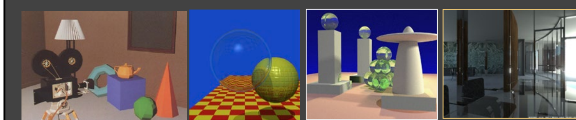
Computer Graphics II: Rendering

CSE 168[Spr 21],Lecture 15: Volumetric Rendering
Ravi Ramamoorthi

http://viscomp.ucsd.edu/classes/cse168/sp21

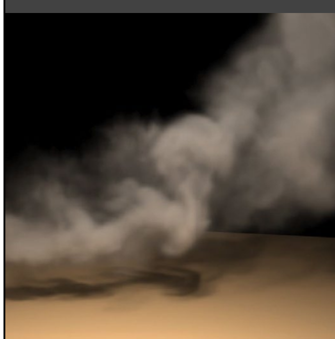


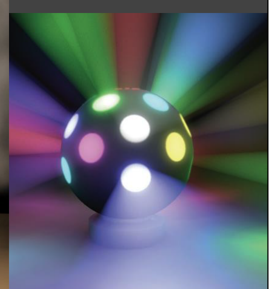
To Do

- Start working on final projects (initial results and proposal due in a week). Ask me if problems
- Volumetric rendering (this lecture) may be one component of the final project (but hard, be careful)
- Increasingly accurate appearance requires volumetric scattering (even for skin, hair, fur)
- Continues to be an active area of research

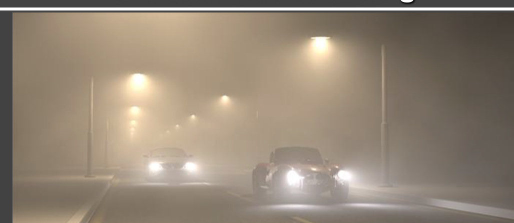
Many slides courtesy Pat Hanrahan/Matt Phārr (Stanford CS 348b) and Steve Rotenberg, Henrik Wann Jensen (UCSD CSE 168)

Volumetric Scattering

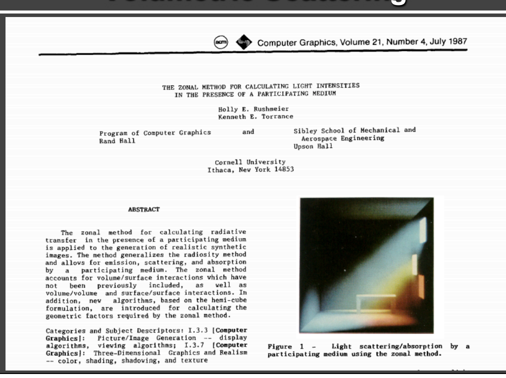




Volumetric Scattering



Volumetric Scattering



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THE ZONAL METHOD FOR CALCULATING LIGHT INTENSITIES
IN THE PRESENCE OF A PARTICIPATING MEDIUM

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ABSTRACT

The zonal method for calculating radiative transfer in the presence of a participating medium is applied to the generation of realistic synthetic images. The method generalizes the radiosity method and allows for emission, scattering, and absorption by a participating medium. The zonal method accounts for volume/surface interactions which have not been previously included, as well as volume/volume and surface/surface interactions. In addition, new algorithms, based on the hemi-cube formulation, are introduced for calculating the geometric factors required by the zonal method.

Categories and Subject Descriptors: I.3.3 [Computer Graphics]: Picture/Image Generation -- display algorithms, viewing algorithms; I.3.7 [Computer Graphics]: Three-Dimensional Graphics and Realism -- color, shading, shadowing, and texture

Figure 1 - Light scattering/absorption by a participating medium using the zonal method.

