Computer Graphics II: Rendering

CSE 168[Spr 21],Lecture 10: Materials and BRDFs
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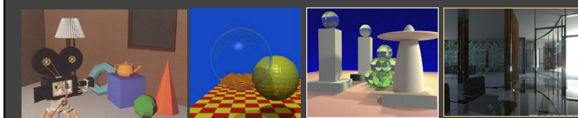
http://viscomp.ucsd.edu/classes/cse168/sp21



To Do

- Start working on homework 3. Ask me if problems
- Also homework 4. Have covered material
- Start thinking about final project

Some slides courtesy Steve Rotenberg and Pat Hanrahan

Materials and BRDFs

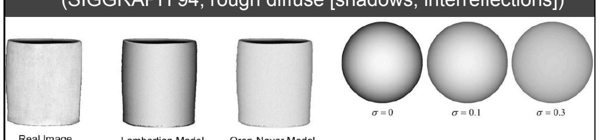
- Key part of renderer: different materials/BRDFs
- Abstract BRDF/Material interface (for MIS)
  - Evaluate (for given incident, outgoing direction)
  - Sample (given outgoing, importance sample incident)
  - PDF (for MIS, evaluate sampling PDF arbitrary direction)
  - Also for value of sample, need to compute eval/PDF (sometimes can simplify this, new value function=eval/PDF)
- Any physical or non-physical BRDF must fit above
  - Evaluation is usually easy (BRDF formula)
  - Can encompass analytic formulae, table measurements
  - Sampling can be hard and is crucial (see my 2004 paper for general importance sampling, special cases for some)
  - PDF function can be non-trivial, make sure math correct

Diffuse Surfaces

- Simplest Case: Lambertian Reflectance
- BRDF is simply a constant: $f = \frac{\rho}{\pi}$
- Note energy conservation, divide albedo by π
- Note cosine incident term in final evaluation $\tilde{f} = \frac{\rho}{\pi}\cos\theta_i$
- Evaluate BRDF is straightforward
- Sample? Sample hemisphere (or cosine-weight)
- PDF is $\frac{1}{2\pi}$ or (if cosine-weight) $\frac{\cos\theta_i}{\pi}$
- Value/weight with cosine sampling is simply ρL_i

Oren-Nayar Model

- Generalization of Lambert's Reflectance Model (SIGGRAPH 94, rough diffuse [shadows, interreflections])



Importance sampling can be complicated (but exact sampling is not required)

Simplest: Lambertian sampling/PDF
But Eval uses Oren-Nayar; Eval/PDF
(will cancel leading Lambertian term only)

$$L_r = \frac{\rho}{\pi} \cdot \cos\theta_i \cdot (A + (B \cdot \max[0, \cos(\phi_i - \phi_r)] \cdot \sin\alpha \cdot \tan\beta)) \cdot E_0$$

where

$$A = 1 - 0.5\frac{\sigma^2}{\sigma^2 + 0.33},$$

$$B = 0.45\frac{\sigma^2}{\sigma^2 + 0.09},$$

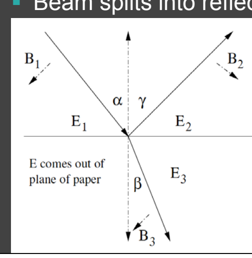
$$\alpha = \max(\theta_i, \theta_r),$$

$$\beta = \min(\theta_i, \theta_r),$$

From Wikipedia

Fresnel Surfaces

- Idealized Fresnel surfaces are perfectly smooth boundary between dielectric (air,glass,water) and another dielectric, or a dielectric and a metal
- Beam splits into reflected/refracted (Snell's law)

