

Computer Graphics

CSE 167 [Win 24], Lecture 10: Curves 2

Ravi Ramamoorthi

<http://viscomp.ucsd.edu/classes/cse167/wi24>

1

Outline of Unit

- Bezier curves (last time)
- deCasteljau algorithm, explicit, matrix (last time)
- *Polar form labeling (blossoms)*
- B-spline curves
- Not well covered in textbooks (especially as taught here). Main reference will be lecture notes. If you do want a printed ref, handouts from CAGD, Seidel

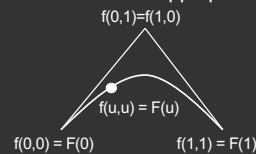
2

Survey Feedback

3

Idea of Blossoms/Polar Forms

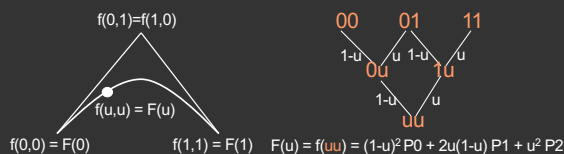
- (Optional) Labeling trick for control points and intermediate deCasteljau points that makes thing intuitive
- E.g. quadratic Bezier curve $F(u)$
 - Define auxiliary function $f(u_1, u_2)$ [number of args = degree]
 - Points on curve simply have $u_1 = u_2$ so that $F(u) = f(u, u)$
 - And we can label control points and deCasteljau points not on curve with appropriate values of (u_1, u_2)



4

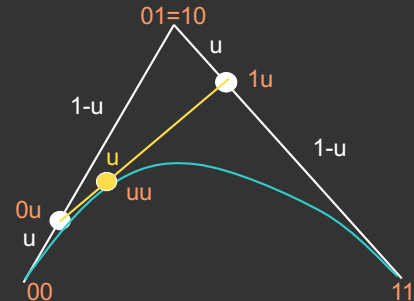
Idea of Blossoms/Polar Forms

- Points on curve simply have $u_1 = u_2$ so that $F(u) = f(u, u)$
- f is symmetric $f(0,1) = f(1,0)$
- Only interpolate linearly between points with one arg different
 - $f(0,u) = (1-u)f(0,0) + uf(0,1)$ Here, interpolate $f(0,0)$ and $f(0,1)=f(1,0)$



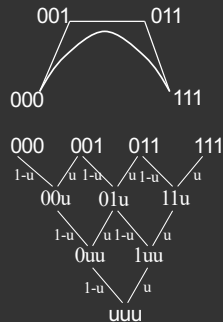
5

Geometric interpretation: Quadratic



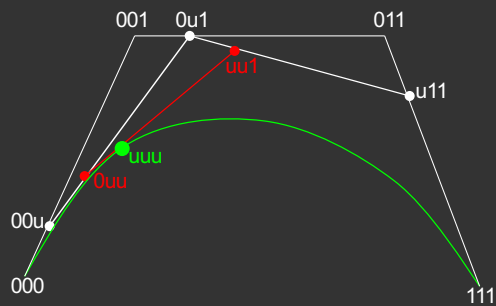
6

Polar Forms: Cubic Bezier Curve



7

Geometric Interpretation: Cubic



8

Why Polar Forms?

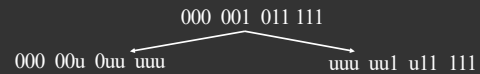
- Simple mnemonic: which points to interpolate and how in deCasteljau algorithm
- Easy to see how to subdivide Bezier curve (next) which is useful for drawing recursively
- Generalizes to arbitrary spline curves (just label control points correctly instead of 00 01 11 for Bezier)
- Easy for many analyses (beyond scope of course)

9

Subdividing Bezier Curves

Drawing: Subdivide into halves ($u = \frac{1}{2}$) Demo: hw3

- Recursively draw each piece
- At some tolerance, draw control polygon
- Trivial for Bezier curves (from deCasteljau algorithm): hence widely used for drawing

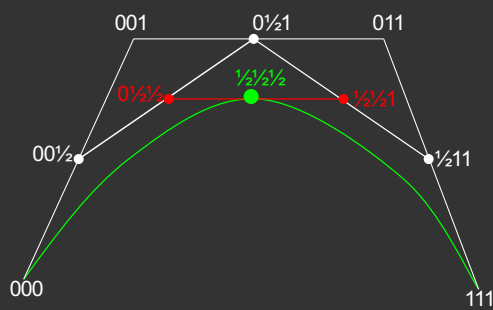


Why specific labels/ control points on left/right?

- How do they follow from deCasteljau?

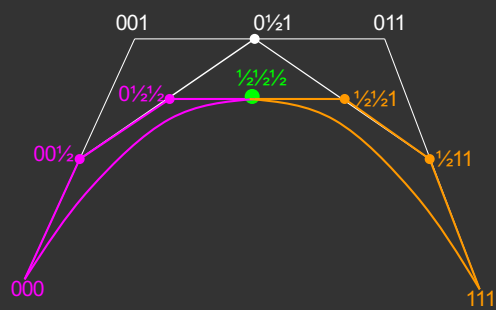
10

Geometrically



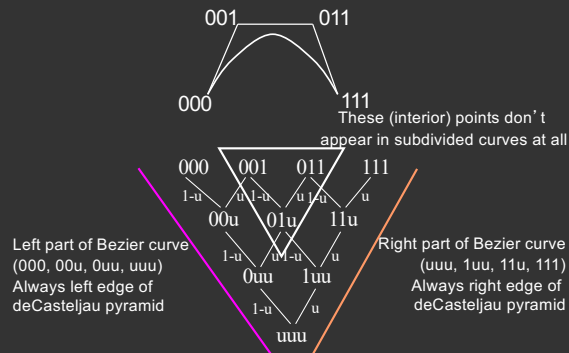
11

Geometrically



12

Subdivision in deCasteljau diagram



13

Summary for HW 3 (with demo)

