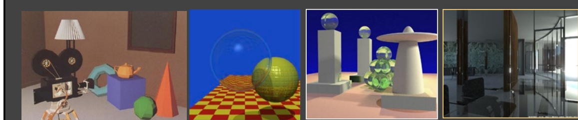
Computer Graphics II: Rendering

CSE 168 [Spr 21], Lecture 3: Illumination and Reflection
Ravi Ramamoorthi

http://viscomp.ucsd.edu/classes/cse168/sp21



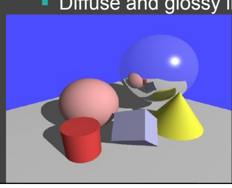
To Do

- Homework 1 (ray tracer) due early next week
- Next assignment direct lighting (on UCSD online). Will cover that material next week

Illumination Models

Local Illumination
- Light directly from light sources to surface
- No shadows (cast shadows are a global effect)

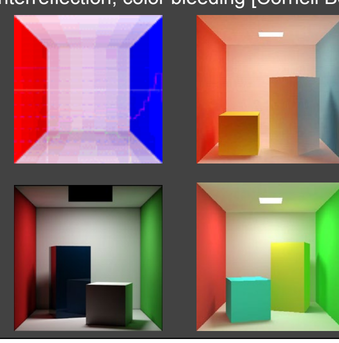
Global Illumination: multiple bounces (indirect light)
- Hard and soft shadows
- Reflections/refractions (already seen in ray tracing)
- Diffuse and glossy interreflections (radiosity, caustics)



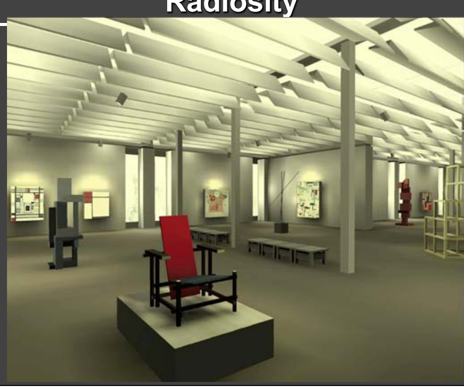
Some images courtesy Henrik Wann Jensen

Diffuse Interreflection

Diffuse interreflection, color bleeding [Cornell Box]

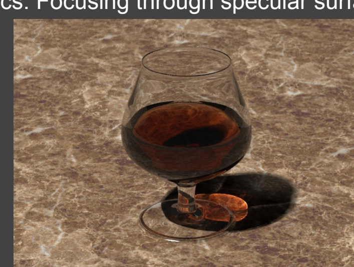


Radiosity



Caustics

Caustics: Focusing through specular surface



- Major research effort in 80s, 90s till today

Motivation: BRDFs, Radiometry

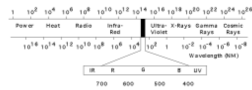
- Basics of Illumination, Reflection
- Formal radiometric analysis (not ad-hoc)
- Reflection Equation (Local Illumination)
- Discussion of BRDFs
- Rendering Equation (Global Illumination) on Thu

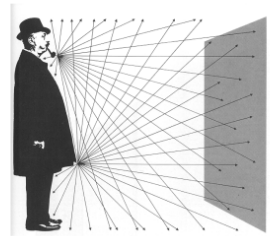
- Formal analysis important for correct implementation

Light

Visible electromagnetic radiation

Power spectrum





Polarization

Photon (quantum effects)

Wave (interference, diffraction)

From London and Upton

Radiometry

- Physical measurement of electromagnetic energy
- *Measure spatial (and angular) properties of light*
  - *Radiance, Irradiance*
  - Reflection functions: Bi-Directional Reflectance Distribution Function or BRDF
  - Reflection Equation
  - Simple BRDF models