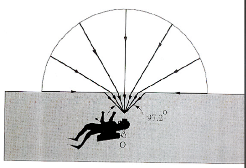


$$\sin\alpha = n\sin\beta$$

$$\alpha = \gamma$$

Optical Manhole

Total internal reflection

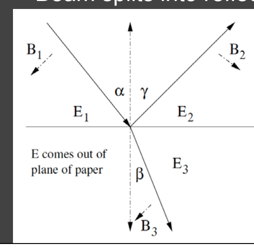


$$n_w = \frac{4}{3}$$

From Livingston and Lynch

Fresnel Surfaces

- Idealized Fresnel surfaces are perfectly smooth boundary between dielectric (air,glass,water) and another dielectric, or a dielectric and a metal
- Beam splits into reflected/refracted (Snell's law)



$$\sin\alpha = n\sin\beta$$

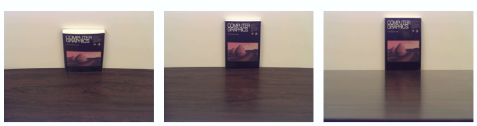
$$\alpha = \gamma$$

$$r_\perp = \frac{\cos\alpha - n\cos\beta}{\cos\alpha + n\cos\beta}$$

$$r_\parallel = \frac{n\cos\alpha - \cos\beta}{n\cos\alpha + \cos\beta}$$

Experiment

Reflections from a shiny floor

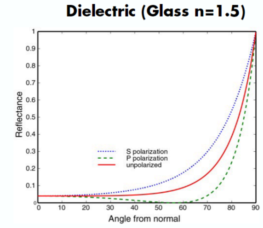


From Lafortune, Foo, Torrance, Greenberg, SIGGRAPH 97

Reflection is greater at glancing angles

Fresnel Reflectance

Dielectric (Glass n=1.5)

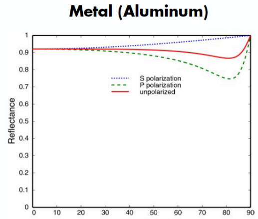


F(0)=0.04

Schlick Approximation $F(\theta) = F(0) + (1 - F(0))(1 - \cos\theta)^5$

Fresnel Reflectance

Metal (Aluminum)

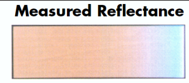


Gold F(0)=0.82
Silver F(0)=0.95

Reflection from Metals

Reflectance of Copper as a function of wavelength and angle of incidence

Measured Reflectance



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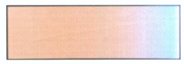
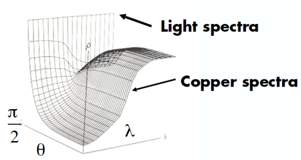
Cook-Torrance Reflection

Reflectance of Copper as a function of wavelength and angle of incidence

Measured Reflectance

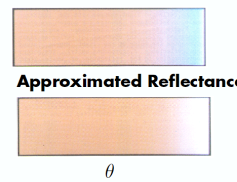


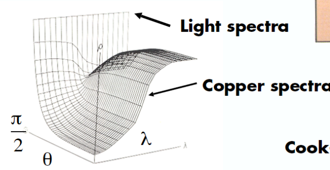


Cook-Torrance Reflection Model

Interpolate between color measured at normal incidence and the light color

Use Fresnel formula to interpolate





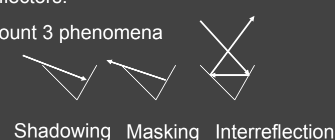
Cook-Torrance approximation

$$R(\theta) = R(0) + (R(\pi/2) - R(0))\max(0, \frac{F(\theta) - F(0)}{F(\pi/2) - F(0)})$$

(Cook-)Torrance-Sparrow

- Assume the surface is made up grooves at the microscopic level. (General *Microfacet Theory*)

- Assume the faces of these grooves (called microfacets) are perfect reflectors.
- Take into account 3 phenomena



(Cook-)Torrance-Sparrow

Fresnel term: allows for wavelength dependency

Geometric Attenuation: reduces the output based on the amount of shadowing or masking that occurs.

$$f = \frac{F(\theta_i)G(\omega_i,\omega_r)D(\theta_h)}{4\cos(\theta_i)\cos(\theta_r)}$$

Distribution: distribution function determines what percentage of microfacets are oriented to reflect in the viewer direction.

