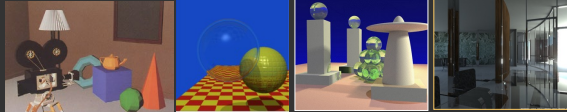


Computer Graphics II: Rendering

CSE 168[Spr 24], Lecture 10: Materials and BRDFs
Ravi Ramamoorthi

<http://viscomp.ucsd.edu/classes/cse168/sp24>



1

To Do

- Start working on homework 3. Ask me if problems
- Also homework 4. Have covered material
- Start thinking about final project

Some slides courtesy Steve Rotenberg and Pat Hanrahan

2

Materials and BRDFs

- Key part of renderer: different materials/BRDFs
- Abstract BRDF/Material interface (for MIS)
 - Evaluate (for given incident, outgoing direction)
 - Sample (given outgoing, importance sample incident)
 - PDF (for MIS, evaluate sampling PDF arbitrary direction)
 - Also for value of sample, need to compute eval/PDF (sometimes can simplify this, new value function=eval/PDF)
- Any physical or non-physical BRDF must fit above
 - Evaluation is usually easy (BRDF formula)
 - Can encompass analytic formulae, table measurements
 - Sampling can be hard and is crucial (see my 2004 paper for general importance sampling, special cases for some)
 - PDF function can be non-trivial, make sure math correct

3

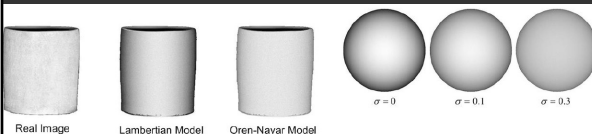
Diffuse Surfaces

- Simplest Case: Lambertian Reflectance
- BRDF is simply a constant: $f = \frac{\rho}{\pi}$
- Note energy conservation, divide albedo by π
- Note cosine incident term in final evaluation $\tilde{f} = \frac{\rho}{\pi} \cos \theta_i$
- Evaluate BRDF is straightforward
- Sample? Sample hemisphere (or cosine-weight)
- PDF is $\frac{1}{2\pi}$ or (if cosine-weight) $\frac{\cos \theta_i}{\pi}$
- Value/weight with cosine sampling is simply ρL_i

4

Oren-Nayar Model

- Generalization of Lambert's Reflectance Model (SIGGRAPH 94, rough diffuse [shadows, interreflections])



Importance sampling can be complicated (but exact sampling is not required)

Simplest: Lambertian sampling/PDF
But Eval uses Oren-Nayar; Eval/PDF (will cancel leading Lambertian term only)

$$L_r = \frac{\rho}{\pi} \cos \theta_i \cdot (A + (B \cdot \max(0, \cos(\phi_i - \phi_r)) \cdot \sin \alpha \cdot \tan \beta)) \cdot E_i$$

where

$$A = 1 - 0.5 \frac{\sigma^2}{\sigma^2 + 0.33}$$

$$B = 0.45 \frac{\sigma^2}{\sigma^2 + 0.09}$$

$$\alpha = \max(\theta_i, \theta_r)$$

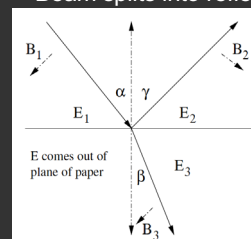
$$\beta = \min(\theta_i, \theta_r)$$

From Wikipedia

5

Fresnel Surfaces

- Idealized Fresnel surfaces are perfectly smooth boundary between dielectric (air, glass, water) and another dielectric, or a dielectric and a metal
- Beam splits into reflected/refracted (Snell's law)



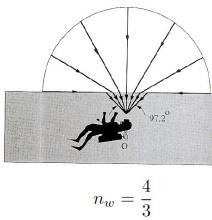
$$\sin \alpha = n \sin \beta$$

$$\alpha = \gamma$$

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Optical Manhole

Total internal reflection



$$n_w = \frac{4}{3}$$

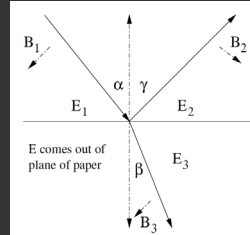


From Livingston and Lynch

7

Fresnel Surfaces

- Idealized Fresnel surfaces are perfectly smooth boundary between dielectric (air, glass, water) and another dielectric, or a dielectric and a metal
- Beam splits into reflected/refracted (Snell's law)



$$\sin \alpha = n \sin \beta$$

$$\alpha = \gamma$$

$$r_{\perp} = \frac{\cos \alpha - n \cos \beta}{\cos \alpha + n \cos \beta}$$

$$r_{\parallel} = \frac{n \cos \alpha - \cos \beta}{n \cos \alpha + \cos \beta}$$

