Computer Graphics II: Rendering

CSE 168[Spr 20],Lecture 15: Volumetric Rendering
Ravi Ramamoorthi

http://viscomp.ucsd.edu/classes/cse168/sp20
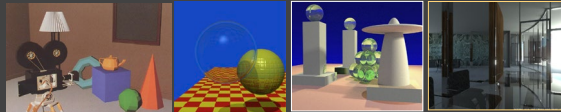
To Do

- Start working on final projects (initial results and proposal due in a week). Ask me if problems

- Volumetric rendering (this lecture) may be one component of the final project (but hard, be careful)

- Increasingly accurate appearance requires volumetric scattering (even for skin, hair, fur)

- Continues to be an active area of research

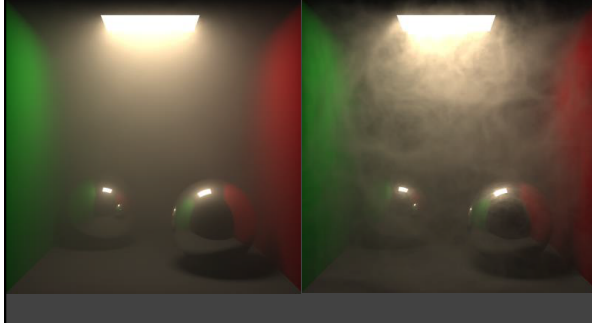## Volumetric Scattering
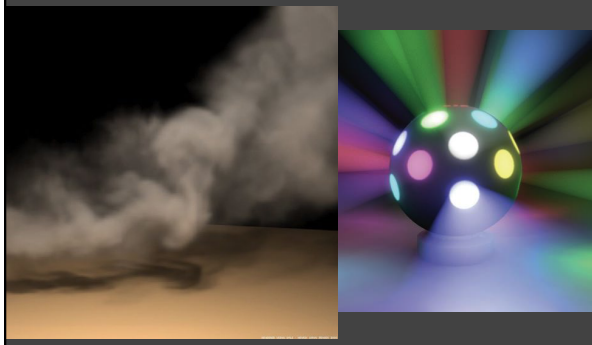
Volumetric Scattering

Volumetric Scattering

- Participating Media (light participates via scattering)
  - Volumetric phenomena like clouds, smoke, fire
  - Subsurface scattering, translucency (wax, human skin)
  - These are not surfaces with well-defined BRDFs
  - Rather volumes where light can scatter
  - Medium is often known as a participating medium

- Surface Rendering: Radiance Constant along Ray
  - Only true in absence of participating media
  - *No longer true for volumetric scattering*
  - *Often replace ray tracing with ray marching in medium*

- Volumetric Properties
  - BRDF replaced by phase function
  - Must consider absorption and scattering in medium

Homogeneous vs Heterogeneous

- Homogeneous: Properties constant everywhere
  - Example: Fog often represented as homogeneous

- Heterogeneous: Varies across space
  - Example: Smoke, fire etc.
  - Sometimes called inhomogeneous

- Homogeneous volumes often easier
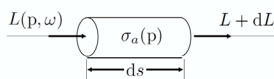 - Some computational shortcuts (transmittance etc.)
  - Some analytic formulae

Homogeneous vs Heterogeneous

Volumetric Interactions

- 4 different processes affect radiance of a beam
  - Absorption
  - Out-Scattering
  - Emission
  - In-Scattering

Absorption

$$dL(p,\omega) = -\sigma_a(p)\, L(p,\omega)\, ds$$

Absorption cross section: $\sigma_a(p)$

- **Probability of being absorbed per unit length**
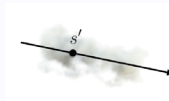- **Units: 1/distance**

Absorption

Transmittance

$$dL(p,\omega) = -\sigma_a(p)\, L(p,\omega)\, ds$$

$$\frac{dL(p,\omega)}{L(p,\omega)} = -\sigma_a(p)\, ds$$

$$\log L(p + s\omega, \omega) = -\int_0^s \sigma_a(p + s'\omega, \omega)\, ds' = -\tau(s)$$

Optical distance (depth): $\tau(s) = \int_0^s \sigma_a(p')\, ds'$
$$p' = p + s'\omega$$
Homogeneous medium–constant σ_a**:** $\tau(s) = \sigma_a s$

