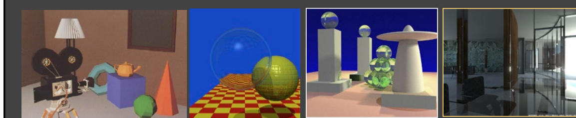
Computer Graphics II: Rendering

CSE 168 [Spr 21], Lecture 6: Direct Lighting Details
Ravi Ramamoorthi

http://viscomp.ucsd.edu/classes/cse168/sp21



To Do

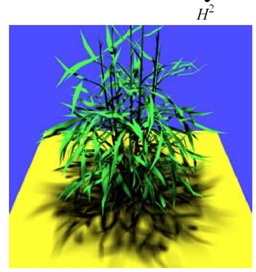
- Homework 2 (Direct Lighting) due Apr 22
- Assignment is on UCSD Online
- START EARLY
- This lecture goes through details of direct lighting, visibility, Monte Carlo for the assignment
- Ask re any questions

Area Lights

- Physically correct (physically no perfect point light)
- Can talk in terms of radiance at surface
- Enable soft shadows, partial visibility/occlusion
- Shoot multiple rays to light source, average
- Need proper rendering equation for accurate result
  - Only direct lighting here, so reflection equation suffices

Example: Soft Shadows

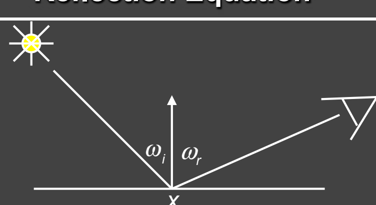
$$E(x) = \int_{H^2} L_i(x,\omega)\cos\theta\, d\omega$$



Challenges
- **Visibility and blockers**
- **Varying light distribution**
- **Complex source geometry**

Source: Agrawala. Ramamoorthi, Heirich, Moll, 2000

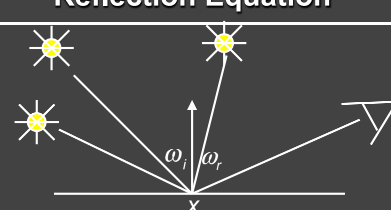
Reflection Equation



$$L_r(x,\omega_r) = L_e(x,\omega_r) + L_i(x,\omega_i) f(x,\omega_i,\omega_r)(\omega_i \bullet n)$$

Reflected Light (Output Image) | Emission | Incident Light (from light source) | BRDF | Cosine of Incident angle

Reflection Equation

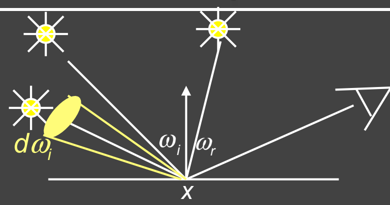


Sum over all light sources

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \sum L_i(x,\omega_i) f(x,\omega_i,\omega_r)(\omega_i \bullet n)$$

Reflected Light (Output Image) | Emission | Incident Light (from light source) | BRDF | Cosine of Incident angle

Reflection Equation

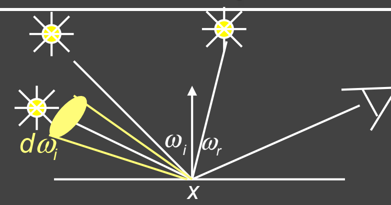


Replace sum with integral

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_{\Omega} L_i(x,\omega_i) f(x,\omega_i,\omega_r) \cos\theta_i \, d\omega_i$$

Reflected Light (Output Image) | Emission | Incident Light (from light source) | BRDF | Cosine of Incident angle

Reflection Equation



As written in assignment (for one pixel)

$$L_r(\omega_o) = L_e(\omega_o) + \int_{\Omega} L_i(\omega_i) f(\omega_i,\omega_o) \max(n \bullet \omega_i, 0) d\omega_i$$

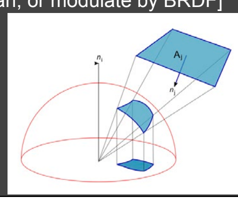
Reflected Light (Outgoing Dirn. Drop spat. Coord.) | Emission | Incident Light (from area light) | BRDF | Cosine of Incident angle (max not explicit hw)