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Experiment

Reflections from a shiny floor



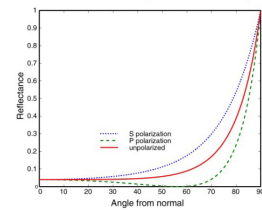
From Lafortune, Foo, Torrance, Greenberg, SIGGRAPH 97

Reflection is greater at glancing angles

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Fresnel Reflectance

Dielectric (Glass $n=1.5$)



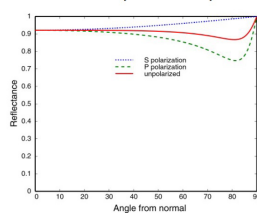
Schlick Approximation

$$F(\theta) = F(0) + (1 - F(0))(1 - \cos \theta)^5$$

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Fresnel Reflectance

Metal (Aluminum)

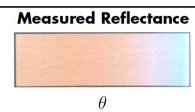


Gold $F(0)=0.82$
Silver $F(0)=0.95$

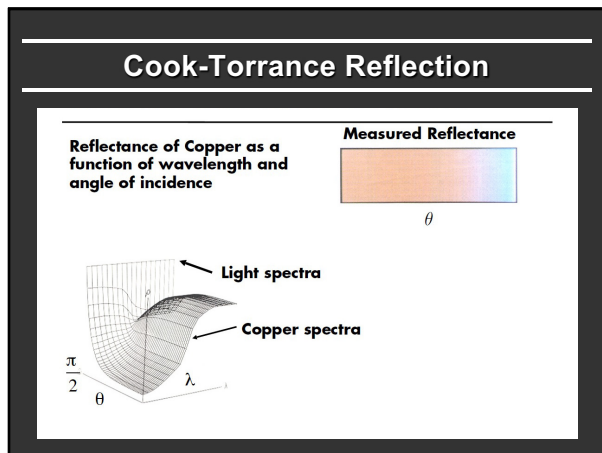
11

Reflection from Metals

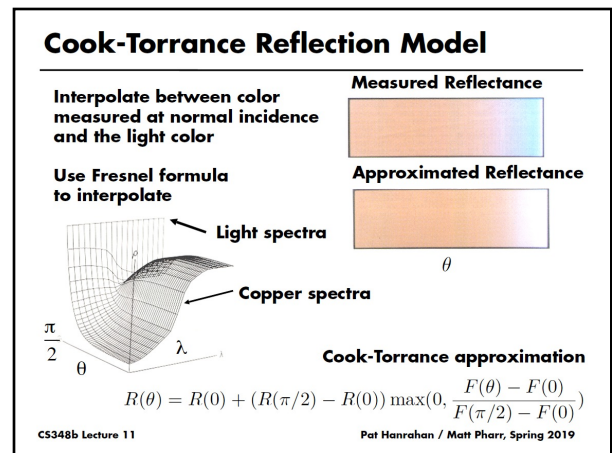
Reflectance of Copper as a function of wavelength and angle of incidence