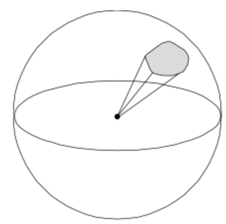
Angles and Solid Angles

- **Angle** $\theta = \frac{l}{r}$

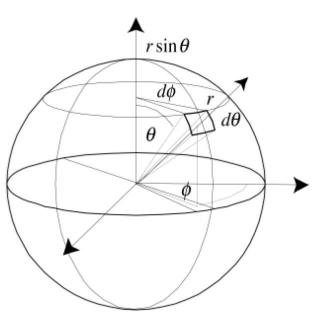
  $\Rightarrow$ **circle has 2π radians**

- **Solid angle** $\Omega = \frac{A}{R^2}$

  $\Rightarrow$ **sphere has 4π steradians**

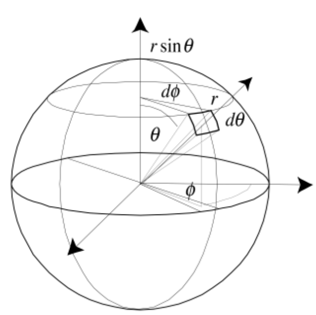


Differential Solid Angles



$$dA = (r\,d\theta)(r\sin\theta\,d\phi) = r^2 \sin\theta\,d\theta\,d\phi$$

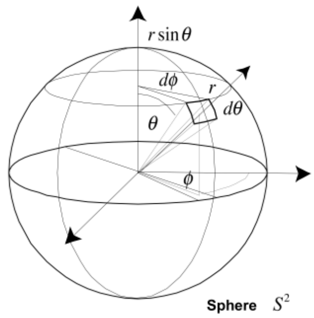
Differential Solid Angles



$$dA = (r\,d\theta)(r\sin\theta\,d\phi) = r^2 \sin\theta\,d\theta\,d\phi$$

$$d\omega = \frac{dA}{r^2} = \sin\theta\,d\theta\,d\phi$$

Differential Solid Angles



$$d\omega = \sin\theta\, d\theta\, d\phi$$

$$\Omega = \int_{S^2} d\omega = \int_0^{\pi}\int_0^{2\pi} \sin\theta\, d\theta\, d\phi = \int_{-1}^{1}\int_0^{2\pi} d\cos\theta\ d\phi = 4\pi$$

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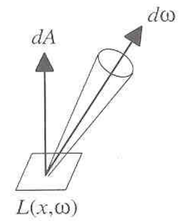
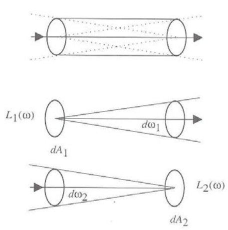
Radiance

- Power per unit projected area perpendicular to the ray per unit solid angle in the direction of the ray
- Symbol: $L(x,\omega)$ (W/m^2 sr)
- Flux given by
  $d\Phi = L(x,\omega) \cos\theta\, d\omega\, dA$



Radiance properties

- Radiance constant as propagates along ray
  - Derived from conservation of flux
  - Fundamental in Light Transport.



$$d\Phi_1 = L_1 d\omega_1 dA_1 = L_2 d\omega_2 dA_2 = d\Phi_2$$

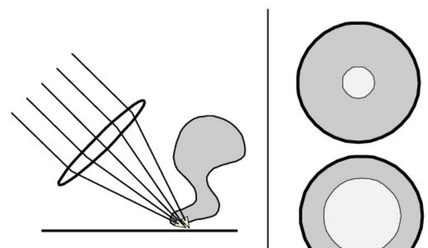
$$d\omega_1 = dA_2 / r^2 \qquad d\omega_2 = dA_1 / r^2$$

$$d\omega_1 dA_1 = \frac{dA_1 dA_2}{r^2} = d\omega_2 dA_2$$

$$\therefore L_1 = L_2$$

Quiz

Does radiance increase under a magnifying glass?



No!!