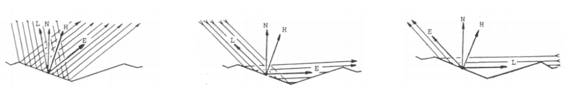
How much of the macroscopic surface is visible to the light source

How much of the macroscopic surface is visible to the viewer

Geometric Attenuation

- *Geometric attenuation* refers to the decrease in light reflection due to both shadowing and masking

$$G = \min\left(1, \frac{2(\mathbf{n}\cdot\mathbf{h})(\mathbf{n}\cdot\mathbf{v})}{(\mathbf{v}\cdot\mathbf{h})}, \frac{2(\mathbf{n}\cdot\mathbf{h})(\mathbf{n}\cdot\mathbf{L})}{(\mathbf{v}\cdot\mathbf{h})}\right)$$



Microfacet Distribution Function

- There have been various functions proposed that describe the distribution of microfacets around the average surface normal
- Gaussian: $D = ce^{-(\alpha/m)^2}$
- Beckmann: $D = \frac{1}{m^2\cos^4\alpha}e^{-\left(\frac{\tan^2\alpha}{m^2}\right)}$

where

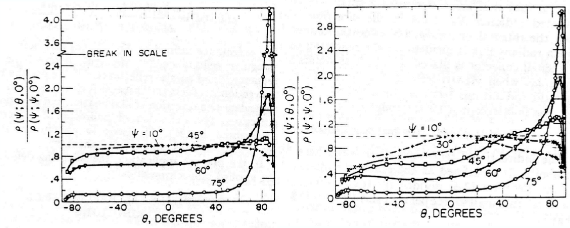
α=acos(**n**·**h**)
m=root mean square slope of microfacets
c=an arbitrary constant (?)

Torrance-Sparrow Model

K. E. Torrance, E. M. Sparrow, Theory of the off-specular reflection from roughened surfaces, JOSA 1967

Experiment: "Off-Specular" Peak

Peak of reflection is not at the angle of reflection



Magnesium Oxide
Dielectric

Aluminum
Metal

Torrance-Sparrow Theory

$$f_r(\omega_i \rightarrow \omega_r) = \frac{F(\theta_i')S(\theta_i)S(\theta_r)D(\alpha)}{4\cos\theta_i \cos\theta_r}$$

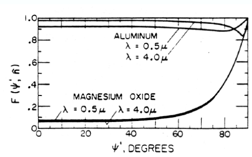


FIG. 6. Fresnel reflectance.

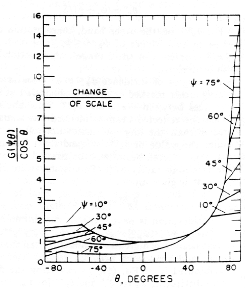
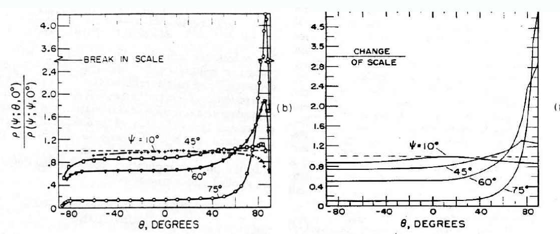


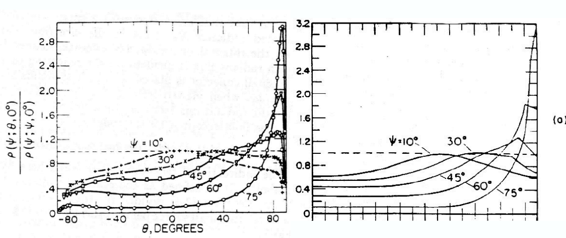
FIG. 7. The factor $G(\psi,\theta)/\cos\theta$ in the plane of incidence for various incidence angles ψ.

