Question 1: Shading (6 points)

Consider the following snippet of shading from the GLSL fragment shader in homework 0 or homework 2. **direction** is the direction to the light, and **halfvec** is the half-vector between light and eye (normalized and properly in eye coordinates). The variables **mydiffuse**, **myspecular** and **myshininess** stand as usual for the diffuse, specular and shininess coefficients (the first two are RGB colors), and **lightcolor** is the (RGBA) color of the light. Note that we only deal with the diffuse and specular terms here, with no ambient or emission.

```
vec4 ComputeLight (   const in vec3 direction ,
                      const in vec4 lightcolor ,
                      const in vec3 normal ,
                      const in vec3 halfvec ,
                      const in vec4 mydiffuse ,
                      const in vec4 myspecular ,
                      const in float myshininess ) {
    1. float nDotL = dot(normal , direction );
    2. vec4 lambert = mydiffuse * lightcolor ;
    3. float nDotH = dot(normal , halfvec );
    4. vec4 phong = myspecular * lightcolor * pow (max(nDotH, 0), myshininess) ;
    5. vec4 retval = lambert + phong ;

    return retval ;
}
```

i. There is a subtle (mathematical) error in line 2. Please identify the error and correct the code. Also provide the mathematical formula for Lambertian reflectance. (3 points)

- Error explanation:

  The N.L term is not included. → nDotL
  We should multiply by max (L·N, 0)
  L = LIGHT DIRECTION, N = NORMAL

- Corrected Code of Line 2:

  vec4 lambert = mydiffuse * lightcolor * max (nDotL, 0.0) ;

- Mathematical Formula:

$$I = k_d \times L_c \times \max(\hat{L} \cdot \hat{N}, 0)$$

I: IMAGE INTENSITY
k_d: DIFFUSE COLOR, mydiffuse
L_c: LIGHT COLOR, lightcolor
$\hat{L}$: LIGHT DIRECTION (UNIT VECTOR) $= \vec{L}/\|\vec{L}\|$
$\hat{N}$ → SURFACE NORMAL UNIT VECTOR

ii. Please complete line 4. Also, provide the mathematical formula for Blinn-Phong reflectance. *nb: getting the exact GLSL syntax right is not important, but you should be able to convey the formula.* (3 points)

- Completed Line 4:

```
vec4 phong = myspecular * lightcolor
             * pow(max(nDotH, 0), myshininess);
                              ↑
                             0.0
```

- Mathematical Formula:

$$I = K_s * L_c * \left[\max(\bar{N} \cdot \bar{H}, 0)\right]^p$$

I: IMAGE INTENSITY

K_s: SPECULAR COLOR, myspecular

L_c: LIGHT COLOR, lightcolor

$\bar{N}$: SURFACE NORMAL (UNIT VECTOR)

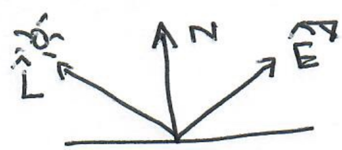
$\bar{H}$: HALF VECTOR (UNIT VEC.)

p = PHONG EXPONENT, myshininess

$$\bar{L} = \frac{\hat{L}}{\|\hat{L}\|}$$ UNIT VECTOR TO LIGHT

$$\bar{H} = \frac{\bar{L} + \bar{E}}{\|\bar{L} + \bar{E}\|}$$ ← NORMALIZE

$$\bar{E} = \frac{\hat{E}}{\|\hat{E}\|}$$ UNIT VECTOR TO EYE



Question 2: Short Answer (14 points)

(i) Assume you are trying to display a color (on a linear scale, not pre-corrected) (0.25, 1.0, 0.64) on a monitor with a gamma of 2.0. What color value should you actually send to the monitor with gamma correction so the correct colors are displayed? (3 points)

$I_0 =$ ORIG COLOR. $I = (I_0)^{1/\gamma} = (0.25, 1.0, 0.64)^{1/2}$

Answer: R: $\boxed{0.5}$ G: $\boxed{1.0}$ B: $\boxed{0.8}$

(ii) In OpenGL, consider the following stages:

- Rasterization
- Fragment Shader
- Vertex Shader

In what order are these stages executed in the graphics pipeline? Which of these stages are programmable versus fixed? (3 points)
(Please put a check after Programmable or Fixed for each stage below.)