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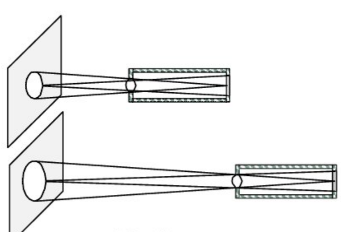
Quiz

Does the brightness that a wall appears to the eye depend on the distance of the viewer to the wall?



No!!

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Radiance properties

- Sensor response proportional to radiance (constant of proportionality is throughput)
  - Far away surface: See more, but subtends smaller angle
  - Wall equally bright across viewing distances

Consequences
  - Radiance associated with rays in a ray tracer
  - Other radiometric quants derived from radiance
  - *This course primarily about computing radiance*

Irradiance, Radiosity

- Irradiance E is radiant power per unit area
- Integrate incoming radiance over hemisphere
  - Projected solid angle (cos θ dω)
  - Uniform illumination: Irradiance = π [CW 24,25]
  - Units: W/m^2
- Radiant Exitance (radiosity)
  - Power per unit area leaving surface (like irradiance)

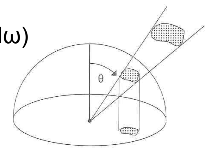
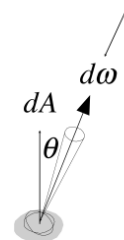


Figure 2.8: *Projection of differential area.*

Directional Power Arriving at a Surface



$L_i(x,\omega)$

$d\omega$

dA

θ

$d^2\Phi_i(x,\omega)$

$$d^2\Phi_i(x,\omega) = L_i(x,\omega)\cos\theta \, dA \, d\omega$$

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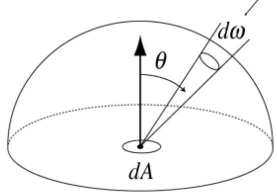
Irradiance from the Environment

$$d^2\Phi_i(x,\omega) = L_i(x,\omega)\cos\theta \, dA \, d\omega$$

$$dE(x,\omega) = L_i(x,\omega)\cos\theta \, d\omega$$

Light meter



$L_i(x,\omega)$, $d\omega$, θ, dA

$$E(x) = \int_{H^2} L_i(x,\omega)\cos\theta \, d\omega$$

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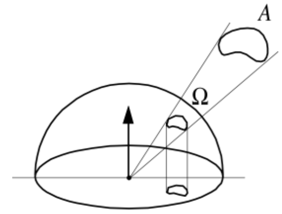
Uniform Area Source

$$E(x) = \int_{H^2} L\cos\theta \, d\omega$$

$$= L\int_{\Omega} \cos\theta \, d\omega$$

$$= L\tilde{\Omega}$$



A, Ω, $\tilde{\Omega}$

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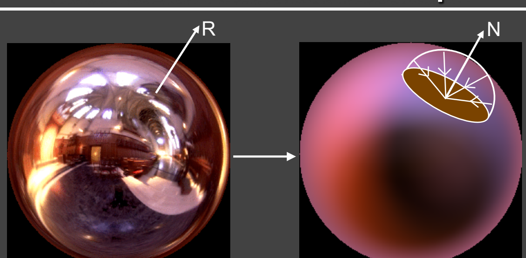
Irradiance Environment Maps



Incident Radiance (Illumination Environment Map) — Irradiance Environment Map

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Radiometry

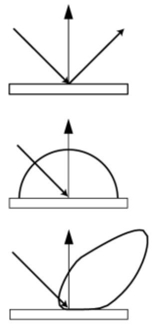
- Physical measurement of electromagnetic energy
- *Measure spatial (and angular) properties of light*
  - Radiance, Irradiance
  - *Reflection functions: Bi-Directional Reflectance Distribution Function or BRDF*
  - *Reflection Equation*
  - Simple BRDF models