- Bezier2 (Bezier discussed last time)
- Given arbitrary degree Bezier curve, recursively subdivide for some levels, then draw control polygon
- Generate deCasteljau diagram; recursively call a routine with left edge and right edge of this diagram
- You are given some code structure; you essentially just need to compute appropriate control points for left, right

14

DeCasteljau: Recursive Subdivision

Input: Control points C_i with $0 \leq i \leq n$ where n is the degree.
Output: L_i, R_i for left and right control points in recursion.

```

1 for (level = n ; level >= 0 ; level --) {
2   if (level == n) { // Initial control points
3      $\forall i : 0 \leq i \leq n : p_i^{level} = C_i$  ; continue ; }
4   for (i = 0 ; i <= level ; i++)
5      $p_i^{level} = \frac{1}{2} * (p_i^{level+1} + p_{i+1}^{level+1})$  ;
6 }
7  $\forall i : 0 \leq i \leq n : L_i = p_0^i ; R_i = p_i^1$  ;

```

- DeCasteljau (from last lecture) for midpoint
- Followed by recursive calls using left, right parts

15

Outline of Unit

- Bezier curves (last time)
- deCasteljau algorithm, explicit, matrix (last time)
- Polar form labeling (blossoms)
- B-spline curves
- Not well covered in textbooks (especially as taught here). Main reference will be lecture notes. If you do want a printed ref, handouts from CAGD, Seidel

16

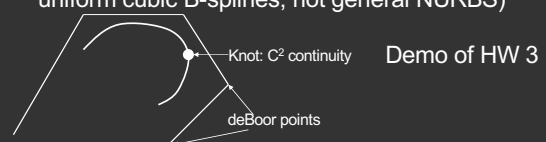
Bezier: Disadvantages

- Single piece, no local control (move a control point, whole curve changes) [Demo of HW 3]
- Complex shapes: can be very high degree, difficult
- In practice, combine many Bezier curve segments
 - But only position continuous at join since Bezier curves interpolate end-points (which match at segment boundaries)
 - Unpleasant derivative (slope) discontinuities at end-points
 - Can you see why this is an issue?

17

B-Splines

- Cubic B-splines have C^2 continuity, local control
- 4 segments / control point, 4 control points/segment
- Knots where two segments join: Knotvector
- Knotvector uniform/non-uniform (we only consider uniform cubic B-splines, not general NURBS)



18

Polar Forms: Cubic Bspline Curve

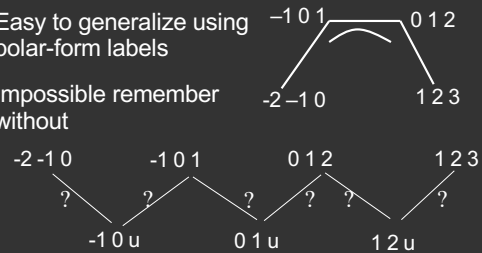
- Labeling little different from in Bezier curve
- No interpolation of end-points like in Bezier
- Advantage of polar forms: easy to generalize



19

deCasteljau: Cubic B-Splines

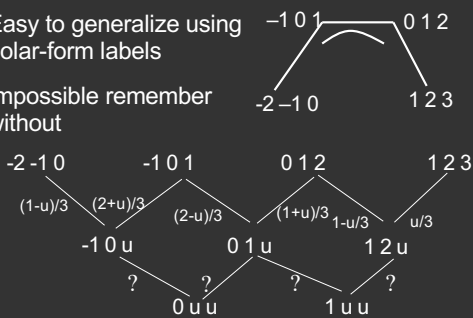
- Easy to generalize using polar-form labels
- Impossible remember without



20

deCasteljau: Cubic B-Splines

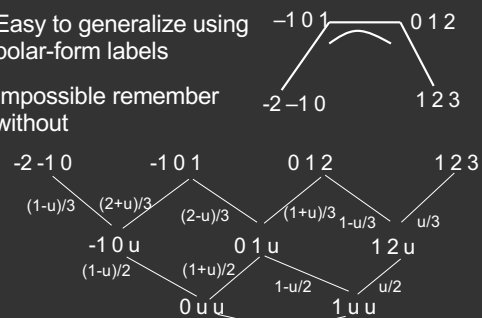
- Easy to generalize using polar-form labels
- Impossible remember without



21

deCasteljau: Cubic B-Splines

- Easy to generalize using polar-form labels
- Impossible remember without



22

Explicit Formula (derive as exercise)

$$F(u) = [u^3 \ u^2 \ u \ 1] M \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix} \quad M = \frac{1}{6} \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix}$$

23

Summary of HW 3

- BSpline Demo hw3
- Arbitrary number of control points / segments
 - Do nothing till 4 control points (see demo)
 - Number of segments = # cpts - 3
- Segment A will have control pts A, A+1, A+2, A+3
- Evaluate Bspline for each segment using 4 control points (at some number of locations, connect lines)
- Use either deCasteljau algorithm (like Bezier) or explicit form [matrix formula on previous slide]
- Questions?

24