Stage 1:	VERTEX SHADER	Programmable ✓	Fixed
Stage 2:	RASTERIZATION	Programmable	Fixed ✓
Stage 3:	FRAGMENT SHADER	Programmable ✓	Fixed

(iii) Briefly describe the difference between raster and vector graphics. (3 points)

IN RASTER GRAPHICS, IMAGES ARE REPRESENTED AS A (2D) GRID OF PIXELS (THE RASTER).

IN VECTOR GRAPHICS, IMAGES ARE STORED AS A SEQUENCE OF LINE SEGMENTS OR VECTORS.

OPTIONAL

NOTE THAT RASTER IMPLIES A PARTICULAR GRID OR PIXEL RESOLUTION. RESIZING WILL CAUSE IMAGE DEGRADATION. COMPLEX IMAGES AND PHOTOGRAPHS CAN BE STORED.

VECTOR GRAPHICS CAN BE STORED IN FLOATING POINT IN A RESOLUTION-INDEPENDENT WAY. BUT IT IS HARD TO REPRESENT PHOTOGRAPHS OR COMPLEX IMAGES.

(iv) For the eye position (0, 1, 0), looking at the origin (0, 0, 0), and with an up vector (1, 1, 0), what is the resulting orthonormal ***uvw*** basis (used for coordinate transforms and gluLookAt)? (5 points) (Please put your answer in the table below and use the room below to work)

P E (0,1,0)

O (0,0,0)

EYE LOOKS DOWN −W DIRN

$$\bar{W} = \frac{\bar{E} - \bar{O}}{\|\bar{E} - \bar{O}\|} = (0,1,0)$$

$$\bar{U} = \frac{\overline{UP} \times \bar{W}}{\|\overline{UP} \times \bar{W}\|} = (0,0,1)$$

$$(1,1,0) \times (0,1,0)$$

$$\begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{vmatrix} = \bar{k} \rightarrow (0,0,1)$$

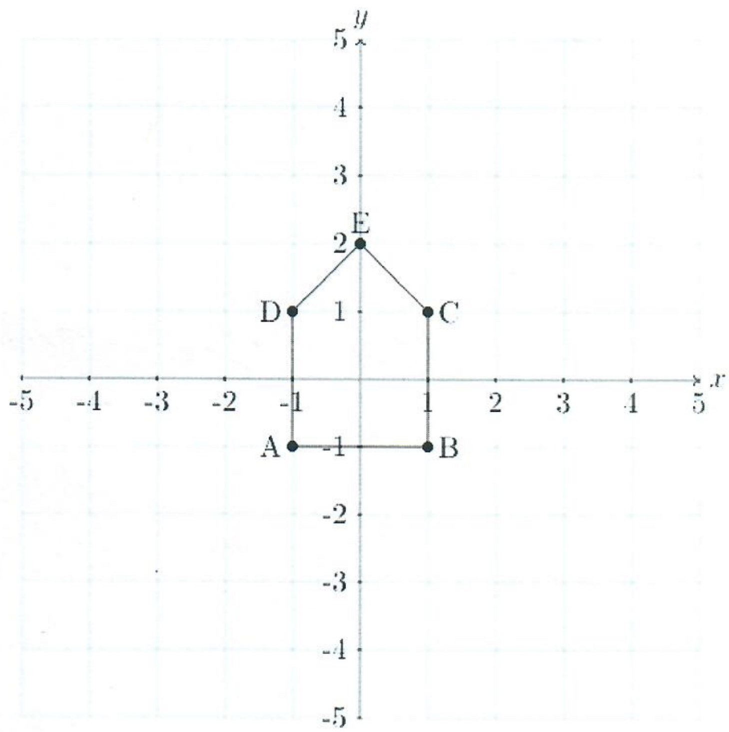
u	(0 , 0 , 1)
v	(1 , 0 , 0)
w	(0 , 1 , 0)

$$\bar{V} = \bar{W} \times \bar{U} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = \bar{i}$$

$$= (1,0,0)$$

Question 3: Transformations (20 points)

Consider a house in the xy-plane ($z = 0$) with vertices on the square ABCD, where $A = (-1, -1)$; $B = (1, -1)$; $C = (1, 1)$; $D = (-1, 1)$; with the apex $E = (0, 2)$.
For each of the following transformations, draw out the transformed house (marking each vertex) and provide the 4 by 4 matrix for the transform.
All of the transforms are independently applied to the rest configuration unless stated otherwise.