Uniform Polygonal Area Source

- Integrate projection of polygon P onto sphere
- Assume uniform radiance incident from light L_i
- Assume no emission (and no occlusion for now)
- Nusselt's analog (wikipedia image) project to (unit hemi)sphere, then circle (unit disk at equator) for cosine term [but only valid for Lambertian, or modulate by BRDF]

$$L_d(\omega_o) = \int_{\Omega_P} L_i \, f(\omega_i,\omega_o)\,(n \bullet \omega_i)\, d\omega_i$$

$$= \frac{k_d}{\pi} L_i \int_{\Omega_P} (n \bullet \omega_i)\, d\omega_i \quad [Lambertian]$$



Analytic Irradiance

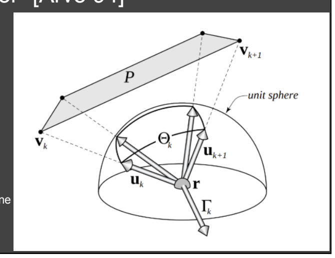
- Analytic irradiance for a polygon [Baum89, Arvo94-95]
- Useful to get precise answer, check Monte Carlo
- Define "irradiance vector" [Arvo 94]

$$\Phi(\mathbf{r}) = \frac{1}{2}\sum_{i=1}^{n} \Theta_i(\mathbf{r}) \Gamma_i(\mathbf{r})$$

where $\Theta_1, \Theta_2, \ldots \Theta_n$ are the angles subtended by the n edges of the polygon as seen from $\mathbf{r}$

$\Theta_k = \angle \mathbf{v}_k \mathbf{r} \mathbf{v}_{k+1}$ where $\mathbf{v}_k$ are vertices of polygon, in cyclic order ($n+1 = 1$)

$\boldsymbol{\Gamma}_1, \boldsymbol{\Gamma}_2, \ldots \boldsymbol{\Gamma}_n$ are the unit normals of the polygonal cone with cross section P and apex at $\mathbf{r}$



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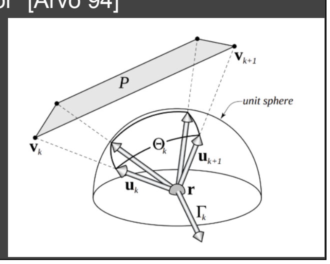
$$\Theta_k(\mathbf{r}) = \cos^{-1}\left(\frac{\mathbf{v}_k - \mathbf{r}}{\|\mathbf{v}_k - \mathbf{r}\|} \bullet \frac{\mathbf{v}_{k+1} - \mathbf{r}}{\|\mathbf{v}_{k+1} - \mathbf{r}\|} \right)$$

$$\boldsymbol{\Gamma}_k(\mathbf{r}) = \left(\frac{(\mathbf{v}_k - \mathbf{r}) \times (\mathbf{v}_{k+1} - \mathbf{r})}{\|(\mathbf{v}_k - \mathbf{r}) \times (\mathbf{v}_{k+1} - \mathbf{r})\|} \right)$$



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$$\Phi(\mathbf{r}) = \frac{1}{2}\sum_{i=1}^{n} \Theta_i(\mathbf{r}) \Gamma_i(\mathbf{r})$$

$$E = L_i \left(\Phi(\mathbf{r}) \bullet \mathbf{n}(\mathbf{r}) \right) \quad [Irradiance]$$

$$L_d(\omega_o) = f\,E = \frac{k_d}{\pi} E = \frac{k_d}{\pi} L_i \left(\Phi(\mathbf{r}) \bullet \mathbf{n}(\mathbf{r}) \right)$$

$$\Theta_k(\mathbf{r}) = \cos^{-1}\left(\frac{\mathbf{v}_k - \mathbf{r}}{\|\mathbf{v}_k - \mathbf{r}\|} \bullet \frac{\mathbf{v}_{k+1} - \mathbf{r}}{\|\mathbf{v}_{k+1} - \mathbf{r}\|} \right)$$

