahan/Matt Pharr (Stanford CS 348b) and Steve Rotenberg, Henrik Wann Jensen (UCSD CSE 168)

2

Volumetric Scattering



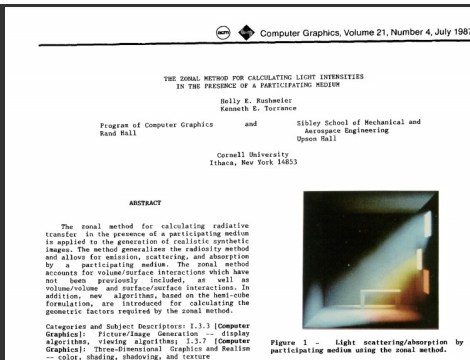
3

Volumetric Scattering



4

Volumetric Scattering



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Volumetric Scattering

- Participating Media (light participates via scattering)
 - Volumetric phenomena like clouds, smoke, fire
 - Subsurface scattering, translucency (wax, human skin)
 - These are not surfaces with well-defined BRDFs
 - Rather volumes where light can scatter
 - Medium is often known as a participating medium
- Surface Rendering: Radiance Constant along Ray
 - Only true in absence of participating media
 - No longer true for volumetric scattering
 - Often replace ray tracing with ray marching in medium
- Volumetric Properties
 - BRDF replaced by phase function
 - Must consider absorption and scattering in medium

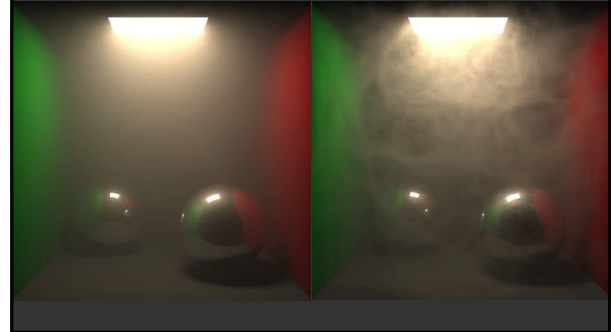
6

Homogeneous vs Heterogeneous

- Homogeneous: Properties constant everywhere
 - Example: Fog often represented as homogeneous
- Heterogeneous: Varies across space
 - Example: Smoke, fire etc.
 - Sometimes called inhomogeneous
- Homogeneous volumes often easier
 - Some computational shortcuts (transmittance etc.)
 - Some analytic formulae

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Homogeneous vs Heterogeneous



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Volumetric Interactions

- 4 different processes affect radiance of a beam
 - Absorption
 - Out-Scattering
 - Emission
 - In-Scattering

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Absorption

$$dL(p, \omega) = -\sigma_a(p) L(p, \omega) ds$$

Absorption cross section: $\sigma_a(p)$

■ **Probability of being absorbed per unit length**

■ **Units: 1/distance**

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Absorption



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Transmittance

$$dL(p, \omega) = -\sigma_a(p) L(p, \omega) ds$$

$$\frac{dL(p, \omega)}{L(p, \omega)} = -\sigma_a(p) ds$$

$$\log L(p + s\omega, \omega) = -\int_0^s \sigma_a(p + s'\omega, \omega) ds' = -\tau(s)$$

Optical distance (depth): $\tau(s) = \int_0^s \sigma_a(p') ds'$
 $p' = p + s'\omega$

Homogeneous medium-constant σ_a : $\tau(s) = \sigma_a s$



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Transmittance and Opacity

$$dL(p, \omega) = -\sigma_a(p) L(p, \omega) ds$$

$$\frac{dL(p, \omega)}{L(p, \omega)} = -\sigma_a(p) ds$$

$$\log L(p + s\omega, \omega) = -\int_0^s \sigma_a(p + s'\omega, \omega) ds' = -\tau(s)$$

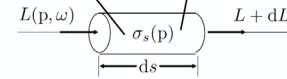
$$L(p + s\omega, \omega) = e^{-\tau(s)} L(p, \omega) = T(s) L(p, \omega)$$

Transmittance: $T(s) = e^{-\tau(s)}$

Opacity: $\alpha(s) = 1 - T(s)$

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Out-Scattering



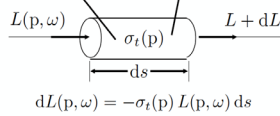
$$dL(p, \omega) = -\sigma_s(p) L(p, \omega) ds$$