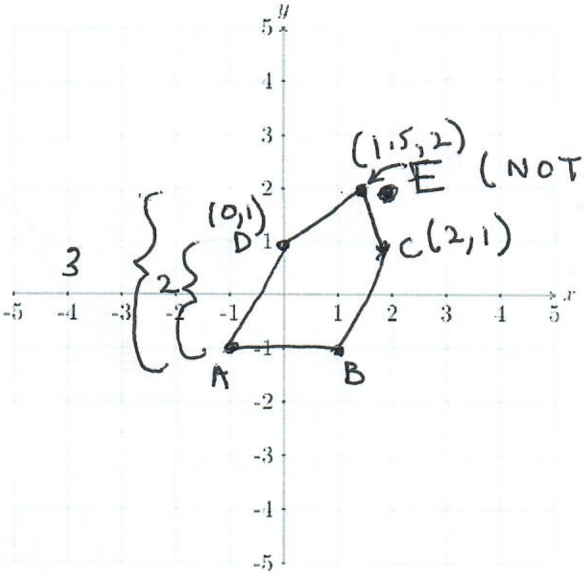


(i) A shear along the x-axis, so that the bottom (at $y = -1$) of the house is not affected, but the top of the square (at $y = +1$) is moved one unit to the right, i.e. $(-1, 1)$ is moved to $(0, 1)$ for example. Remember to provide the 4 by 4 transform.

Hint: first apply a translation in y to bring the lower part of the house to $y = 0$. (7 points)

Please make use of the coordinate grid and the answer box on the next page. Use the extra space on the next page to show your work.

Draw the transformed figure:



(NOTE MOVE BY $\frac{3}{2}$ AS MUCH AS CD = 1·5)

Transformation matrix:

$$\begin{bmatrix} \boxed{1} & \boxed{1/2} & \boxed{0} & \boxed{1/2} \\ \boxed{0} & \boxed{1} & \boxed{0} & \boxed{0} \\ \boxed{0} & \boxed{0} & \boxed{1} & \boxed{0} \\ \boxed{0} & \boxed{0} & \boxed{0} & \boxed{1} \end{bmatrix}$$

T(y, +1) $\longrightarrow \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

TRANSLATE Y +1

SHEAR IN X $\begin{pmatrix} 1 & 1/2 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

2 UNITS Y → 1 UNIT X

T(y, −1) $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

TRANSLATE BACK Y −1

MULTIPLY

$$\begin{pmatrix} 1 & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & +1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

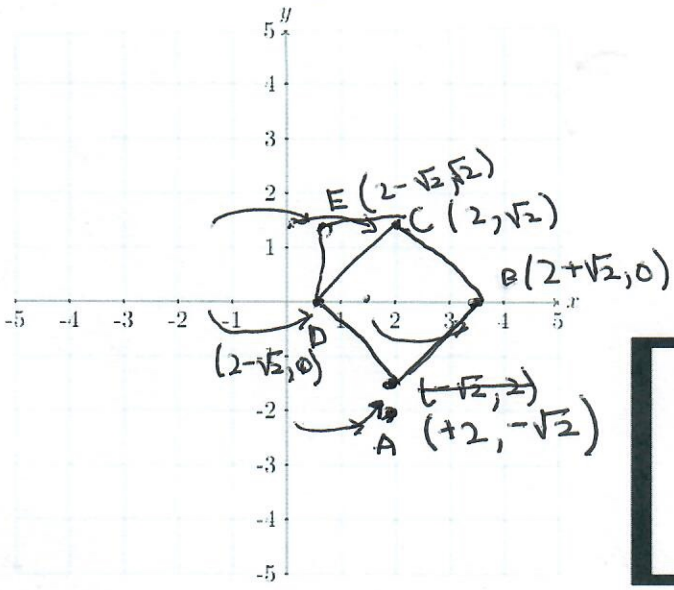
$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