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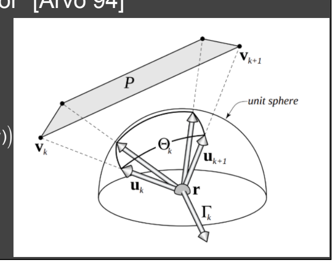
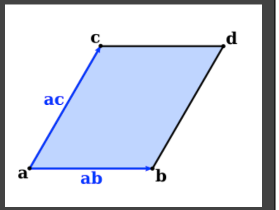
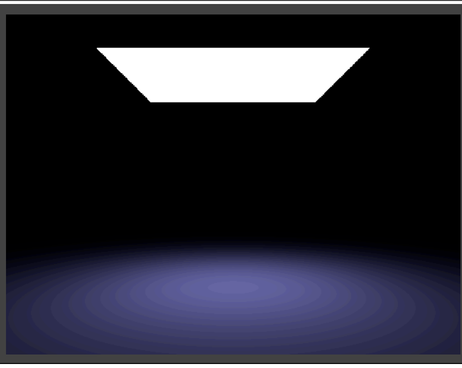


Implementing Analytic Irradiance

- New parallelogram light quadLight <a> <ab> <ac> <radiance>
- quadLight 0 0 0 1 0 0 0 0 1 1 1 1
- Primary rays like ray tracer
- Use analytic formula for shading
- Add contribs. from all lights
- No emission, visibility (yet)



Example (No Noise)



General Direct Lighting

- Include (partial) visibility by tracing multiple rays

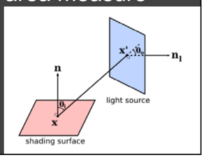
$$L_d(\omega_o) = L_i \int_{\Omega_P} f(\omega_i, \omega_o)\,(n \cdot \omega_i)\,V(\omega_i)\,d\omega_i$$

- Include non-Lambertian reflectance (Phong)

$$f(\omega_i, \omega_o) = \frac{k_d}{\pi} + k_s \frac{(s+2)}{2\pi} \max(\mathbf{r} \bullet \omega_\iota, 0)^s$$

- Integrate over area light, change area measure

$$d\omega_i = \frac{\mathbf{n}_l \bullet \omega_i}{R^2} dA$$



General Direct Lighting

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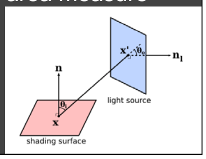
- Integrate over area light, change area measure

$$d\omega_i = \frac{\mathbf{n}_l \bullet \omega_i}{R^2} dA$$

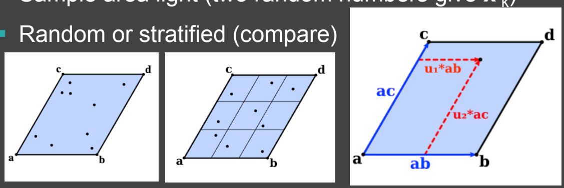
$$L_d(\mathbf{x}, \omega_o) = L_i \int_P f(\mathbf{x}, \omega_i, \omega_o)\,G(\mathbf{x}, \mathbf{x'})\,V(\mathbf{x}, \mathbf{x'})\,dA(\mathbf{x'})$$

$$G(\mathbf{x}, \mathbf{x'}) = \frac{(\mathbf{n} \cdot \omega_i)(\mathbf{n}_l \bullet \omega_i)}{R^2} = \frac{\cos\theta_i \cos\theta_o}{|\mathbf{x} - \mathbf{x'}|^2}$$



Monte Carlo Solution

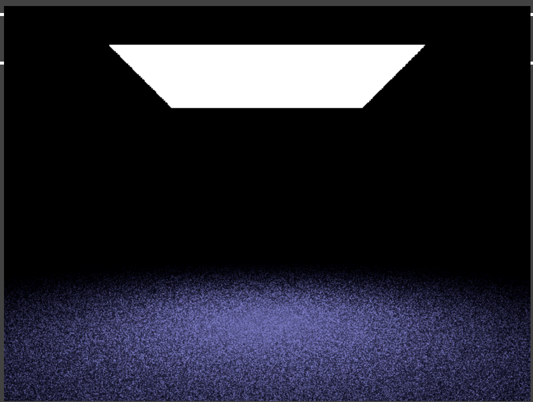
- Sample area light (two random numbers give $\mathbf{x'}_k$)
- Random or stratified (compare)