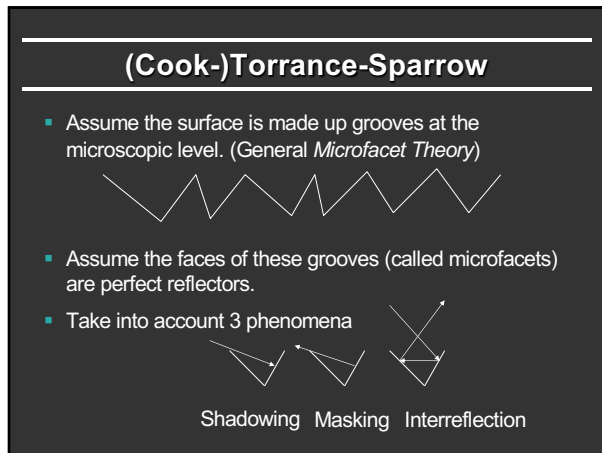
12



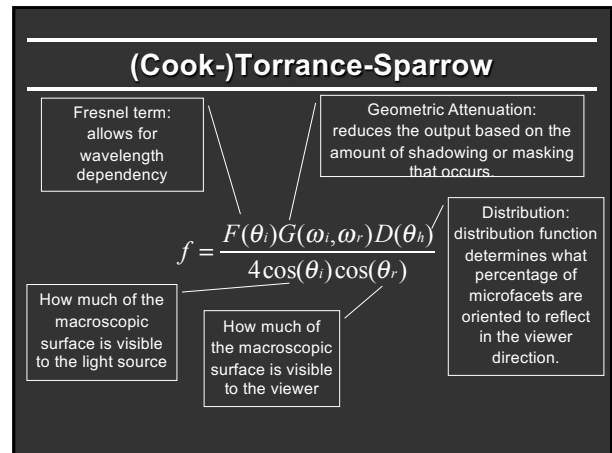
13



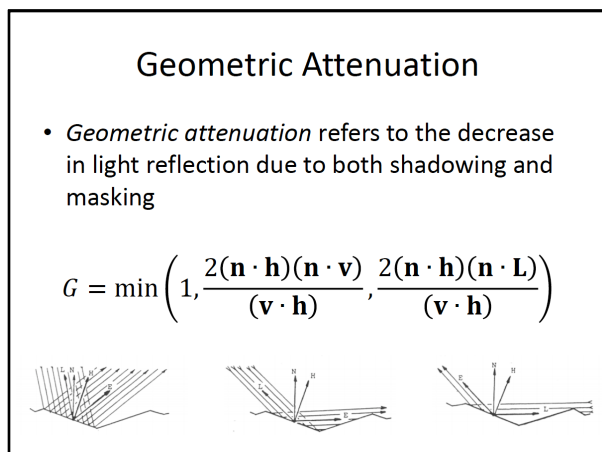
14



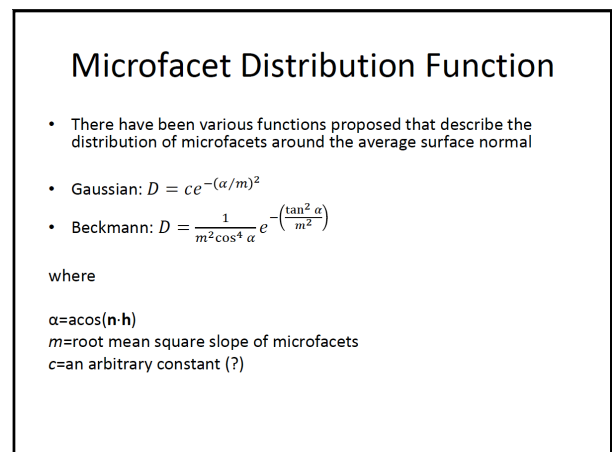
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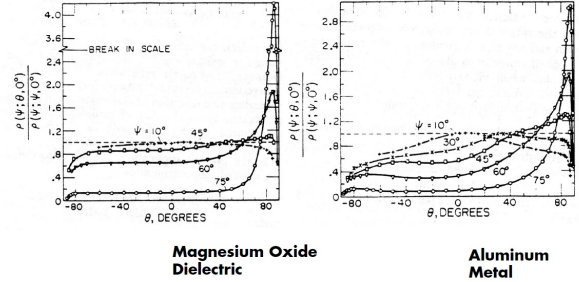
Torrance-Sparrow Model

K. E. Torrance, E. M. Sparrow,
Theory of the off-specular reflection
from roughened surfaces,
JOSA 1967

19

Experiment: "Off-Specular" Peak

Peak of reflection is not at the angle of reflection



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Torrance-Sparrow Theory

$$f_r(\omega_i \rightarrow \omega_r) = \frac{F(\theta_i)S(\theta_i)S(\theta_r)D(\alpha)}{4 \cos \theta_i \cos \theta_r}$$

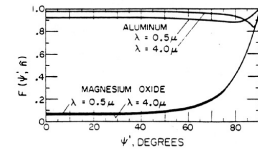


FIG. 6. Fresnel reflectance.

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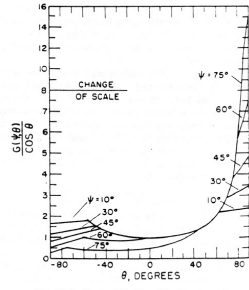
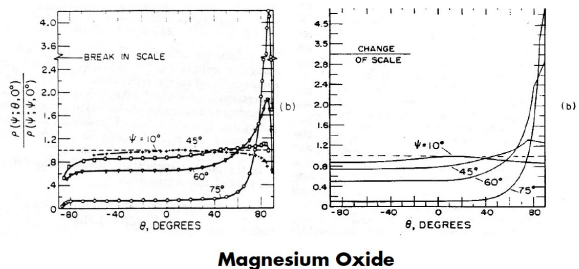


FIG. 7. The factor $G(\psi, \theta) / \cos \theta$ in the plane of incidence for various incidence angles ψ .