CAN VERIFY MULT BY ABCDE GIVES RESULT IN FIGURE

$$= \begin{pmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(ii) [Independent question, not carried over from (i)]. Now, consider a rotation by 45 degrees counterclockwise (in the plane), followed by a translation of +2 units on the x-axis. Draw the resulting picture for the house, and write down the 4 by 4 transformation matrix. (7 points)

Draw the transformed figure:



Transformation matrix:

$$\begin{bmatrix} \boxed{1/\sqrt{2}} & \boxed{-1/\sqrt{2}} & \boxed{0} & \boxed{2} \\ \boxed{\frac{1}{\sqrt{2}}} & \boxed{\frac{1}{\sqrt{2}}} & \boxed{0} & \boxed{0} \\ \boxed{0} & \boxed{0} & \boxed{1} & \boxed{0} \\ \boxed{0} & \boxed{0} & \boxed{0} & \boxed{1} \end{bmatrix}$$

(Space for working)

$$R_{45} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \rightarrow \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$T_{+2} = \begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$TR = \begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 & 2 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

SINCE TRANSLATE SECOND, ROTATE SUBMATRIX TRANS ON RIGHT

(iii) Provide the inverse 4 by 4 transformation matrix for (ii). (6 points)

Inverse transformation matrix:

$$\begin{bmatrix} \boxed{1/\sqrt{2}} & \boxed{1/\sqrt{2}} & \boxed{0} & \boxed{-\sqrt{2}} \\ \boxed{-1/\sqrt{2}} & \boxed{1/\sqrt{2}} & \boxed{0} & \boxed{\sqrt{2}} \\ \boxed{0} & \boxed{0} & \boxed{1} & \boxed{0} \\ \boxed{0} & \boxed{0} & \boxed{0} & \boxed{1} \end{bmatrix}$$

(Space for working.)

ORIG MATRIX IS TR

INVERSE IS $R^{-1}T^{-1} \rightarrow R^t(-T)$

ROTATION IS $\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$ $\begin{pmatrix} -2 \\ 0 \end{pmatrix} \leftarrow$ TRANS

$$T' \Rightarrow \begin{pmatrix} -\sqrt{2} \\ \sqrt{2} \end{pmatrix}$$

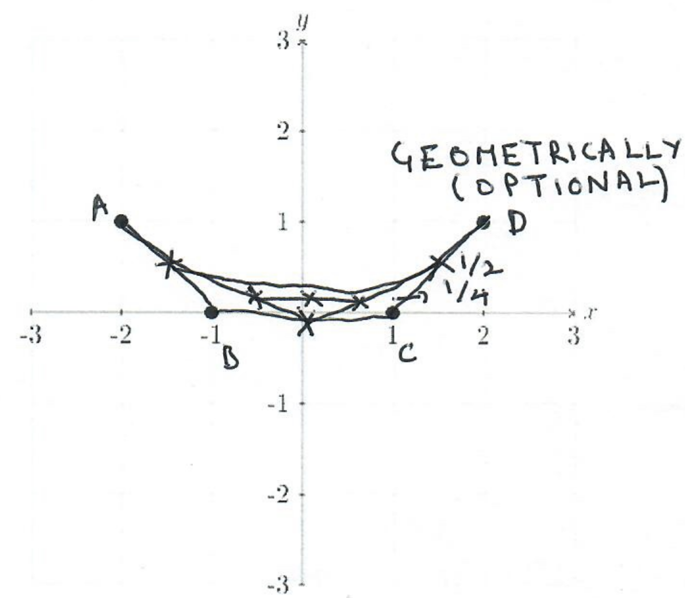
NET MATRIX $\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & -\sqrt{2} \\ \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & \sqrt{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ NOTE M = $\begin{pmatrix} R^T_{3\times3} & -R^T T_{3\times1} \\ 0_{1\times3} & 1_{1\times1} \end{pmatrix}$