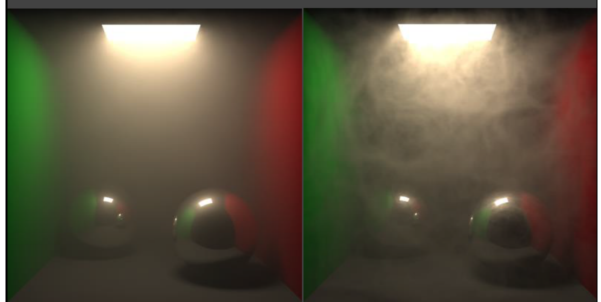
Volumetric Scattering

- Participating Media (light participates via scattering)
  - Volumetric phenomena like clouds, smoke, fire
  - Subsurface scattering, translucency (wax, human skin)
  - These are not surfaces with well-defined BRDFs
  - Rather volumes where light can scatter
  - Medium is often known as a participating medium
- Surface Rendering: Radiance Constant along Ray
  - Only true in absence of participating media
  - *No longer true for volumetric scattering*
  - *Often replace ray tracing with ray marching in medium*
- Volumetric Properties
  - BRDF replaced by phase function
  - Must consider absorption and scattering in medium

Homogeneous vs Heterogeneous

- Homogeneous: Properties constant everywhere
  - Example: Fog often represented as homogeneous
- Heterogeneous: Varies across space
  - Example: Smoke, fire etc.
  - Sometimes called inhomogeneous
- Homogeneous volumes often easier
  - Some computational shortcuts (transmittance etc.)
  - Some analytic formulae

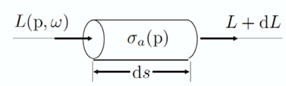
Homogeneous vs Heterogeneous



Volumetric Interactions

- 4 different processes affect radiance of a beam
  - Absorption
  - Out-Scattering
  - Emission
  - In-Scattering

Absorption



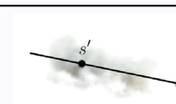
$$\mathrm{d}L(\mathrm{p},\omega) = -\sigma_a(\mathrm{p})\, L(\mathrm{p},\omega)\, \mathrm{d}s$$

Absorption cross section: $\sigma_a(\mathrm{p})$

- **Probability of being absorbed per unit length**
- **Units: 1/distance**

Absorption

Transmittance



$$\mathrm{d}L(\mathrm{p},\omega) = -\sigma_a(\mathrm{p})\, L(\mathrm{p},\omega)\, \mathrm{d}s$$

$$\frac{\mathrm{d}L(\mathrm{p},\omega)}{L(\mathrm{p},\omega)} = -\sigma_a(\mathrm{p})\, \mathrm{d}s$$

$$\log L(\mathrm{p}+s\omega,\omega) = -\int_0^s \sigma_a(\mathrm{p}+s'\omega,\omega)\, \mathrm{d}s' = -\tau(s)$$

Optical distance (depth): $\tau(s) = \int_0^s \sigma_a(\mathrm{p}')\, \mathrm{d}s'$

$$\mathrm{p}' = \mathrm{p} + s'\omega$$

Homogeneous medium–constant σ_a**:** $\tau(s) = \sigma_a s$

Transmittance and Opacity

$$\mathrm{d}L(\mathrm{p},\omega) = -\sigma_a(\mathrm{p})\, L(\mathrm{p},\omega)\, \mathrm{d}s$$

$$\frac{\mathrm{d}L(\mathrm{p},\omega)}{L(\mathrm{p},\omega)} = -\sigma_a(\mathrm{p})\, \mathrm{d}s$$

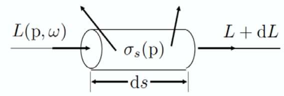
$$\log L(\mathrm{p}+s\omega,\omega) = -\int_0^s \sigma_a(\mathrm{p}+s'\omega,\omega)\, \mathrm{d}s' = -\tau(s)$$

$$L(\mathrm{p}+s\omega,\omega) = \mathrm{e}^{-\tau(s)} L(\mathrm{p},\omega) = T(s)\, L(\mathrm{p},\omega)$$