Types of Reflection Functions

Ideal Specular
- Reflection Law
- Mirror

Ideal Diffuse
- Lambert's Law
- Matte

Specular
- Glossy
- Directional diffuse



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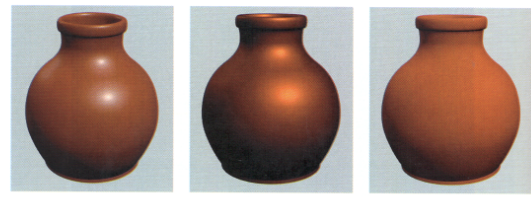
Materials



Plastic | Metal | Matte

From Apodaca and Gritz, *Advanced RenderMan*

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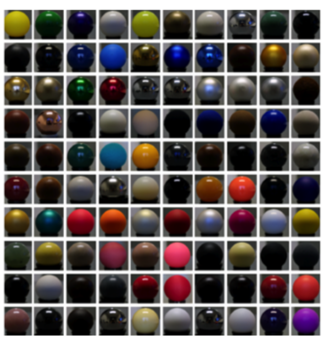
Spheres [Matusik et al.]



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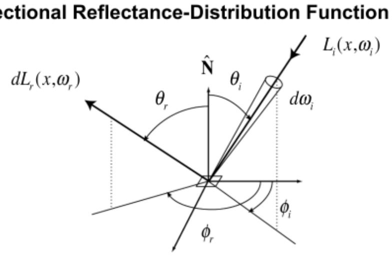
Building up the BRDF

- Bi-Directional Reflectance Distribution Function [Nicodemus 77]
- Function based on incident, view direction
- Relates incoming light energy to outgoing
- Unifying framework for many materials

The BRDF

Bidirectional Reflectance-Distribution Function



$$f_r(\omega_i \to \omega_r) \equiv \frac{dL_r(\omega_i \to \omega_r)}{dE_i} \quad \left[\frac{1}{sr}\right]$$

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BRDF

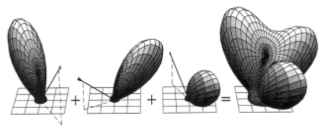
- Reflected Radiance proportional Irradiance
- Constant proportionality: BRDF
- Ratio of outgoing light (radiance) to incoming light (irradiance)
  - Bidirectional Reflection Distribution Function
  - (4 Vars) units 1/sr

$$f(\omega_i, \omega_r) = \frac{L_r(\omega_r)}{L_i(\omega_i)\cos\theta_i d\omega_i}$$

$$L_r(\omega_r) = L_i(\omega_i) f(\omega_i, \omega_r) \cos\theta_i d\omega_i$$

Properties of BRDF's

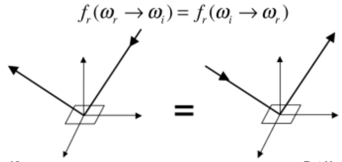
1. Linearity



From Sillion, Arvo, Westin, Greenberg

2. Reciprocity principle

$$f_r(\omega_r \rightarrow \omega_i) = f_r(\omega_i \rightarrow \omega_r)$$



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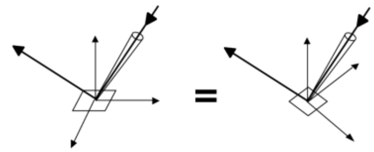
Properties of BRDF's

3. Isotropic vs. anisotropic

$$f_r(\theta_i, \varphi_i; \theta_r, \varphi_r) = f_r(\theta_i, \theta_r, \varphi_r - \varphi_i)$$



Reciprocity *and* isotropy

$$f_r(\theta_i, \theta_r, \varphi_r - \varphi_i) = f_r(\theta_r, \theta_i, \varphi_i - \varphi_r) = f_r(\theta_i, \theta_r, |\varphi_r - \varphi_i|)$$

4. Energy conservation

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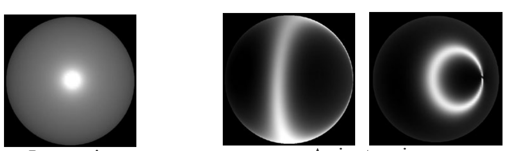
Isotropic vs Anisotropic

