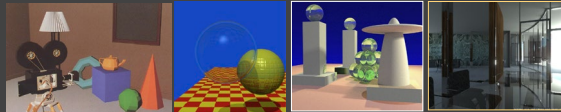


## Computer Graphics II: Rendering

CSE 168[Spr 21], Lecture 9: Importance Sampling  
Ravi Ramamoorthi

<http://viscomp.ucsd.edu/classes/cse168/sp21>



## To Do

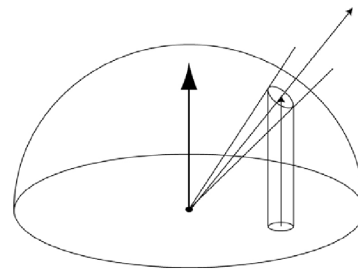
- Start working on homework 3. Ask me if problems
- Also homework 4. This lecture covers material (Lecture is designed to follow assignment closely)
- Start thinking about final project

## Importance Sampling

- Talked about in Monte Carlo Path Tracing
- This assignment: implement at each bounce
- Use “good” pdf for sampling instead of uniform
- Extension to Multiple Importance Sampling (Veach 95)
  - Allows considering both lighting and BRDF sampling
  - Key development in production rendering (Academy Award)
  - Remains active topic of research (many papers in 2019)

## Sampling Projected Solid Angle

Generate cosine weighted distribution



CS348B Lecture 6

Pat Hanrahan, Spring 2004

## Cosine Importance Sampling

- Include cosine term in PDF (for indirect lighting)
- Previously, uniformly integrate over hemisphere  $pdf(\omega_i) = \frac{1}{2\pi}$

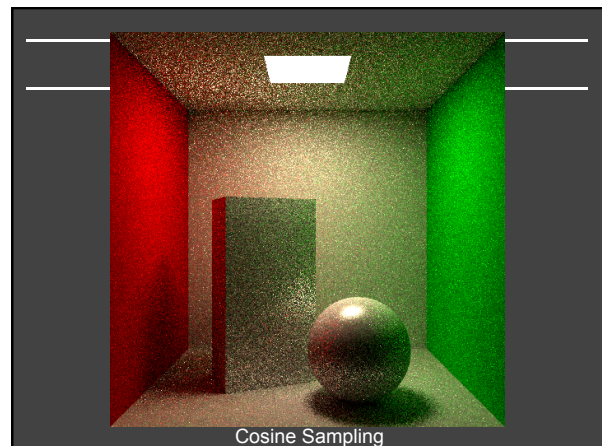
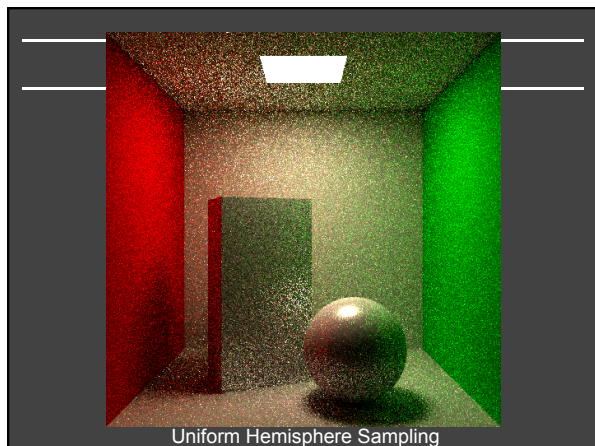
$$\frac{1}{N} \sum_{i=1}^N \frac{L(\omega_{i,k}) f(\omega_{i,k}, \omega_o)(n \cdot \omega_{i,k})}{pdf(\omega_i)} = \frac{2\pi}{N} \sum_{i=1}^N L(\omega_{i,k}) f(\omega_{i,k}, \omega_o)(n \cdot \omega_{i,k})$$