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Torrance-Sparrow Model Prediction



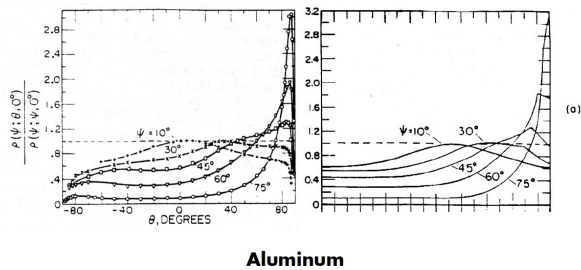
Magnesium Oxide

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Torrance-Sparrow Model Prediction



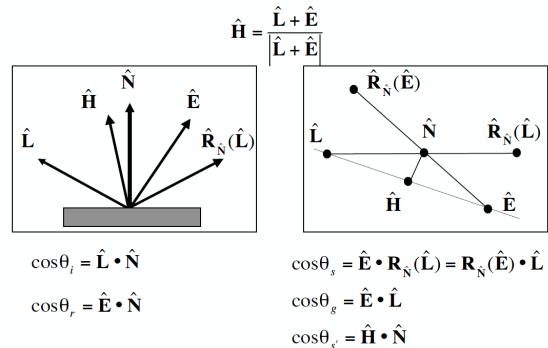
Aluminum

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Reflection Geometry

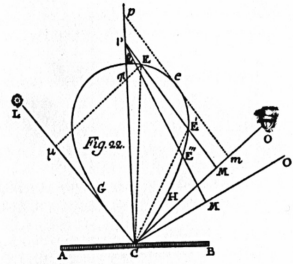


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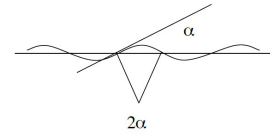
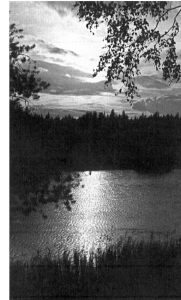
Microfacet BRDFs ("Little Faces")



P. Bouguer, Treatise on Optics, 1760

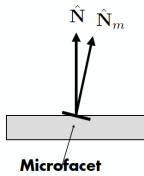
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Reflection of the Sun from Waves



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Microfacet Distributions



Normalize projected area

$$\int_{H^2} dA(\omega_m) \cos \theta_m d\omega_m = dA$$

Probability distribution

$$\int_{H^2} D(\omega_m) \cos \theta_m d\omega_m = 1$$

Area distribution $dA(\omega_m)$

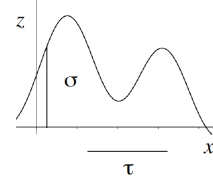
Microfacet distribution $D(\omega_m) = dA(\omega_m)/dA$

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Beckmann Distribution

Gaussian distribution of heights

$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{z^2}{2\sigma^2}}$$



Beckmann distribution of normals (mirrors)

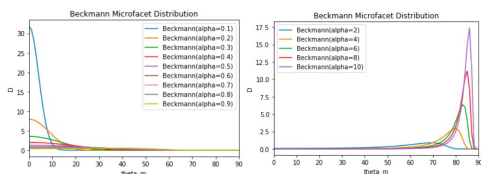
$$D(\omega_m) = \frac{e^{-\frac{\tan^2 \theta_m}{\alpha^2}}}{\pi \alpha^2 \cos^4 \theta_m}$$

$$\alpha = \sqrt{2} \frac{\sigma}{\tau}$$

mean slope

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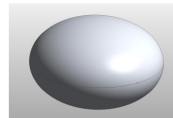
Beckmann Distribution



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Trowbridge-Reitz (GGX) Distribution

Ellipsoidal



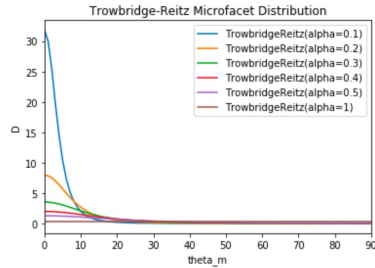
$$z = \alpha(1 - x^2 - y^2)^{1/2}$$

GGX distribution of normals

$$D(\omega_m) = \frac{1}{\pi \alpha^2 \cos^4 \theta_m (1 + \frac{\tan^2 \theta_m}{\alpha^2})^2}$$

