

Computer Graphics

CSE 167 [Win 23], Lectures 16, 17:

Nuts and bolts of Ray Tracing

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<http://viscomp.ucsd.edu/classes/cse167/wi23>

Acknowledgements: Thomas Funkhouser and Greg Humphreys

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Heckbert's Business Card Ray Tracer

```
typedef struct {double x,y,z;}vec;vec U,black,amb=[.02,.02,.02];struct sphere{ vec cen,color;
double rad,kd,ks,kt,kl,ir} *s,*best_sph;[0.6,.5,1.,1.,1.,.9,.05,.2,.85,0.,1.7,-1.,8,-.5,1.,.5,.2,1.,
.7,.3,0.,.05,1.2,1.,8,-.5,1.,.8,8,1.,.3,.7,0.,.0,1.2,3,-.6,.15,.1.,.8,1.,.7,.0.,.0.,.6,1.5,-.3,-.3,.12,
.8,1.,.1,5.,.0.,.0.,.5,1.5];yx;double u,b,tmin,sqrt(),tan();double vdot(A,B){vec A,B;(return A.x
*B.x+A.y*B.y+A.z*B.z);}vec vcomb(a,B){double a;vec A,B;(B.x+=a*A.x;B.y+=a*A.y;B.z+=a*A.z;
return B);}vec vunit(A){vec A;(return vcomb(1./sqrt(vdot(A,A)),A,black);}struct sphere*intersect
(P,D){vec P,D;(best=0;tmin=1e30;s= sph+5;while(s-->sph)b=vdot(D,U=vcomb(-1.,P,->cen)),
u=b*b-vdot(U,U)+s->rad*s->rad,u=u>0?sqrt(u):1e31,u=b-u>1e-7?b-u:b+u,tmin=u>1e-7&&
u<tmin?best=s,u:tmin;return best;}vec trace(level,P,D){vec P,D;(double d,eta,e;vec N,color;
struct sphere *s;if(!level-->return black;if(s=intersect(P,D));else return amb;color=amb;eta=
s->ir;d= -vdot(D,N=vunit(vcomb(-1.,P=vcomb(tmin,D,P),s->cen)));if(d<0)N=vcomb(-1.,N,black),
eta=1/eta,d= -d;|=sph+5;while(l-->sph)if((e=1->kt)*vdot(N,U=vunit(vcomb(-1.,P,->cen)))>0&&
intersect(P,U)=e)color=vcomb(e,->color,color);U=s->color;color.x*=U.x;color.y*=U.y;color.z
*=U.z;e=1-eta*eta*(1-d*d);return vcomb(s->ks,trace(level,P,vcomb(eta,D,vcomb(eta*d-
sqrt(e),N,black))););black,vcomb(s->ks,trace(level,P,vcomb(2*d,N,D)),vcomb(s->kd,color,vcomb
(s->kl,U,black))););main(){printf("%d %d\n",32,32);while(yx<32*32) U.x=yx%32*32/2,U.z=32/2-
yx%32,U.y=32/2-tan(25/114.5915590261),U=vcomb(255.,trace(3,black,vunit(U)),black),printf
("%%.0f %%.0f %%.0f\n",U);/"minray!"}
```

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To Do

- START EARLY on HW 4
- Milestone is due on Mar 10

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Outline

- Camera Ray Casting (choose ray directions)
- Ray-object intersections
- Ray-tracing transformed objects
- Lighting calculations
- Recursive ray tracing

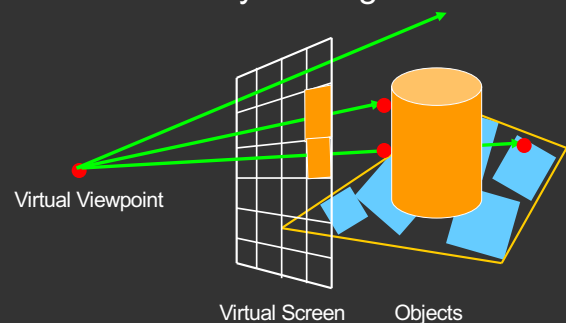
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Outline in Code

```
Image Raytrace (Camera cam, Scene scene, int width, int height)
{
    Image image = new Image (width, height) ;
    for (int i = 0 ; i < height ; i++)
        for (int j = 0 ; j < width ; j++) {
            Ray ray = RayThruPixel (cam, i, j) ;
            Intersection hit = Intersect (ray, scene) ;
            image[i][j] = FindColor (hit) ;
        }
    return image ;
}
```