- Now, consider a cosine PDF  $pdf(\omega_i) = \frac{n \cdot \omega_i}{\pi}$

$$\frac{1}{N} \sum_{i=1}^N \frac{L(\omega_{i,k}) f(\omega_{i,k}, \omega_o)(n \cdot \omega_{i,k})}{pdf(\omega_i)} = \frac{\pi}{N} \sum_{i=1}^N \frac{L(\omega_{i,k}) f(\omega_{i,k}, \omega_o)(n \cdot \omega_{i,k})}{n \cdot \omega_{i,k}} = \frac{\pi}{N} \sum_{i=1}^N L(\omega_{i,k}) f(\omega_{i,k}, \omega_o)$$

## Cosine Sampling Upper Hemisphere

- Inversion method
  - In polar coords, density must be proportional to  $\cos \theta \sin \theta$  (remember  $d(\text{solid angle}) = \sin \theta d\theta d\phi$ )
  - Integrate, invert  $\rightarrow \cos^{-1}(\text{sqrt}(\dots))$
- Recipe is (start with two random numbers  $\xi_1, \xi_2$  in  $0 \dots 1$ )
  - Generate  $\phi$  in  $0 \dots 2\pi$   $\phi = 2\pi\xi_2$
  - Generate  $z$  in  $0 \dots 1$   $z = \text{sqrt}(\xi_1)$  // Note extra sqrt wrt uniform
  - Let  $\theta = \cos^{-1} z$   $\theta = \text{acos}(\text{sqrt}(\xi_1))$
  - $(x, y, z) = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$
- Rotate according to surface normal (z goes to normal)
  - Or create coordinate frame (as you did for uniform sampling)
- Modify indirect lighting estimator (remove  $n \cdot \omega_i$ ) and replace  $2\pi$  with  $\pi$  (indirect lighting, Russian Roulette)



### Specular BRDFs

- Cosine importance sampling works well for near-Lambertian BRDFs (modest improvement)
- But more sophisticated sampling for specular BRDFs
- Will talk about general BRDFs next lecture
- For now, for assignment: Modified Phong, GGX
- Sampling BRDFs in general is non-trivial
  - Can simply normalize to get PDF, but sampling non-trivial
  - For now, sample a simpler BRDF, then divide by PDF
  - (This procedure is always guaranteed to work)

### BRDF Importance Sampling

- Phong BRDF:  $f_r \sim \cos^s \beta$  where  $\beta$  is angle between outgoing ray and ideal mirror direction
- Constant scale =  $k_s(s+2)/(2\pi)$
- Can't sample this times  $\cos \theta_i$ 
  - Can only sample BRDF itself, then multiply by  $\cos \theta_i$
  - That's OK – still better than random sampling

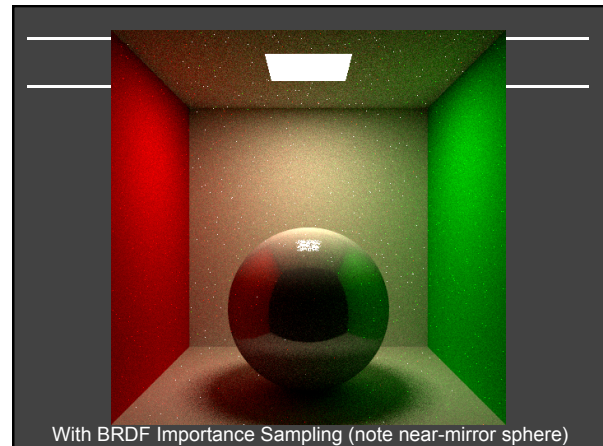
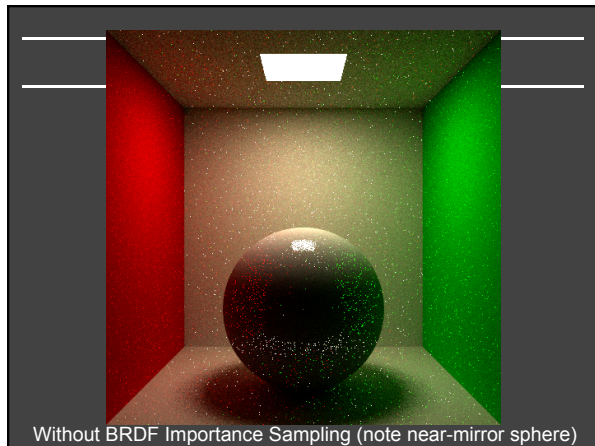
### BRDF Importance Sampling