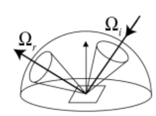
- Isotropic: Most materials (you can rotate about normal without changing reflections)
- Anisotropic: brushed metal etc. preferred tangential direction



Isotropic — Anisotropic

Energy Conservation

$$\frac{d\Phi_r}{d\Phi_i} = \frac{\int_{\Omega_r} L_r(\omega_r)\cos\theta_r \, d\omega_r}{\int_{\Omega_i} L_i(\omega_i)\cos\theta_i \, d\omega_i}$$



$$= \frac{\int_{\Omega_r}\int_{\Omega_i} f_r(\omega_i \rightarrow \omega_r) L_i(\omega_i)\cos\theta_i \, d\omega_i \cos\theta_r \, d\omega_r}{\int_{\Omega_i} L_i(\omega_i)\cos\theta_i \, d\omega_i}$$

$$\leq 1$$

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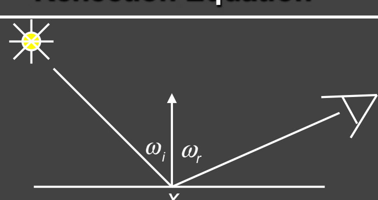
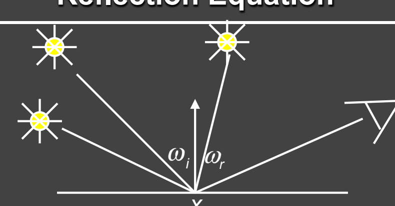
Reflection Equation



$$L_r(x,\omega_r) = L_e(x,\omega_r) + L_i(x,\omega_i) f(x,\omega_i,\omega_r)(\omega_i \bullet n)$$

Reflected Light (Output Image) — Emission — Incident Light (from light source) — BRDF — Cosine of Incident angle

Reflection Equation

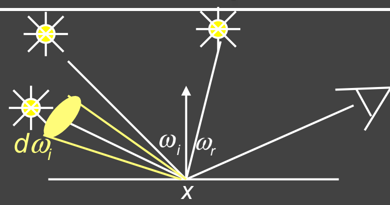


Sum over all light sources

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \sum L_i(x,\omega_i) f(x,\omega_i,\omega_r)(\omega_i \bullet n)$$

Reflected Light (Output Image) — Emission — Incident Light (from light source) — BRDF — Cosine of Incident angle

Reflection Equation



Replace sum with integral

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_\Omega L_i(x,\omega_i) f(x,\omega_i,\omega_r)\cos\theta_i d\omega_i$$

Reflected Light (Output Image) | Emission | Incident Light (from light source) | BRDF | Cosine of Incident angle

Environment Maps

- Light as a function of direction, from entire environment
- Captured by photographing a chrome steel or mirror sphere
- Accurate only for one point, but distant lighting same at other scene locations (typically use only one env. map)

Blinn and Newell 1976, Miller and Hoffman, 1984
Later, Greene 86, Cabral et al. 87

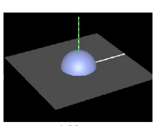
Environment Maps

- Environment maps widely used as lighting representation
- Many modern methods deal with offline and real-time rendering with environment maps
- Image-based complex lighting + complex BRDFs

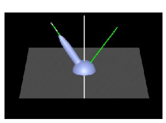
Radiometry

- Physical measurement of electromagnetic energy
- *Measure spatial (and angular) properties of light*
  - Radiance, Irradiance
  - Reflection functions: Bi-Directional Reflectance Distribution Function or BRDF
  - Reflection Equation
  - *Simple BRDF models*