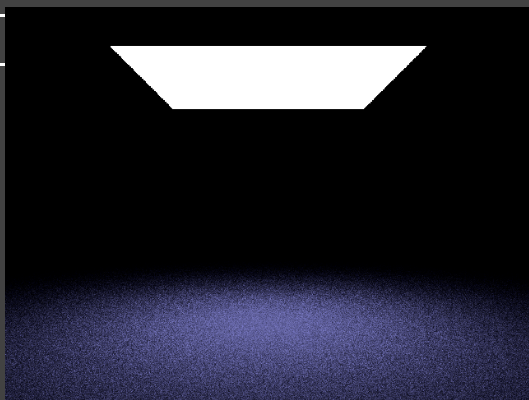
- Monte Carlo evaluation of direct lighting (N samples)

$$L_d(\mathbf{x}, \omega_o) = L_i \int_P f(\mathbf{x}, \omega_i, \omega_o)\,G(\mathbf{x}, \mathbf{x'})\,V(\mathbf{x}, \mathbf{x'})\,dA(\mathbf{x'})$$

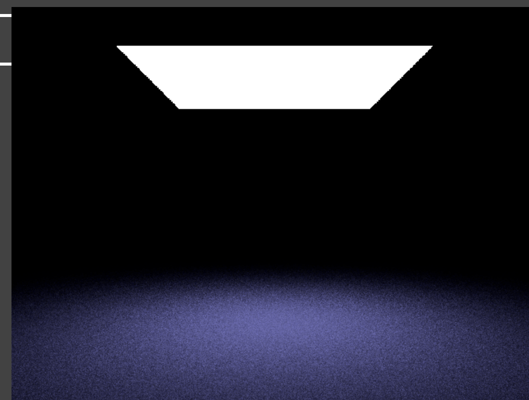
$$L_d(\mathbf{x}, \omega_o) \approx L_i \frac{A}{N} \sum_{k=1}^{N} f(\mathbf{x}, \omega_i(k), \omega_o)\,G(\mathbf{x}, \mathbf{x'}_k)\,V(\mathbf{x}, \mathbf{x'}_k)$$



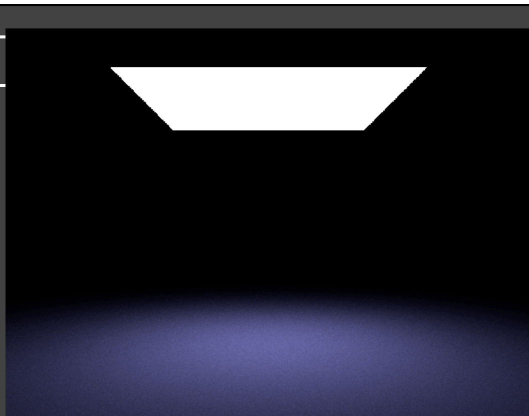
Number of Samples 1 (Random)



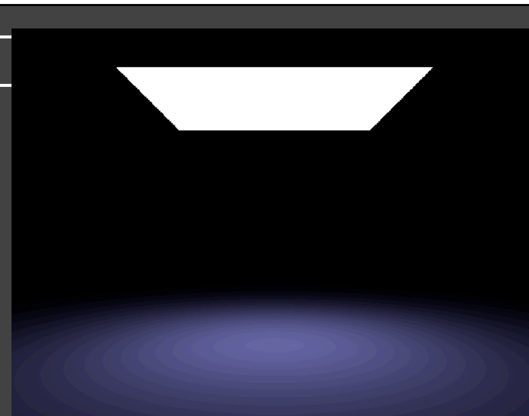
Number of Samples 3 (Random)



Number of Samples 9 (Random)



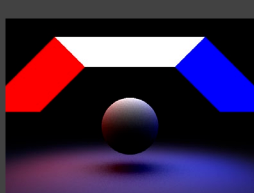
Number of Samples 9 (Stratified)

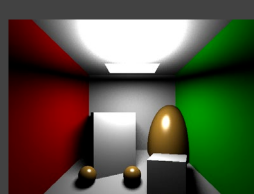


Analytic

More Complex Scenes

- For assignment, render with 5x5 stratified
- Direct lighting only, turn off recursive raytracing etc





Submission (bias, variance)

- Monte Carlo introduces noise
  - So auto-feedback can't just pixel difference
- Bias is on average how far from (analytic?) result
  - Monte Carlo rendering is an unbiased (bias 0) technique
  - High bias indicates an error in the program
  - Analytic should be exact (no bias or variance)
- Variance is squared pixel error (RMS = stdev)
  - All Monte Carlo results will have some variance
  - May differ in variance from our solution
  - But our bounds are reasonable (too far off = issues?)
- Focus on being correct, not just pretty
  - Can submit to autofeedback as many times as you want
  - Scores only suggestive, will grade manually