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Ray Casting



Multiple objects are used to build the scene (as does OpenGL)

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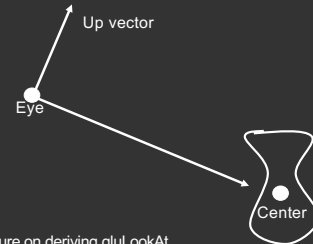
Finding Ray Direction

- Goal is to find ray direction for given pixel i and j
- Many ways to approach problem
 - Objects in world coord, find dirn of each ray (we do this)
 - Camera in canonical frame, transform objects (OpenGL)
- Basic idea
 - Ray has origin (camera center) and direction
 - Find direction given camera params and i and j
- Camera params as in gluLookAt
 - Lookfrom[3], LookAt[3], up[3], fov

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Similar to gluLookAt derivation

- `gluLookAt(eyex, eyey, eyez, centerx, centery, centerz, upx, upy, upz)`
- Camera at eye, looking at center, with up direction being up



From earlier lecture on deriving gluLookAt

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Constructing a coordinate frame?

- We want to associate w with a , and v with b
- But a and b are neither orthogonal nor unit norm
 - And we also need to find u

$$w = \frac{a}{\|a\|}$$

$$u = \frac{b \times w}{\|b \times w\|}$$

$$v = w \times u$$

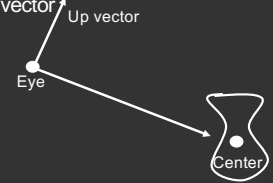
From basic math lecture - Vectors: Orthonormal Basis Frames

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Camera coordinate frame

$$w = \frac{a}{\|a\|} \quad u = \frac{b \times w}{\|b \times w\|} \quad v = w \times u$$

- We want to position camera at origin, looking down $-Z$ dirn
- Hence, vector a is given by **eye** – **center**
- The vector b is simply the **up** vector



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Canonical viewing geometry

$$ray = eye + t \left[\alpha u + \beta v - w \right]$$

$$\alpha = \tan\left(\frac{fov_x}{2}\right) \times \left(\frac{j - (width / 2)}{width / 2}\right) \quad \beta = \tan\left(\frac{fov_y}{2}\right) \times \left(\frac{(height / 2) - i}{height / 2}\right)$$

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Outline

- Camera Ray Casting (choosing ray directions)
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- Ray-tracing transformed objects
- Lighting calculations
- Recursive ray tracing

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Outline in Code

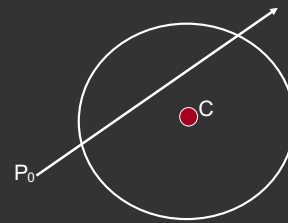
```
Image Raytrace (Camera cam, Scene scene, int width, int height)
{
    Image image = new Image (width, height) ;
    for (int i = 0 ; i < height ; i++)
        for (int j = 0 ; j < width ; j++) {
            Ray ray = RayThruPixel (cam, i, j) ;
            Intersection hit = Intersect (ray, scene) ;
            image[i][j] = FindColor (hit) ;
        }
    return image ;
}
```

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Ray-Sphere Intersection

$$\text{ray} \equiv \vec{P} = \vec{P}_0 + \vec{P}_1 t$$

$$\text{sphere} \equiv (\vec{P} - \vec{C}) \cdot (\vec{P} - \vec{C}) - r^2 = 0$$



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Ray-Sphere Intersection

$$\text{ray} \equiv \vec{P} = \vec{P}_0 + \vec{P}_1 t$$

$$\text{sphere} \equiv (\vec{P} - \vec{C}) \cdot (\vec{P} - \vec{C}) - r^2 = 0$$

Substitute

$$\text{ray} \equiv \vec{P} = \vec{P}_0 + \vec{P}_1 t$$

$$\text{sphere} \equiv (\vec{P}_0 + \vec{P}_1 t - \vec{C}) \cdot (\vec{P}_0 + \vec{P}_1 t - \vec{C}) - r^2 = 0$$