Transmittance: $T(s) = \mathrm{e}^{-\tau(s)}$

Opacity: $\alpha(s) = 1 - T(s)$

Out-Scattering

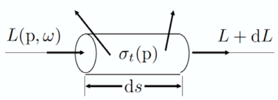


$$\mathrm{d}L(\mathrm{p},\omega) = -\sigma_s(\mathrm{p})\, L(\mathrm{p},\omega)\, \mathrm{d}s$$

Scattering cross-section: σ_s

- **Probability of being scattered per unit length**

Extinction



$$\mathrm{d}L(\mathrm{p},\omega) = -\sigma_t(\mathrm{p})\, L(\mathrm{p},\omega)\, \mathrm{d}s$$

Total cross section: $\sigma_t = \sigma_a + \sigma_s$

Albedo: $W = \frac{\sigma_s}{\sigma_t} = \frac{\sigma_s}{\sigma_a + \sigma_s}$

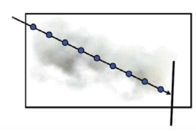
Optical distance from absorption and scattering:

$$\tau(s) = \int_0^s \sigma_t(\mathrm{p}')\, \mathrm{d}s'$$

Ray Marching for Transmittance

$$\tau(s) = \int_0^s \sigma_t(x + s'\omega)\, \mathrm{d}s'$$

$$T(s) = \mathrm{e}^{-\tau(s)}$$

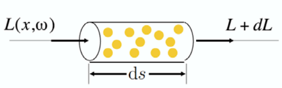


Monte Carlo not necessary for 1D—can use a Riemann sum:

$$\tau(s) \approx \frac{s}{N} \sum_i^N \sigma_t(x_i)$$

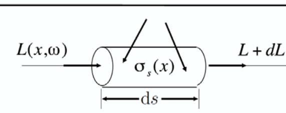
$$x_i = x + \frac{i + 0.5}{N}\omega$$

Emission



$$\mathrm{d}L(\mathrm{p},\omega) = \sigma_a(\mathrm{p}) L_\mathrm{e}(\mathrm{p},\omega)\, \mathrm{d}s$$

In-Scattering



$$S(\mathrm{p},\omega) = \sigma_s(\mathrm{p}) \int_{S^2} p(\omega' \to \omega)\, L(\mathrm{p},\omega')\, \mathrm{d}\omega'$$

Phase function: $p(\omega' \to \omega)$

Reciprocity: $p(\omega' \to \omega) = p(\omega \to \omega')$

Energy conservation: $\int_{S^2} p(\omega' \to \omega)\, \mathrm{d}\omega' = 1$

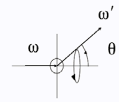
Scattering Phase Functions

- Light interacts with volume, scatters in some spherical distribution
- Similar to light scattering off a surface
- Phase function analogous to a surface BRDF
- Depends only on cosine of incident-outgoing
- Like BRDFs, volumetric phase functions must be reciprocal and conserve energy
- Similar to BRDFs, we will want to do importance sampling and evaluation of phase functions

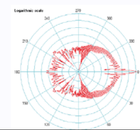
Phase Functions

Phase angle $\cos\theta = \omega \cdot \omega'$

Phase functions



- **Isotropic:** $p(\cos\theta) = \frac{1}{4\pi}$
- **Rayleigh:** $p(\cos\theta) = \frac{3}{4}(1+\cos^2\theta)$ **with** $\sigma_s \propto \frac{1}{\lambda^4}$
- **Mie:**



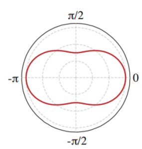
[Philip Laven]

Rayleigh Scattering

- Rayleigh scattering describes the scattering of light by particles much smaller than the wavelength

$$p(\cos\theta) = \frac{3}{16\pi}(1+\cos^2\theta)$$

$$\sigma_s = \frac{2\pi^5}{3}\frac{d^6}{\lambda^4}\left(\frac{n^2-1}{n^2+2}\right)^2$$