Transmittance and Opacity

$$dL(p,\omega) = -\sigma_a(p)\, L(p,\omega)\, ds$$

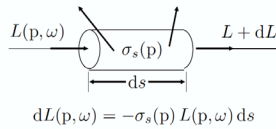$$\frac{dL(p,\omega)}{L(p,\omega)} = -\sigma_a(p)\, ds$$

$$\log L(p + s\omega, \omega) = -\int_0^s \sigma_a(p + s'\omega, \omega)\, ds' = -\tau(s)$$

$$L(p + s\omega, \omega) = e^{-\tau(s)} L(p,\omega) = T(s)\, L(p,\omega)$$

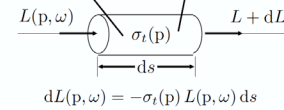**Transmittance:** $T(s) = e^{-\tau(s)}$

Opacity: $\alpha(s) = 1 - T(s)$

Out-Scattering

$$dL(p,\omega) = -\sigma_s(p)\,L(p,\omega)\,ds$$

Scattering cross-section: σ_s

- **Probability of being scattered per unit length**

Extinction

$$dL(p,\omega) = -\sigma_t(p)\,L(p,\omega)\,ds$$

Total cross section: $\sigma_t = \sigma_a + \sigma_s$

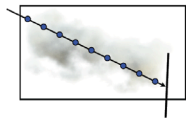**Albedo:** $W = \dfrac{\sigma_s}{\sigma_t} = \dfrac{\sigma_s}{\sigma_a + \sigma_s}$

Optical distance from absorption and scattering:

$$\tau(s) = \int_0^s \sigma_t(p')\,ds'$$

Ray Marching for Transmittance

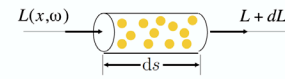$$\tau(s) = \int_0^s \sigma_t(x + s'\omega)\,ds'$$

$$T(s) = e^{-\tau(s)}$$

Monte Carlo not necessary for 1D—can use a Riemann sum:

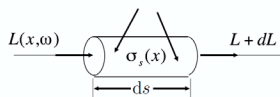$$\tau(s) \approx \frac{s}{N}\sum_i^N \sigma_t(x_i)$$

$$x_i = x + \frac{i + 0.5}{N}\omega$$

Emission

$$dL(p,\omega) = \sigma_a(p)L_e(p,\omega)\,ds$$

In-Scattering

$$S(p,\omega) = \sigma_s(p)\int_{S^2} p(\omega' \to \omega)\,L(p,\omega')\,d\omega'$$

Phase function: $p(\omega' \to \omega)$

Reciprocity: $p(\omega' \to \omega) = p(\omega \to \omega')$

Energy conservation: $\int_{S^2} p(\omega' \to \omega)\,d\omega' = 1$

Scattering Phase Functions

- Light interacts with volume, scatters in some spherical distribution
- Similar to light scattering off a surface
- Phase function analogous to a surface BRDF
- Depends only on cosine of incident-outgoing
- Like BRDFs, volumetric phase functions must be reciprocal and conserve energy
- Similar to BRDFs, we will want to do importance sampling and evaluation of phase functions
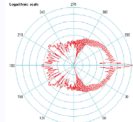
Phase Functions

Phase angle $\cos\theta = \omega \cdot \omega'$

Phase functions

- **Isotropic:** $p(\cos\theta) = \dfrac{1}{4\pi}$

- **Rayleigh:** $p(\cos\theta) = \dfrac{3}{4}(1 + \cos^2\theta)$ **with** $\sigma_s \propto \dfrac{1}{\lambda^4}$
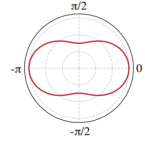
- **Mie:**

[Philip Laven]

Rayleigh Scattering

- Rayleigh scattering describes the scattering of light by particles much smaller than the wavelength

$$p(\cos\theta) = \frac{3}{16\pi}(1 + \cos^2\theta)$$

$$\sigma_s = \frac{2\pi^5}{3}\frac{d^6}{\lambda^4}\left(\frac{n^2 - 1}{n^2 + 2}\right)^2$$

- Where λ is the wavelength of light, d is the diameter of the particle, and n is the index of refraction of the particle
- The strong dependence on wavelength (λ^{-4}) causes greater scattering towards the blue end of the spectrum
- The blue color of the sky is caused by Rayleigh scattering of sunlight by air molecules

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Rayleigh Scattering: Blue Sky, Red Sunset

From Greenler: Rainbows, Halos, and Glories
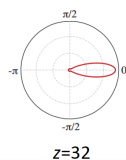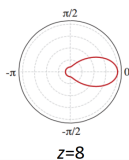

Mie Scattering

- Scatter electromagnetic waves by spherical particles
- Size of particles same scale as wavelength of light
- Water droplets in atmosphere, fat droplets in milk
- After Gustave Mie, Ludvig Lorenz