Scattering cross-section: σ_s

■ Probability of being scattered per unit length

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Extinction



$$dL(p, \omega) = -\sigma_t(p) L(p, \omega) ds$$

Total cross section: $\sigma_t = \sigma_a + \sigma_s$

Albedo: $W = \frac{\sigma_s}{\sigma_t} = \frac{\sigma_s}{\sigma_a + \sigma_s}$

Optical distance from absorption and scattering:

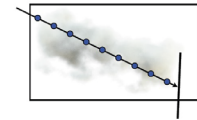
$$\tau(s) = \int_0^s \sigma_t(p') ds'$$

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Ray Marching for Transmittance

$$\tau(s) = \int_0^s \sigma_t(x + s'\omega) ds'$$

$$T(s) = e^{-\tau(s)}$$



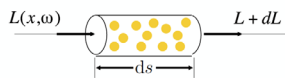
Monte Carlo not necessary for 1D—can use a Riemann sum:

$$\tau(s) \approx \frac{s}{N} \sum_{i=1}^N \sigma_t(x_i)$$

$$x_i = x + \frac{i + 0.5}{N} \omega$$

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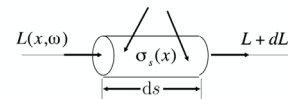
Emission



$$dL(p, \omega) = \sigma_a(p) L_e(p, \omega) ds$$

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In-Scattering



$$S(p, \omega) = \sigma_s(p) \int_{S^2} p(\omega' \rightarrow \omega) L(p, \omega') d\omega'$$

Phase function: $p(\omega' \rightarrow \omega)$

Reciprocity: $p(\omega' \rightarrow \omega) = p(\omega \rightarrow \omega')$

Energy conservation: $\int_{S^2} p(\omega' \rightarrow \omega) d\omega' = 1$

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Scattering Phase Functions

- Light interacts with volume, scatters in some spherical distribution
- Similar to light scattering off a surface
- Phase function analogous to a surface BRDF
- Depends only on cosine of incident-outgoing
- Like BRDFs, volumetric phase functions must be reciprocal and conserve energy
- Similar to BRDFs, we will want to do importance sampling and evaluation of phase functions

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Phase Functions

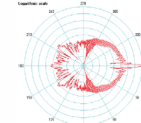
Phase angle $\cos \theta = \omega \cdot \omega'$

Phase functions

■ **Isotropic:** $p(\cos \theta) = \frac{1}{4\pi}$

■ **Rayleigh:** $p(\cos \theta) = \frac{3}{4}(1 + \cos^2 \theta)$ with $\sigma_s \propto \frac{1}{\lambda^4}$

■ **Mie:**



(Philip Leven)

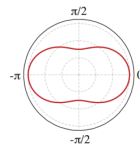
20

Rayleigh Scattering

- Rayleigh scattering describes the scattering of light by particles much smaller than the wavelength

$$p(\cos \theta) = \frac{3}{16\pi}(1 + \cos^2 \theta)$$

$$\sigma_s = \frac{2\pi^5 d^6}{3 \lambda^4} \left(\frac{n^2 - 1}{n^2 + 2} \right)^2$$



- Where λ is the wavelength of light, d is the diameter of the particle, and n is the index of refraction of the particle
- The strong dependence on wavelength (λ^{-4}) causes greater scattering towards the blue end of the spectrum
- The blue color of the sky is caused by Rayleigh scattering of sunlight by air molecules

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Rayleigh Scattering: Blue Sky, Red Sunset



From Greenler: Rainbows, Halos, and Glories

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Mie Scattering

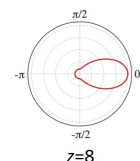
- Scatter electromagnetic waves by spherical particles
- Size of particles same scale as wavelength of light
- Water droplets in atmosphere, fat droplets in milk
- After Gustave Mie, Ludvig Lorenz

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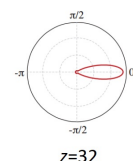
Empirical Mie Approximation

- The following empirical function is often used to approximate the shape of Mie scattering

$$p(\cos \theta) = \frac{1}{4\pi} \left(\frac{1}{2} + \frac{(z+1)}{2} \left(\frac{1 + \cos \theta}{2} \right)^z \right)$$