- Where λ is the wavelength of light, d is the diameter of the particle, and n is the index of refraction of the particle
- The strong dependence on wavelength (λ^{-4}) causes greater scattering towards the blue end of the spectrum
- The blue color of the sky is caused by Rayleigh scattering of sunlight by air molecules

Rayleigh Scattering: Blue Sky, Red Sunset

From Greenler: Rainbows, Halos, and Glories

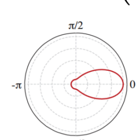
Mie Scattering

- Scatter electromagnetic waves by spherical particles
- Size of particles same scale as wavelength of light
- Water droplets in atmosphere, fat droplets in milk
- After Gustave Mie, Ludvig Lorenz

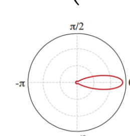
Empirical Mie Approximation

- The following empirical function is often used to approximate the shape of Mie scattering

$$p(\cos\theta) = \frac{1}{4\pi}\left(\frac{1}{2} + \frac{(z+1)}{2}\left(\frac{1+\cos\theta}{2}\right)^z\right)$$



z=8

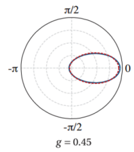


z=32

Henyey-Greenstein Function

- The Henyey-Greenstein phase function is an empirical function originally designed to model the scattering in galactic dust clouds

$$p(\cos\theta) = \frac{1-g^2}{4\pi(1+g^2-2g\cos\theta)^{1.5}}$$



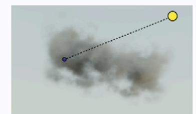
- It uses an anisotropy parameter *g* that ranges between -1 (full backscatter) and 1 (full forward scatter), and is isotropic for *g*=0

Direct Illumination in a Volume

$$S_d(p', \omega) = \sigma_s(p') \int_{S^2} p(\omega' \to \omega)\, L_d(p', \omega')\, d\omega'$$

Can treat like direct illumination at a surface

- **Sample from phase function's distribution**
- **Sample from light source distributions**
- **Weight using multiple importance sampling**



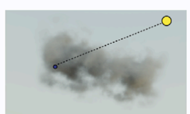
Direct Illumination in a Volume

$$S_d(p', \omega) = \sigma_s(p') \int_{S^2} p(\omega' \to \omega)\, L_d(p', \omega')\, d\omega'$$

Estimator: $\sigma_s(p') \frac{1}{N} \sum_i^N \frac{p(\omega_i \to \omega)\, L_d(p', \omega_i)}{p(\omega_i)}$

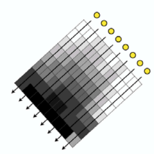
Computing direct lighting, L_d can be expensive

- **Not just a shadow ray–need to compute transmittance**

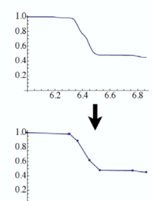


Transmittance for Shadow Rays

Besides Monte Carlo, precomputed transmittance can be faster for point, distant lights



3D grid [Kajiya and von Herzen 1984]

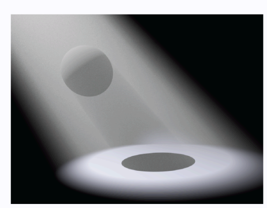


Deep Shadow Maps [Lokovic & Veach 2000], Adaptive Volumetric Shadow Maps [Salvi et al. 2010]

Single-Scattering

Single-Scattering

Minneart: Color and Light In The Open Air



pbrt: Spot-Lit Ball In The Fog

The Volume Rendering Equation

Integro-differential equation:

$$\frac{\partial L(\mathrm{p},\omega)}{\partial s} = -\sigma_t L(\mathrm{p},\omega) + S(\mathrm{p},\omega)$$