Question 4: Curves (20 points)

(i) Consider a cubic Bezier curve in the plane with control points (−2, 1), (−1, 0), (1, 0), (2, 1). What are the end-points and mid-point of this Bezier curve? (8 points)

The grid is provided as an optional working sheet and does not form part of the answer.

	x	y
Left end-point	−2	1
Mid-point	0	1/4
Right end-point	2	1



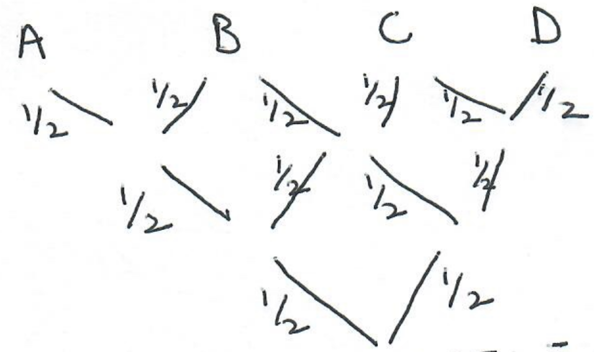
(Space for working)

LEFT END PT = A = (−2, 1)

RIGHT END PT = D = (2, 1)

MID PT → de CASTELJAU WITH WEIGHTS ½

COORDS SHOWN NOT POLAR FORM LABELS



$$\text{MID PT} = \frac{1}{8}A + \frac{3}{8}B + \frac{3}{8}C + \frac{1}{8}D$$

$$= \frac{1}{8}\left[(-2,1) + \frac{3}{\cancel{8}}(-1,0) + 3(1,0) + (2,1)\right]$$

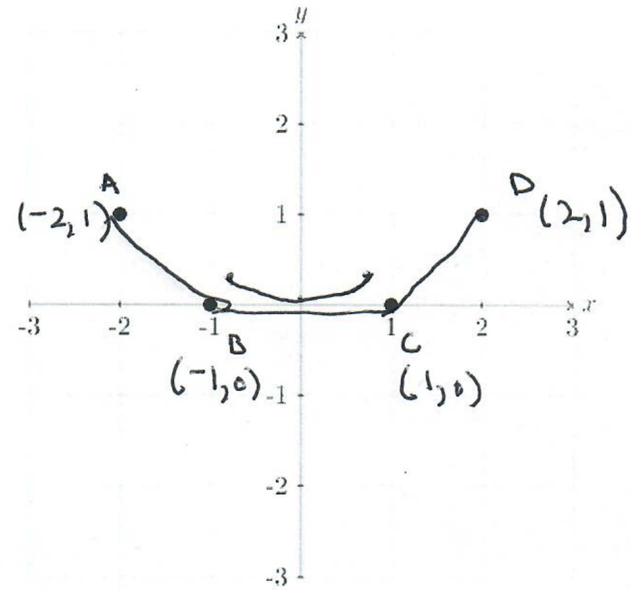
$$= \frac{1}{8}\left[(0,2)\right]$$

$$= \left(0, \frac{1}{4}\right)$$

(ii) Consider a uniform cubic B-spline curve with the same control points. What are the end-points and mid-point of the B-spline curve? (8 points)

The grid is provided as an optional working sheet and does not form part of the answer.

	x	y
Left end-point	$-5/6$	$1/6$
Mid-point	0	$1/24$
Right end-point	$5/6$	$1/6$



(Space for working)

SHOWING POLAR FORM LABELS $-2\ -1\ 0\ 1\ 2\ 3$

A B C D

-2-10 -101 012 123

LEFT A B C D

$-2 \frac{2}{}\ \frac{1}{0}\ +1$ $\ 1/3 \quad 2/3 \quad 2/3 \quad 1/3$

-100 001

$1/2 \quad 1/2$

000

LEFT END PT $= \frac{1}{6}A + \frac{4}{6}B + \frac{1}{6}C = \frac{1}{6}\left((-2,1) + 4(-1,0) + (1,0)\right)$