$z=8$



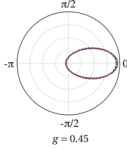
$z=32$

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Henyey-Greenstein Function

- The Henyey-Greenstein phase function is an empirical function originally designed to model the scattering in galactic dust clouds

$$p(\cos \theta) = \frac{1 - g^2}{4\pi(1 + g^2 - 2g\cos \theta)^{1.5}}$$



- It uses an anisotropy parameter g that ranges between -1 (full backscatter) and 1 (full forward scatter), and is isotropic for $g=0$

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Direct Illumination in a Volume

$$S_d(p', \omega) = \sigma_s(p') \int_{S^2} p(\omega' \rightarrow \omega) L_d(p', \omega') d\omega'$$

Can treat like direct illumination at a surface

- Sample from phase function's distribution
- Sample from light source distributions
- Weight using multiple importance sampling



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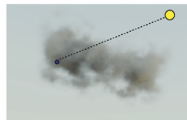
Direct Illumination in a Volume

$$S_d(p', \omega) = \sigma_s(p') \int_{S^2} p(\omega' \rightarrow \omega) L_d(p', \omega') d\omega'$$

Estimator: $\sigma_s(p') \frac{1}{N} \sum_i \frac{p(\omega_i \rightarrow \omega) L_d(p', \omega_i)}{p(\omega_i)}$

Computing direct lighting, L_d can be expensive

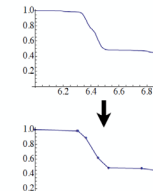
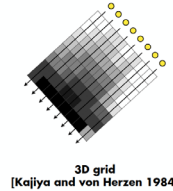
- Not just a shadow ray - need to compute transmittance



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Transmittance for Shadow Rays

Besides Monte Carlo, precomputed transmittance can be faster for point, distant lights



Deep Shadow Maps [Lokovic & Veach 2000]
Adaptive Volumetric Shadow Maps [Salvi et al. 2010]

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Single-Scattering

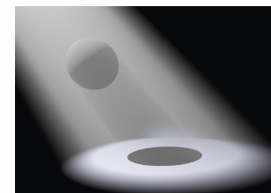


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Single-Scattering



Minneart: Color and Light In The Open Air



pbrt: Spot-Lit Ball In The Fog

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The Volume Rendering Equation

Integro-differential equation:

$$\frac{\partial L(p, \omega)}{\partial s} = -\sigma_t L(p, \omega) + S(p, \omega)$$

Integro-integral equation:

$$L(p, \omega) = \int_0^\infty T(p') S(p', \omega) ds'$$

Attenuation: absorption and scattering
 $e^{-\int_0^{s'} \sigma_t(p'') ds''}$

Source: in-scattering (and emission)
 $\sigma_s(p') \int_{S^2} p(\omega' \rightarrow \omega) L(p', \omega') d\omega'$

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Volumetric Path Tracing

Integro-integral equation:

$$L(p, \omega) = \int_0^\infty T(p') S(p', \omega) ds'$$

Monte Carlo integration: sample $s' \sim p(s)$

Estimator: $\frac{T(p') S(p', \omega)}{p(s')}$

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Evaluating the Estimator: S

Include indirect illumination in the source term:

$$S(x, \omega) = \sigma_s(x) \int_{S^2} p(\omega' \rightarrow \omega) L(x, \omega') d\omega'$$

↓

$$L(x, \omega') = L_d(x, \omega') + L_i(x, \omega')$$

- Compute direct lighting as before
- Sample incident direction from the phase function's distribution, trace a ray recursively...

$$L_i(x, \omega') \approx \frac{p(\omega'' \rightarrow \omega') L(x, \omega'')}{p(\omega'')} \quad \text{Uniform spherical directions: } p(\omega'') = \frac{1}{4\pi}$$

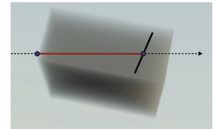
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Linear Sampling of T

We want samples along a finite ray $[0, t_{\max}]$.