Empirical Mie Approximation

- The following empirical function is often used to approximate the shape of Mie scattering
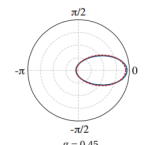
$$p(\cos\theta) = \frac{1}{4\pi}\left(\frac{1}{2} + \frac{(z+1)}{2}\left(\frac{1 + \cos\theta}{2}\right)^z\right)$$

z=8   z=32

Henyey-Greenstein Function

- The Henyey-Greenstein phase function is an empirical function originally designed to model the scattering in galactic dust clouds

$$p(\cos\theta) = \frac{1 - g^2}{4\pi(1 + g^2 - 2g\cos\theta)^{1.5}}$$

- It uses an anisotropy parameter g that ranges between -1 (full backscatter) and 1 (full forward scatter), and is isotropic for $g=0$

Direct Illumination in a Volume

$$S_{\mathrm{d}}(\mathrm{p}',\omega) = \sigma_s(\mathrm{p}') \int_{S^2} p(\omega' \to \omega)\, L_{\mathrm{d}}(\mathrm{p}',\omega')\, \mathrm{d}\omega'$$

Can treat like direct illumination at a surface

- Sample from phase function's distribution
- Sample from light source distributions
- Weight using multiple importance sampling

Direct Illumination in a Volume

$$S_{\mathrm{d}}(\mathrm{p}',\omega) = \sigma_s(\mathrm{p}') \int_{S^2} p(\omega' \to \omega)\, L_{\mathrm{d}}(\mathrm{p}',\omega')\, \mathrm{d}\omega'$$

Estimator: $\sigma_s(\mathrm{p}')\dfrac{1}{N}\sum_{i}^{N} \dfrac{p(\omega_i \to \omega)\, L_d(\mathrm{p}',\omega_i)}{p(\omega_i)}$
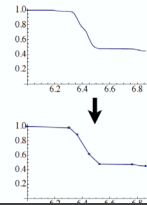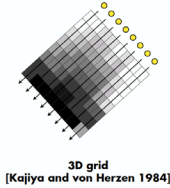
Computing direct lighting, L_d can be expensive

- Not just a shadow ray– need to compute transmittance

Transmittance for Shadow Rays

Besides Monte Carlo, precomputed transmittance can be faster for point, distant lights

Deep Shadow Maps
[Lokovic & Veach 2000],
Adaptive Volumetric Shadow Maps
[Salvi et al. 2010]

3D grid
[Kajiya and von Herzen 1984]

Single-Scattering

Single-Scattering

Minneart: Color and Light In The Open Air

pbrt: Spot-Lit Ball In The Fog

The Volume Rendering Equation

Integro-differential equation:

$$\frac{\partial L(\mathrm{p},\omega)}{\partial s} = -\sigma_t L(\mathrm{p},\omega) + S(\mathrm{p},\omega)$$

Integro-integral equation:

$$L(\mathrm{p},\omega) = \int_0^\infty T(\mathrm{p}')\, S(\mathrm{p}',\omega)\, \mathrm{d}s'$$

Attenuation: absorption and scattering

$$\mathrm{e}^{-\int_0^{s'} \sigma_t(\mathrm{p}'')\,\mathrm{d}s''}$$

Source: in-scattering (and emission)

$$\sigma_s(\mathrm{p}') \int_{S^2} p(\omega' \to \omega)\, L(\mathrm{p}',\omega')\, \mathrm{d}\omega'$$

Volumetric Path Tracing

Integro-integral equation:

$$L(\mathrm{p}, \omega) = \int_0^\infty T(\mathrm{p}') \, S(\mathrm{p}', \omega) \, \mathrm{d}s'$$

Monte Carlo integration: sample $s' \sim p(s)$

Estimator: $\dfrac{T(\mathrm{p}')S(\mathrm{p}', \omega)}{p(s')}$

Evaluating the Estimator: S

Include indirect illumination in the source term:

$$S(x, \omega) = \sigma_s(x) \int_{S^2} p(\omega' \to \omega) \, L(x, \omega') \, \mathrm{d}\omega'$$

$$\Downarrow$$

$$L(x, \omega') = L_d(x, \omega') + L_i(x, \omega')$$

- **Compute direct lighting as before**
- **Sample incident direction from the phase function's distribution, trace a ray recursively...**
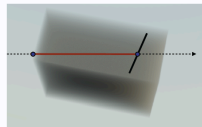
$$L_i(x, \omega') \approx \frac{p(\omega'' \to \omega') \, L(x, \omega'')}{p(\omega'')}$$

Uniform spherical directions: $p(\omega'') = \dfrac{1}{4\pi}$

Linear Sampling of T

We want samples along a finite ray [0,t_{max}].