Torrance-Sparrow Model Prediction



Magnesium Oxide

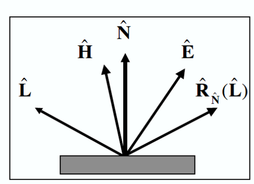
Torrance-Sparrow Model Prediction

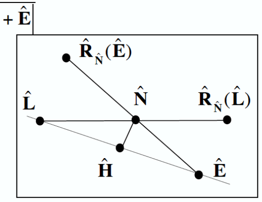


Aluminum

Reflection Geometry

$$\hat{\mathbf{H}} = \frac{\hat{\mathbf{L}} + \hat{\mathbf{E}}}{\left|\hat{\mathbf{L}} + \hat{\mathbf{E}}\right|}$$





$\cos\theta_i = \hat{\mathbf{L}} \bullet \hat{\mathbf{N}}$

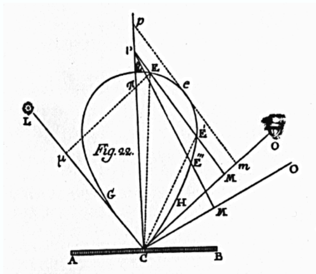
$\cos\theta_r = \hat{\mathbf{E}} \bullet \hat{\mathbf{N}}$

$\cos\theta_s = \hat{\mathbf{E}} \bullet \mathbf{R}_{\hat{N}}(\hat{\mathbf{L}}) = \mathbf{R}_{\hat{N}}(\hat{\mathbf{E}}) \bullet \hat{\mathbf{L}}$

$\cos\theta_g = \hat{\mathbf{E}} \bullet \hat{\mathbf{L}}$

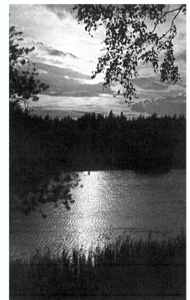
$\cos\theta_{s'} = \hat{\mathbf{H}} \bullet \hat{\mathbf{N}}$

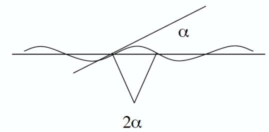
Microfacet BRDFs ("Little Faces")



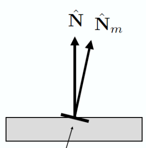
P. Bouguer, Treatise on Optics, 1760

Reflection of the Sun from Waves





Microfacet Distributions



Normalize projected area

$$\int_{H^2} dA(\omega_m)\, cos\theta_m\, d\omega_m = dA$$

Probability distribution

$$\int_{H^2} D(\omega_m)\, cos\theta_m\, d\omega_m = 1$$

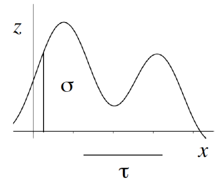
Area distribution $dA(\omega_m)$

Microfacet distribution $D(\omega_m) = dA(\omega_m)/dA$

Beckmann Distribution

Gaussian distribution of heights

$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{z^2}{2\sigma^2}}$$

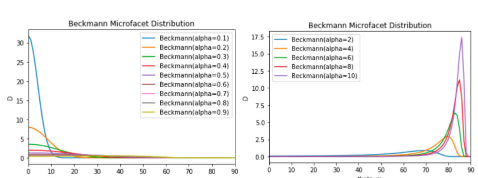


Beckmann distribution of normals (mirrors)

$$D(\omega_m) = \frac{e^{\frac{-\tan^2\theta_m}{\alpha^2}}}{\pi\alpha^2\cos^4\theta_m} \qquad \alpha = \sqrt{2}\frac{\sigma}{\tau}$$

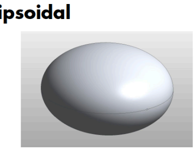
mean slope

Beckmann Distribution



Trowbridge-Reitz (GGX) Distribution

Elipsoidal

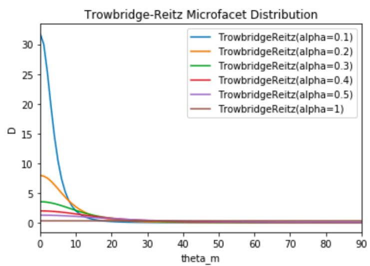


$$z = \alpha(1 - x^2 - y^2)^{(1/2)}$$

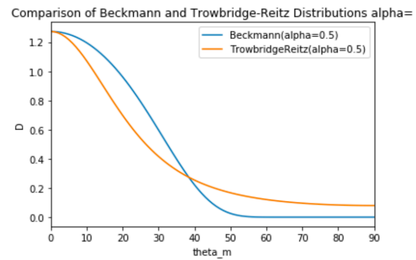
GGX distribution of normals

$$D(\omega_m) = \frac{1}{\pi\alpha^2\cos^4\theta_m(1 + \frac{\tan^2\theta_m}{\alpha^2})^2}$$