- Recipe for sampling specular term:
  - Generate  $z$  in  $0..1$
  - Let  $\gamma = \cos^{-1}(z^{1/(s+1)})$
  - Generate  $\phi_\gamma$  in  $0..2\pi$
  - This gives direction w.r.t. ideal mirror direction
- Convert to  $(x,y,z)$ , then rotate such that  $z$  points along mirror dir.

### Formal Modified Phong Sampling

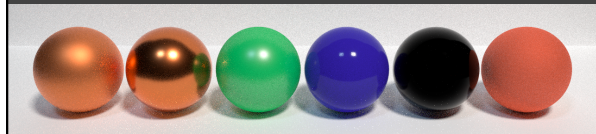
- Multiply by cosine, transport function (note colors)
- Modified Phong is an approximation  $t = \frac{\bar{k}_s}{k_d + \bar{k}_s}$ 

$$(1-t) \frac{n \cdot \omega_i}{\pi} + t \frac{s+1}{2\pi} (r \cdot \omega_i)^s$$
- Generate 3 random numbers:  $\xi_0, \xi_1, \xi_2$  in  $0..1$
- Use  $\xi_0$  to decide diffuse ( $>t$ ) or specular ( $\leq t$ )
- Generate  $\phi$  in  $0..2\pi$   $\phi = 2\pi\xi_2$
- If diffuse  $\theta = \text{acos}(\text{sqrt}(\xi_1))$  [coord. frame normal  $n$ ]
- If specular  $\theta = \text{acos}(\xi_1^{1/(s+1)})$  [coord. frame refl  $r$ ]
- Compute BRDF / PDF (if below visible, BRDF = 0)



## GGX Microfacet Model

- Physically-Based Reflectance Model
- Widely used in practice
- Will discuss BRDFs in more detail next time
- Brief review here, see assignment for details



## Experiment

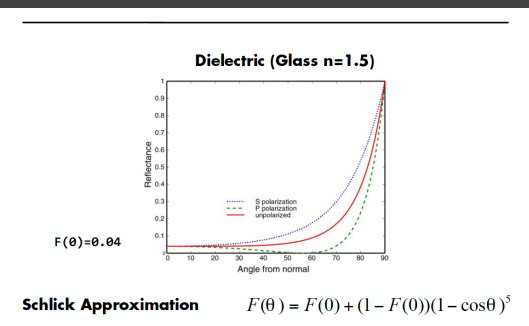
### Reflections from a shiny floor



From Lafortune, Foo, Torrance, Greenberg, SIGGRAPH 97

Reflection is greater at glancing angles

## Fresnel Reflectance



## (Cook-)Torrance-Sparrow

- Assume the surface is made up of grooves at the microscopic level. (General *Microfacet Theory*)
- Assume the faces of these grooves (called microfacets) are perfect reflectors.
- Take into account 3 phenomena



Shadowing Masking Interreflection

### (Cook-)Torrance-Sparrow

Fresnel term:  
allows for  
wavelength  
dependency

Geometric Attenuation:  
reduces the output based on the  
amount of shadowing or masking  
that occurs.

$$f = \frac{F(\theta_i)G(\omega_i, \omega_r)D(\theta_h)}{4\cos(\theta_i)\cos(\theta_r)}$$

How much of the  
macroscopic  
surface is visible  
to the light source

How much of  
the macroscopic  
surface is visible  
to the viewer

Distribution:  
distribution function  
determines what  
percentage of  
microfacets are  
oriented to reflect  
in the viewer  
direction.