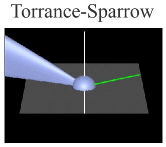
Brdf Viewer plots

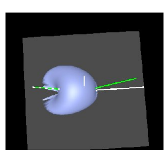


Diffuse



Torrance-Sparrow



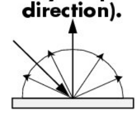


Anisotropic

bv written by Szymon Rusinkiewicz

Ideal Diffuse Reflection

Assume light is equally likely to be reflected in any output direction (independent of input direction).



$$L_{r,d}(\omega_r) = \int f_{r,d} L_i(\omega_i)\cos\theta_i \, d\omega_i$$
$$= f_{r,d}\int L_i(\omega_i)\cos\theta_i \, d\omega_i$$
$$= f_{r,d}E$$

$$M = \int L_r(\omega_r)\cos\theta_r \, d\omega_r = L_r\int\cos\theta_r \, d\omega_r = \pi L_r$$

$$\rho_d = \frac{M}{E} = \frac{\pi L_r}{E} = \frac{\pi f_{r,d}E}{E} = \pi f_{r,d} \quad\Rightarrow\quad f_{r,d} = \frac{\rho_d}{\pi}$$

Lambert's Cosine Law $M = \rho_d E = \rho_d E_s \cos\theta_s$

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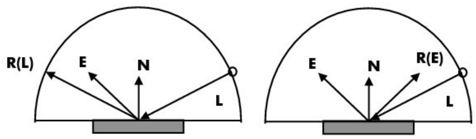
Phong Model



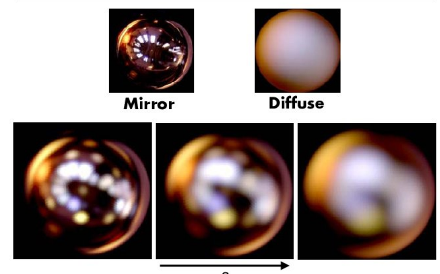
$\left(\hat{\mathbf{E}} \bullet \mathbf{R}(\hat{\mathbf{L}})\right)^s$ $\left(\hat{\mathbf{L}} \bullet \mathbf{R}(\hat{\mathbf{E}})\right)^s$

Reciprocity: $\left(\hat{\mathbf{E}} \bullet \mathbf{R}(\hat{\mathbf{L}})\right)^s = \left(\hat{\mathbf{L}} \bullet \mathbf{R}(\hat{\mathbf{E}})\right)^s$

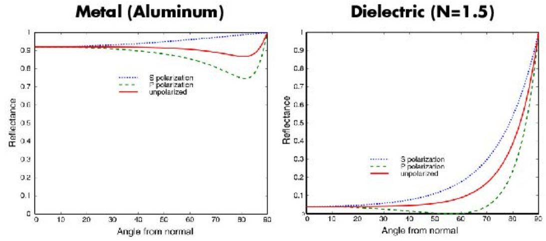
Distributed light source!

Specular Term (Phong)

Phong Model



Fresnel Reflectance



Gold F(0)=0.82
Silver F(0)=0.95

Glass n=1.5 F(0)=0.04
Diamond n=2.4 F(0)=0.15

Schlick Approximation $F(\theta) = F(0) + (1 - F(0))(1 - \cos\theta)^5$

Experiment

Reflections from a shiny floor

From Lafortune, Foo, Torrance, Greenberg, SIGGRAPH 97

Torrance-Sparrow

Fresnel term: allows for wavelength dependency

Geometric Attenuation: reduces the output based on the amount of shadowing or masking that occurs.

$$f = \frac{F(\theta_i)G(\omega_i,\omega_r)D(\theta_h)}{4\cos(\theta_i)\cos(\theta_r)}$$

Distribution: distribution function determines what percentage of microfacets are oriented to reflect in the viewer direction.

How much of the macroscopic surface is visible to the light source

How much of the macroscopic surface is visible to the viewer

Other BRDF models

- Empirical: Measure and build a 4D table
- Anisotropic models for hair, brushed steel
- Cartoon shaders, funky BRDFs
- Capturing spatial variation
- Very active area of research