Trowbridge-Reitz (GGX) Distribution



Comparison

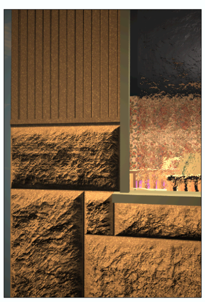


Trowbridge-Reitz has a longer tail

Trowbridge-Reitz matches experimental data better

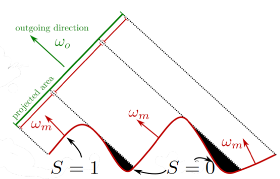
Shadowing Reduces Reflected Energy

Without self-shadowing **With self-shadowing**



Visible Projected Area

From Heitz 2014

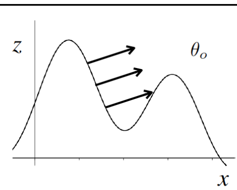


$$\int S(\omega_o)\max(0, \omega_m \cdot \omega_o)\, D(\omega_m)\, d\omega_m = \cos\theta_o$$

The sum of the *visible* areas of the rough surface as viewed from the outgoing direction should equal the projected area of the underlying mean surface

Smith Self-Shadowing Function

Assume probability of shadowing is independent of the normal



$$S(\theta_o) = \frac{1}{1 + \Lambda(\theta_o)}$$

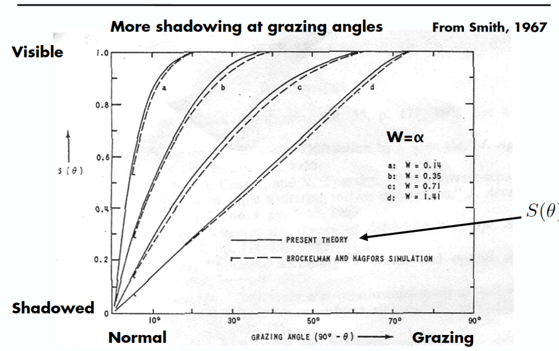
$$\Lambda(\theta_o) = \frac{\operatorname{erf}(a) - 1}{2} + \frac{1}{2a\sqrt{\pi}}\exp(-a^2) \qquad a = \frac{1}{\alpha\tan\theta_o}$$

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\, dt$$

From Smith, 1967

Smith Self-Shadowing Function

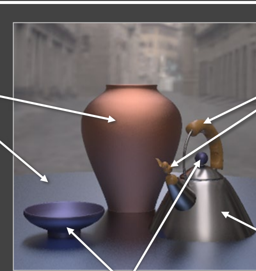
More shadowing at grazing angles **From Smith, 1967**



BRDF Sampling

- Have dealt with BRDF evaluation, need importance sampling and PDF functions for MIS
- In 2004, no good importance sampling schemes for most BRDFs, including common Torrance-Sparrow
- From Lawrence et al. 04, factor BRDF into data-driven terms that can each be importance sampled
- Now some form of light/BRDF sampling common in production (standard in RenderMan 16, 2011-)

Motivation



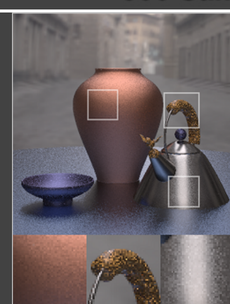
Key Idea

- Project 4D BRDF into sum of products of 2D function dependent on ω_o and 2D function dependent on ω_i :

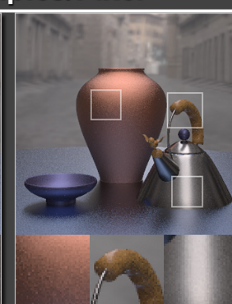
$$f_r(\omega_o,\omega_i)(n\cdot\omega_i) = \sum_{j=1}^{J} F_j(\omega_o)\; G_j(\omega_p)$$

ω_p depends **only** on the incoming direction and some re-parameterization of the hemisphere.

300 Samples/Pixel



Sampling Lafortune Fit



Our Method