### GGX Microfacet Model

- Specular term (see assignment for G, F)
 
$$f(\omega_i, \omega_r) = \frac{k_s}{\pi} + f_{\text{GGX}}(\omega_i, \omega_r)$$

$$f_{\text{GGX}}(\omega_i, \omega_r) = \frac{F(\omega_i, \mathbf{h}, k_s)G(\omega_i, \omega_r)D(\mathbf{h})}{4(\omega_i \cdot \mathbf{n})(\omega_r \cdot \mathbf{n})}$$

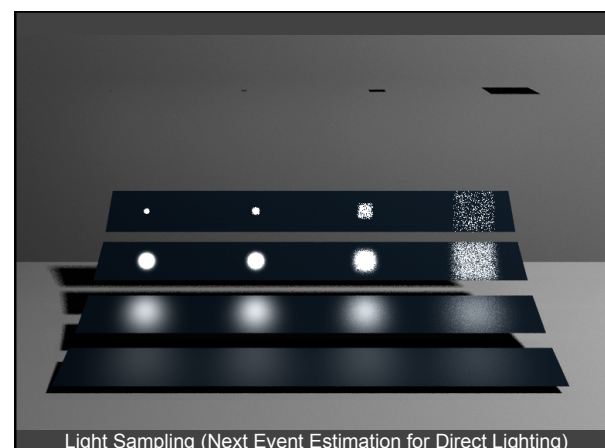
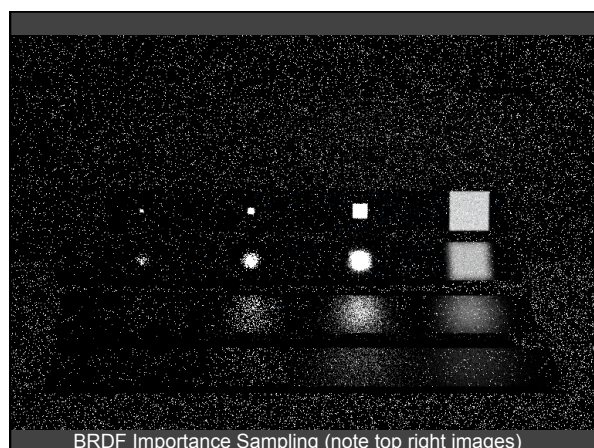
$$D(\mathbf{h}) = \frac{\alpha^2}{\pi \cos^4 \theta_h (\alpha^2 + \tan^2 \theta_h)^2}$$
- Importance Sampling PDF (includes cosine term)
  - Neglects F and G terms, must do BRDF / PDF
$$pdf(\omega_i | \omega_o) = (1-t) \frac{\mathbf{n} \cdot \omega_i}{\pi} + t \frac{D(\mathbf{h})(\mathbf{n} \cdot \mathbf{h})}{4(\omega_i \cdot \mathbf{h})}$$
  - Note that t is clamped at a min of 0.25 to give some specular samples even for low  $k_s$  (because of Fresnel)