30

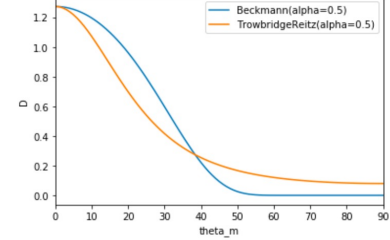
Trowbridge-Reitz (GGX) Distribution



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Comparison

Comparison of Beckmann and Trowbridge-Reitz Distributions alpha=



Trowbridge-Reitz has a longer tail

Trowbridge-Reitz matches experimental data better

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Shadowing Reduces Reflected Energy

Without self-shadowing



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With self-shadowing

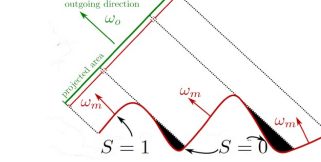


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Visible Projected Area

From Heitz 2014



$$\int S(\omega_o) \max(0, \omega_m \cdot \omega_o) D(\omega_m) d\omega_m = \cos \theta_o$$

The sum of the visible areas of the rough surface as viewed from the outgoing direction should equal

the projected area of the underlying mean surface

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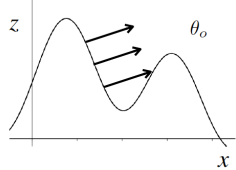
Smith Self-Shadowing Function

Assume probability of shadowing is independent of the normal

$$S(\theta_o) = \frac{1}{1 + \Lambda(\theta_o)}$$

$$\Lambda(\theta_o) = \frac{\text{erf}(a) - 1}{2} + \frac{1}{2a\sqrt{\pi}} \exp(-a^2)$$

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$



$$a = \frac{1}{\alpha \tan \theta_o}$$

From Smith, 1967

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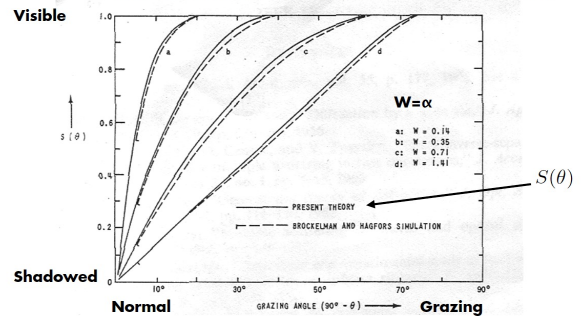
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Smith Self-Shadowing Function

More shadowing at grazing angles

From Smith, 1967



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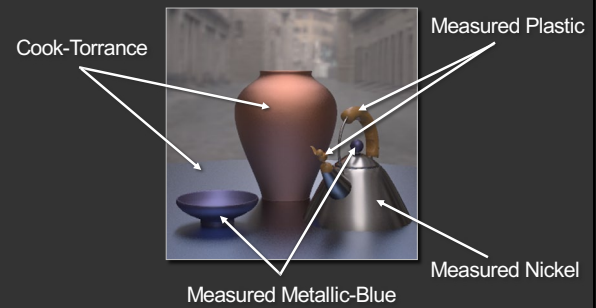
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BRDF Sampling

- Have dealt with BRDF evaluation, need importance sampling and PDF functions for MIS
- In 2004, no good importance sampling schemes for most BRDFs, including common Torrance-Sparrow
- From Lawrence et al. 04, factor BRDF into data-driven terms that can each be importance sampled
- Now some form of light/BRDF sampling common in production (standard in RenderMan 16, 2011-)

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Motivation



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Key Idea

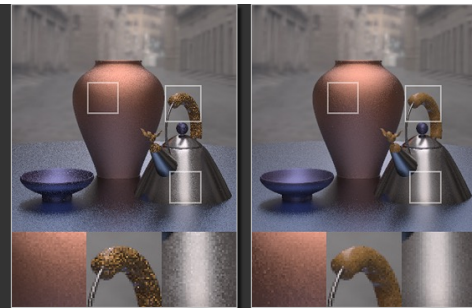
- Project 4D BRDF into sum of products of 2D function dependent on ω_o and 2D function dependent on ω_i :

$$f_r(\omega_o, \omega_i)(n \cdot \omega_i) = \sum_{j=1}^J F_j(\omega_o) G_j(\omega_i)$$

ω_p depends **only** on the incoming direction and some re-parameterization of the hemisphere.

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300 Samples/Pixel



Sampling Lafortune Fit

Our Method

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