Integro-integral equation:

$$L(\mathrm{p},\omega) = \int_0^\infty T(\mathrm{p}')\, S(\mathrm{p}',\omega)\, \mathrm{d}s'$$

Attenuation: absorption and scattering

$$\mathrm{e}^{-\int_0^{s'} \sigma_t(\mathrm{p}'')\, \mathrm{d}s''}$$

Source: in-scattering (and emission)

$$\sigma_s(\mathrm{p}') \int_{S^2} p(\omega' \to \omega)\, L(\mathrm{p}',\omega')\, \mathrm{d}\omega'$$

Volumetric Path Tracing

Integro-integral equation:

$$L(\mathrm{p},\omega) = \int_0^\infty T(\mathrm{p}')\, S(\mathrm{p}',\omega)\, \mathrm{d}s'$$

Monte Carlo integration: sample $s' \sim p(s)$

Estimator: $\dfrac{T(\mathrm{p}')S(\mathrm{p}',\omega)}{p(s')}$

Evaluating the Estimator: S

Include indirect illumination in the source term:

$$S(x,\omega) = \sigma_s(x) \int_{S^2} p(\omega' \to \omega)\, L(x,\omega')\, \mathrm{d}\omega'$$

$$L(x,\omega') = L_d(x,\omega') + L_i(x,\omega')$$

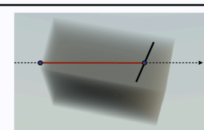
- **Compute direct lighting as before**
- **Sample incident direction from the phase function's distribution, trace a ray recursively...**

$$L_i(x,\omega') \approx \frac{p(\omega'' \to \omega')\, L(x,\omega'')}{p(\omega'')}$$

Uniform spherical directions: $p(\omega'') = \dfrac{1}{4\pi}$

Linear Sampling of T

We want samples along a finite ray [0,t_{max}].



- **Uniform probability along the ray:**

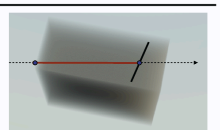
$$p(t) = \frac{1}{t_{\max}}$$

- **Sampling recipe:**

$$\xi = \int_0^t p(t)\, \mathrm{d}t \qquad t = \xi t_{\max}$$

Exact Sampling of Uniform T

We want samples along a finite ray [0,t_{max}], $p(t) \propto \mathrm{e}^{-\sigma t}$



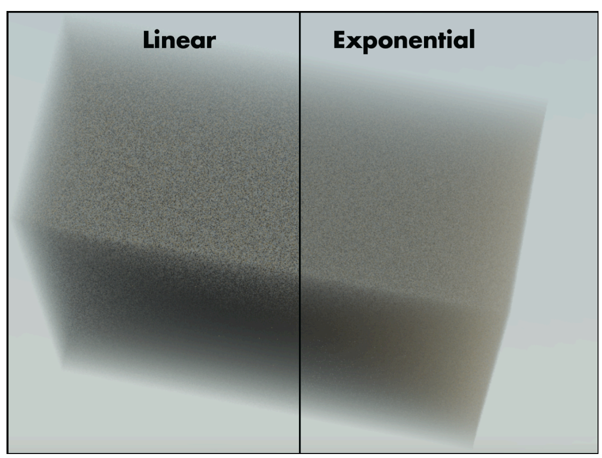
- **Normalize to find PDF:**

$$\int_0^{t_{\max}} \mathrm{e}^{-\sigma t}\, \mathrm{d}t = -\frac{1}{\sigma}(\mathrm{e}^{-\sigma t_{\max}} - 1) = c \qquad p(t) = c\mathrm{e}^{-\sigma t}$$

- **Invert to find t for a random sample:**

$$\xi = \int_0^t p(t)\, \mathrm{d}t$$

$$t = -\frac{1}{\sigma} \log(1 - \xi(1 - \mathrm{e}^{-\sigma t_{\max}}))$$



Volumetric Path Tracing

Integro-integral equation:

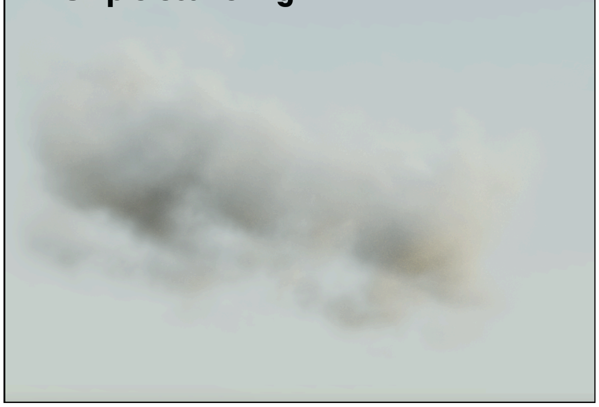
$$L(\mathrm{p}, \omega) = \int_0^\infty T(\mathrm{p}')\, S(\mathrm{p}', \omega)\, \mathrm{d}s'$$

Monte Carlo integration: sample $s' \sim p(s)$

Estimator: $\dfrac{T(\mathrm{p}')S(\mathrm{p}', \omega)}{p(s')}$

Single-Scattering

Multiple Scattering



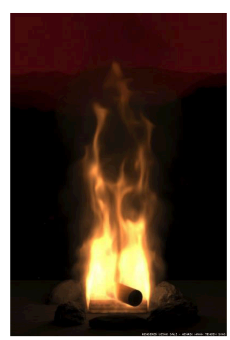
Clouds

Translucency

- Translucency is a volumetric lighting effect with additional effects at the surface (usually rough dielectric type interaction)
- These can be modeled through standard volumetric lighting techniques, or can be optimized through some further methods designed specifically for sub-surface scattering

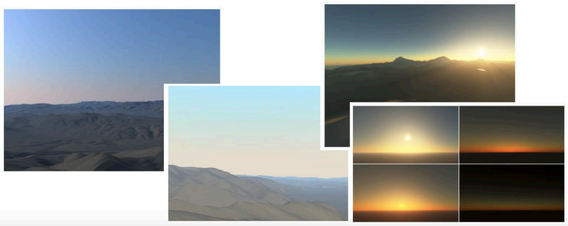
Fire

- "Physically Based Modeling and Animation of Fire", Nguyen, Fedkiw, Jensen, 2003



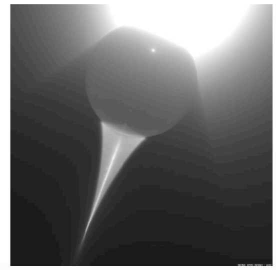
Sky Rendering

- "A Practical Analytical Model for Daylight", Preetham, Shirley, Smits, 1999
- "A Physically Based Night Sky Model", Jensen, Durand, Stark, Premoze, Dorsey, Shirley, 2001
- "Precomputed Atmospheric Scattering", Bruneton, Neyret, 2008
- "An Analytic Model for Full Spectral Sky-Dome Radiance", Hosek, Wilkie, 2012



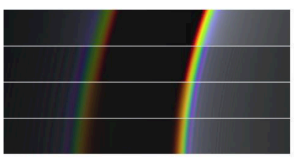
Volumetric Caustics

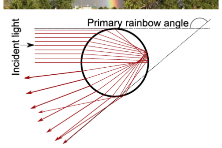
- "Efficient Simulation of Light Transport in Scenes with Participating Media using Photon Maps", Jensen, Christensen, 1998



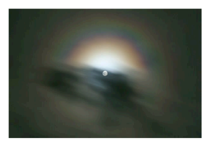
Rainbows

- "Physically Based Simulation of Rainbows", Sadeghi, Munoz, Laven, Jarosz, Seron, Gutierrez, Jensen, 2012





Atmospheric Phenomena



Corona

Ice Crystal Halo

Glory