- Uniform probability along the ray:

$$p(t) = \frac{1}{t_{\max}}$$

- Sampling recipe:

$$\xi = \int_0^t p(t) dt \quad t = \xi t_{\max}$$

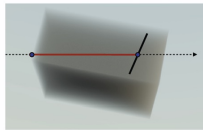
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Exact Sampling of Uniform T

We want samples along a finite ray $[0, t_{\max}]$, $p(t) \propto e^{-\sigma t}$



- Normalize to find PDF:

$$\int_0^{t_{\max}} e^{-\sigma t} dt = -\frac{1}{\sigma} (e^{-\sigma t_{\max}} - 1) = c \quad p(t) = c e^{-\sigma t}$$

- Invert to find t for a random sample:

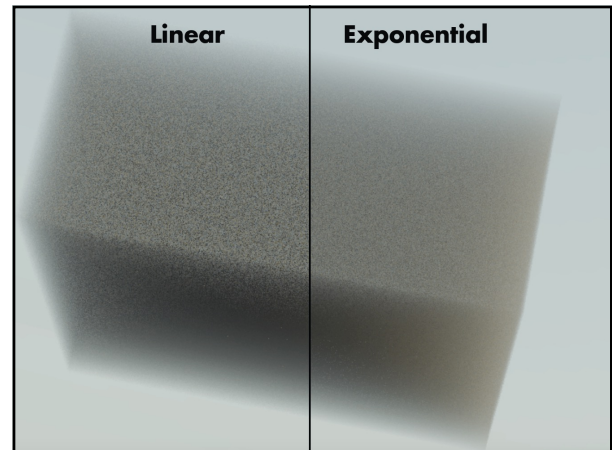
$$\xi = \int_0^t p(t) dt$$

$$t = -\frac{1}{\sigma} \log(1 - \xi(1 - e^{-\sigma t_{\max}}))$$

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Volumetric Path Tracing

Integro-integral equation:

$$L(p, \omega) = \int_0^\infty T(p') S(p', \omega) ds'$$

Monte Carlo integration: sample $s' \sim p(s)$

Estimator: $\frac{T(p')S(p', \omega)}{p(s')}$

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Single-Scattering



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Multiple Scattering



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Clouds



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Translucency

- Translucency is a volumetric lighting effect with additional effects at the surface (usually rough dielectric type interaction)
- These can be modeled through standard volumetric lighting techniques, or can be optimized through some further methods designed specifically for sub-surface scattering



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Fire

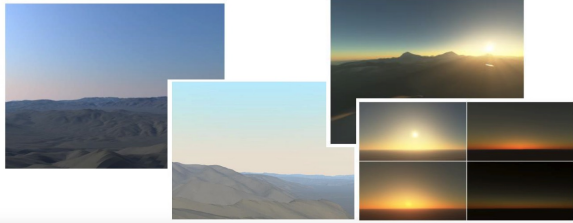
- “Physically Based Modeling and Animation of Fire”, Nguyen, Fedkiw, Jensen, 2003



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Sky Rendering

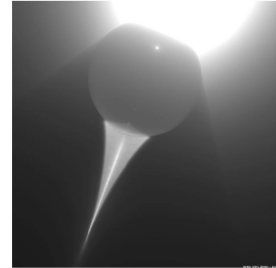
- "A Practical Analytical Model for Daylight", Preetham, Shirley, Smits, 1999
- "A Physically Based Night Sky Model", Jensen, Durand, Stark, Premoze, Dorsey, Shirley, 2001
- "Precomputed Atmospheric Scattering", Bruneton, Neyret, 2008
- "An Analytic Model for Full Spectral Sky-Dome Radiance", Hosek, Wilkie, 2012



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Volumetric Caustics

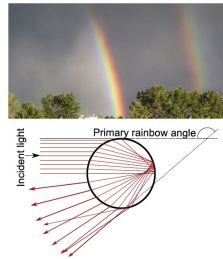
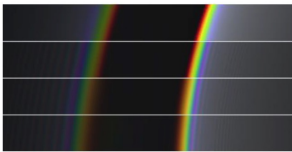
- "Efficient Simulation of Light Transport in Scenes with Participating Media using Photon Maps", Jensen, Christensen, 1998



44

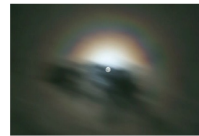
Rainbows

- "Physically Based Simulation of Rainbows", Sadeghi, Munoz, Laven, Jarosz, Seron, Gutierrez, Jensen, 2012



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Atmospheric Phenomena



Corona



Ice Crystal Halo



Glory

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