Submission (bias, variance)

- Monte Carlo introduces noise
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- Bias is on average how far from result (close to 0)
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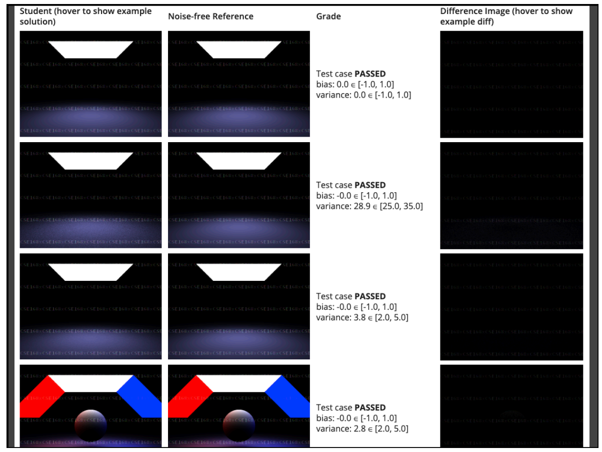

Extra: Reducing Variance

- Increase bias slightly by evaluating everything except visibility (at center [of stratum], or analytic)

$$L_d(\mathbf{x},\omega_o) \approx L_i \frac{A}{N}\sum_{k=1}^{N} f(\mathbf{x},\omega_i(k),\omega_o)\, G(\mathbf{x},\mathbf{x'}_k)\, V(\mathbf{x},\mathbf{x'}_k)$$

$$L_d(\mathbf{x},\omega_o) \approx L_i \frac{A}{N}\sum_{k=1}^{N} f(\mathbf{x},\omega_i(\mathbf{c_k}),\omega_o)\, G(\mathbf{x},\mathbf{c_k})\, V(\mathbf{x},\mathbf{x'}_k)$$

$\mathbf{c_k}$ is center of stratum, no noise (not Monte Carlo)

- Noise now only from visibility, none otherwise
  - No noise if no occlusions/shadows
  - Exact if use analytic formula (to evaluate unshadowed irradiance from each strata) instead of stratum center

Extra: Point Lights

- Point lights not physical (but widely used)
- How to include in Monte Carlo Rendering?
- Units aren't even same (can't use radiance)
  - Power per solid angle, not power per unit area per solid angle
- Think of as delta functions (area goes to 0, total radiance integrated over light I = L_i A remains same)
  - No cosine term for light, inverse square falloff, no integral

$$L_d(\omega_o) = L_i \int_{\Omega_p} f(\omega_i,\omega_o)\,(n\bullet\omega_i)\,V(\omega_i)\,d\omega_i$$

$$L_d(\omega_o) = \frac{I}{R^2} f(\omega_i,\omega_o)\,(n\bullet\omega_i)\,V(\omega_i)$$

Extra: Directional Lights

- First, assume small angular extent (even sun subtends an angle of 0.5 degrees) $L_d(\omega_o) = L_i \int_{\Omega_p} f(\omega_i,\omega_o)\,(n\bullet\omega_i)\,V(\omega_i)\,d\omega_i$

$$L_d(\omega_o) \approx L_i \frac{\Delta\omega}{N}\sum_{k=1}^{N} f(\omega_i(k),\omega_o)(n\bullet\omega_i)V(\omega_i(k))$$

- If pure directional light (delta function), then fix product $E = L_i \Delta\omega$ (same units as irradiance)

$$L_d(\omega_o) = Ef(\omega_i,\omega_o)\,(n\bullet\omega_i)\,V(\omega_i)$$

Combining Point, Directional, Area

- Small angle, area expressed in radiance L_i
  - Use physical units $Wm^{-2}\ sr^{-1}$
- Point is in intensity Wsr^{-1}
  - This is not in same units (e.g. 0…1) as area lights
  - Need to normalize by multiplying by an area unit
  - Point light same intensity as area light: multiply by area
- Directional is in irradiance units Wm^{-2}
  - This is not in same units (e.g. 0…1) as area/point lights
  - Need to normalize by multiplying by solid angle
  - Directional light same intensity as area light: multiply by average solid angle subtended by area light
  - Directional same intensity as point: divide by average squared distance to point light, mult by 2 for avg cosine