- **Uniform probability along the ray:**

$$p(t) = \frac{1}{t_{max}}$$
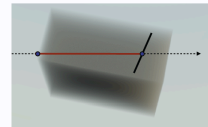
- **Sampling recipe:**

$$\xi = \int_0^t p(t) \, \mathrm{d}t \qquad t = \xi t_{max}$$

Exact Sampling of Uniform T

We want samples along a finite ray [0,t_{max}], $p(t) \propto e^{-\sigma t}$

- **Normalize to find PDF:**

$$\int_0^{t_{max}} e^{-\sigma t} \, \mathrm{d}t = -\frac{1}{\sigma}(e^{-\sigma t_{max}} - 1) = c \qquad p(t) = c e^{-\sigma t}$$
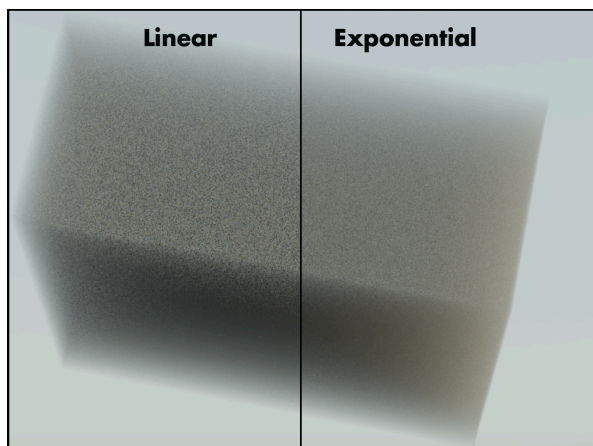
- **Invert to find t for a random sample:**

$$\xi = \int_0^t p(t) \, \mathrm{d}t$$

$$t = -\frac{1}{\sigma} \log(1 - \xi(1 - e^{-\sigma t_{max}}))$$

Linear	Exponential

Volumetric Path Tracing

Integro-integral equation:

$$L(\mathrm{p}, \omega) = \int_0^\infty T(\mathrm{p}') \, S(\mathrm{p}', \omega) \, \mathrm{d}s'$$

Monte Carlo integration: sample $s' \sim p(s)$

Estimator: $\dfrac{T(\mathrm{p}')S(\mathrm{p}', \omega)}{p(s')}$

Single-Scattering

Multiple Scattering

Clouds

Translucency

- Translucency is a volumetric lighting effect with additional effects at the surface (usually rough dielectric type interaction)
- These can be modeled through standard volumetric lighting techniques, or can be optimized through some further methods designed specifically for sub-surface scattering


Fire

- "Physically Based Modeling and Animation of Fire", Nguyen, Fedkiw, Jensen, 2003


Sky Rendering

- "A Practical Analytical Model for Daylight", Preetham, Shirley, Smits, 1999
- "A Physically Based Night Sky Model", Jensen, Durand, Stark, Premoze, Dorsey, Shirley, 2001
- "Precomputed Atmospheric Scattering", Bruneton, Neyret, 2008
- "An Analytic Model for Full Spectral Sky-Dome Radiance", Hosek, Wilkie, 2012
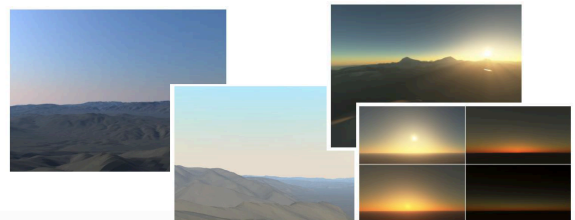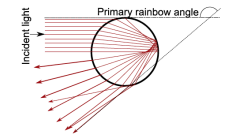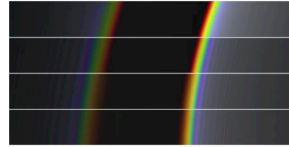
Volumetric Caustics

- "Efficient Simulation of Light Transport in Scenes with Participating Media using Photon Maps", Jensen, Christensen, 1998
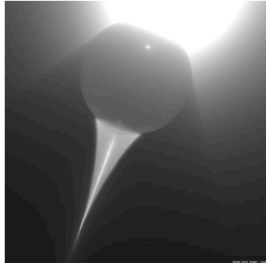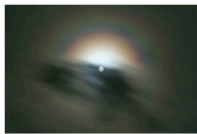

Rainbows

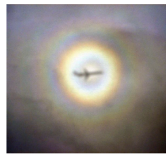- "Physically Based Simulation of Rainbows", Sadeghi, Munoz, Laven, Jarosz, Seron, Gutierrez, Jensen, 2012

Atmospheric Phenomena

Corona

Ice Crystal Halo

Glory