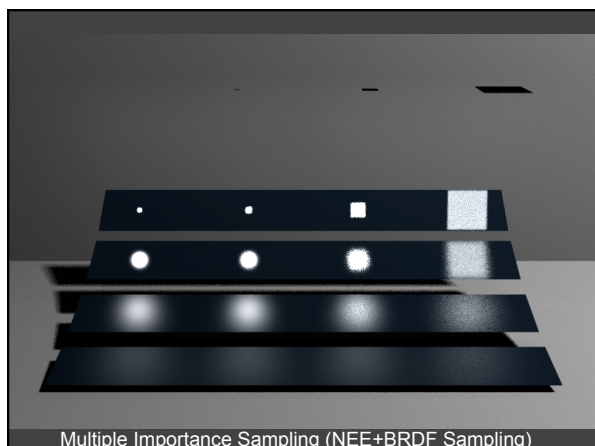
### Importance Sampling GGX

- High-Level idea similar to modified Phong
- Generate 3 random numbers:  $\xi_0, \xi_1, \xi_2$  in  $0 \dots 1$
- Use  $\xi_0$  to decide diffuse ( $>t$ ) or specular ( $\leq t$ )
- Generate  $\phi$  in  $0 \dots 2\pi$   $\phi = 2\pi\xi_2$  (if specular, this is  $\phi_h$ )
- If diffuse  $\theta = \text{acos}(\text{sqrt}(\xi_1))$  [coord. frame normal  $\mathbf{n}$ ]
- If specular**  $\theta_h = \arctan\left(\frac{\alpha\sqrt{\xi_1}}{\sqrt{1-\xi_1}}\right)$  [coord. frame halfvector  $\mathbf{h}$ ]
  - Must compute incident direction from outgoing, half-vector
  - Rotate  $\mathbf{h}$  about normal, reflect outgoing about half-vector
- Compute BRDF / PDF (if below visible, BRDF = 0)

### Multiple Importance Sampling

- Veach 95 classic scene (4 lights, 4 glossiness)
- BRDF importance sampling only (no NEE, so no explicit direct lighting or light sampling pass)
  - Mostly noisy but sharper reflections handled well
- Compare with light sampling (NEE)
  - Mostly better but noisy for sharp reflections
- Can we combine BRDF, Light(NEE) sampling?
  - MIS (Veach95) provides a way, bounds
  - Very robust, works well shiny/rough etc.
  - Key development in production rendering
  - Remains topic of interest (many papers in 2019)





## Multiple Importance Sampling

- MIS relies on NEE almost everywhere, but relies on BRDF importance sampling when needed
- Multi-sample: sample both distributions at each intersection
- General case: N sampling techniques (inner summation is unbiased estimator each technique)

$$\int_x f(x) dx \approx \sum_{i=1}^N \frac{1}{N_i} \sum_{j=1}^{N_i} w_i(x_j) \frac{f(x_j)}{pdf_i(x_j)}$$

- Weights must sum to 1, unbiased

## Multiple Importance Sampling

- General case: N sampling techniques (inner summation is unbiased estimator each technique)

$$\int_x f(x) dx \approx \sum_{i=1}^N \frac{1}{N_i} \sum_{j=1}^{N_i} w_i(x_j) \frac{f(x_j)}{pdf_i(x_j)}$$

- Weights must sum to 1, unbiased
  - Interesting theory (ongoing, papers in 2019)
  - Veach and Guibas 95 proposed balance, power heuristics (provably "good" under certain assumptions)
  - We use power heuristic with  $\beta = 2$
  - Subtle point: PDF must be able to be evaluated anywhere (not just own samples)
- Natural abstract interface for sampling and MIS
  - Eval(), Sample(), PDF() [sometimes Value() = Eval/PDF]

$$w_i(\omega) = \frac{pdf_i^\beta(\omega)}{\sum_{k=1}^N pdf_k^\beta(\omega)}$$

## Lighting/BRDF Sampling

- For now, 1 sample on light (NEE), 1 from BRDF
  - We already know BRDF PDF
  - Light PDF implicitly on light, convert to angle  $d\omega = dA \frac{\cos \theta}{R^2}$

$$pdf_{light}(\omega) = \frac{R^2}{(\mathbf{n}_{light} \cdot \omega)A}$$

- For multiple lights, simple normalization (see homework)
- Combine NEE and BRDF sampling (power heuristic)

$$w_i(\omega) = \frac{pdf_i^\beta(\omega)}{\sum_{k=1}^N pdf_k^\beta(\omega)}$$

## MIS Implementation

- Can be tricky, see assignment
- First disable NEE, BRDF sampling for direct
  - Separate NEE function, toggle light/BRDF sampling
- Now implement pdf(nee)
  - Beware divide by zero, see assignment for specifics
- Implement weight function
  - Visualize weighted lighting, weighted BRDF
  - Then combine them with MIS, enable both techniques
- See assignment carefully
  - MIS for direct lighting only (Veach scene no indirect)
  - Note gamma correction for this assignment

## MIS weights