$= \frac{1}{6}(-5, 1) = \left(-\frac{5}{6}, \frac{1}{6}\right)$ LT

BY SYMMETRY RIGHT END PT

$= \frac{1}{6}B + \frac{4}{6}C + \frac{1}{6}D \rightarrow \left(\frac{5}{6}, \frac{1}{6}\right)$

RT $= \frac{1}{6}\left((-1,0) + 4(1,0) + (2,1)\right)$

RIGHT END PT $= \left(+\frac{5}{6}, \frac{1}{6}\right)$

FOR MID, deCASTELJAU

-2-10 -101 012 123

A B C D

$1/6 \quad 5/6 \quad 1/2 \quad 1/2 \quad 5/6 \quad 1/6$

$-10\tfrac{1}{2} \quad 01\tfrac{1}{2} \quad 12\tfrac{1}{2}$

$1/4 \quad 3/4 \quad 3/4 \quad 1/4$

$0\tfrac{1}{2}\tfrac{1}{2} \quad 1\tfrac{1}{2}\tfrac{1}{2}$

$1/2 \quad 1/2$

$\tfrac{1}{2}\tfrac{1}{2}\tfrac{1}{2}$

SHOW POLAR FORM LABELS

$\text{MID} = \frac{1}{48}A + \frac{23}{48}B + \frac{23}{48}C + \frac{1}{48}D$

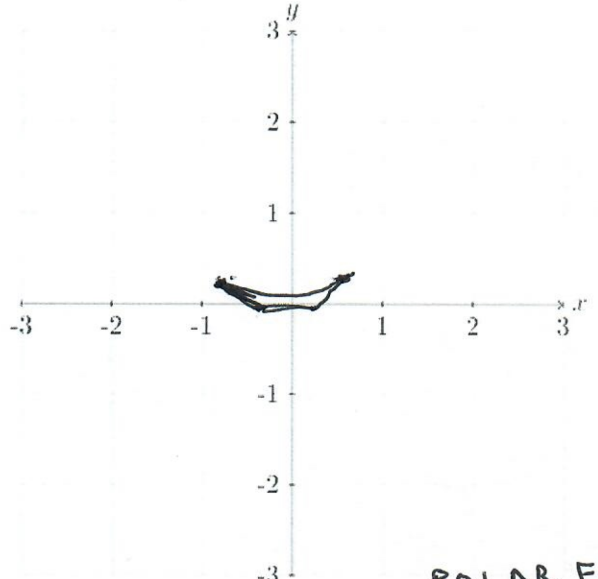
$= \frac{1}{48}\left[(-2,1) + 23(-1,0) + 23(1,0) + (2,1)\right]$

$= \frac{1}{48}(0,2) = 0, \frac{1}{24}$

(iii) What would be the control points of a cubic Bezier curve that reproduces (is identical to) the B-spline curve in (ii)? Provide the control points from left to right. (4 points)

The grid is provided as an optional working sheet and does not form part of the answer.

	x	y
Control point 1	$-5/6$	$1/6$
Control point 2	$-1/3$	0
Control point 3	$1/3$	0
Control point 4	$5/6$	$1/6$



(Space for working)

LEFT END PT = CPT 1 OF BEZIER = $(-5/6, 1/6)$ POLAR FORM 000

RIGHT END PT = CPT 4 OF BEZIER = $(5/6, 1/6)$ POLAR FORM 111

CPT 2 NEEDS 001 POLAR FORM LABEL BEZIER

CAN DO FROM B, C IN BSPLINE POLAR FORM LABELS

-1 01 B   012 C

1 2 / -1 0 2

$2/3$ \ / $1/3$ 001

$\frac{2}{3} B + \frac{1}{3} C$

COORDS $\frac{2}{3}(-1, 0) + \frac{1}{3}(1, 0)$

$= \left(-\frac{1}{3}, 0\right)$

SIMILARLY CPT 3 LABEL ← 011

-101 B   012 C   2 1 / 1 2

$1/3$ \ / $2/3$ 011

$\frac{1}{3} B + \frac{2}{3} C$

COORDS $\frac{1}{3}(-1, 0) + \frac{2}{3}(1, 0)$

$= \left(\frac{1}{3}, 0\right)$

END OF EXAMINATION