Simplify

$$t^2(\vec{P}_1 \cdot \vec{P}_1) + 2t\vec{P}_1 \cdot (\vec{P}_0 - \vec{C}) + (\vec{P}_0 - \vec{C}) \cdot (\vec{P}_0 - \vec{C}) - r^2 = 0$$

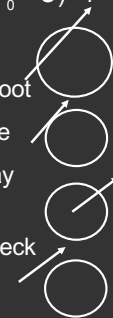
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Ray-Sphere Intersection

$$t^2(\vec{P}_1 \cdot \vec{P}_1) + 2t\vec{P}_1 \cdot (\vec{P}_0 - \vec{C}) + (\vec{P}_0 - \vec{C}) \cdot (\vec{P}_0 - \vec{C}) - r^2 = 0$$

Solve quadratic equations for t

- 2 real positive roots: pick smaller root
- Both roots same: tangent to sphere
- One positive, one negative root: ray origin inside sphere (pick + root)
- Complex roots: no intersection (check discriminant of equation first)



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Ray-Sphere Intersection

- Intersection point: $\text{ray} \equiv \vec{P} = \vec{P}_0 + \vec{P}_1 t$
- Normal (for sphere, this is same as coordinates in sphere frame of reference, useful other tasks)

$$\text{normal} = \frac{\vec{P} - \vec{C}}{|\vec{P} - \vec{C}|}$$

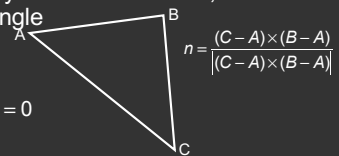
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Ray-Triangle Intersection

- One approach: Ray-Plane intersection, then check if inside triangle

- Plane equation:

$$\text{plane} \equiv \vec{P} \cdot \vec{n} - \vec{A} \cdot \vec{n} = 0$$



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Ray-Triangle Intersection

- One approach: Ray-Plane intersection, then check if inside triangle

- Plane equation:

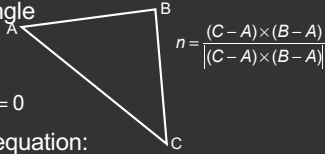
$$\text{plane} \equiv \vec{P} \cdot \vec{n} - \vec{A} \cdot \vec{n} = 0$$

- Combine with ray equation:

$$\text{ray} \equiv \vec{P} = \vec{P}_0 + \vec{P}_1 t$$

$$(\vec{P}_0 + \vec{P}_1 t) \cdot \vec{n} = \vec{A} \cdot \vec{n}$$

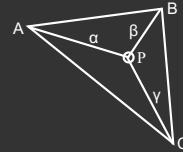
$$t = \frac{\vec{A} \cdot \vec{n} - \vec{P}_0 \cdot \vec{n}}{\vec{P}_1 \cdot \vec{n}}$$



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Ray inside Triangle

- Once intersect with plane, still need to find if in triangle
- Many possibilities for triangles, general polygons (point in polygon tests)
- We find parametrically [barycentric coordinates]. Also useful for other applications (texture mapping)



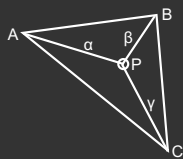
$$P = \alpha A + \beta B + \gamma C$$

$$\alpha \geq 0, \beta \geq 0, \gamma \geq 0$$

$$\alpha + \beta + \gamma = 1$$

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Ray inside Triangle



$$P = \alpha A + \beta B + \gamma C$$

$$\alpha \geq 0, \beta \geq 0, \gamma \geq 0$$

$$\alpha + \beta + \gamma = 1$$

$$P - A = \beta(B - A) + \gamma(C - A)$$

$$0 \leq \beta \leq 1, 0 \leq \gamma \leq 1$$

$$\beta + \gamma \leq 1$$

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Other primitives

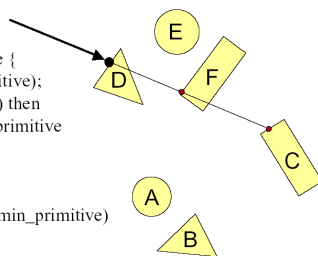
- Much early work in ray tracing focused on ray-primitive intersection tests
- Cones, cylinders, ellipsoids
- Boxes (especially useful for bounding boxes)
- General planar polygons
- Many more
- Consult chapter in Glassner (handed out) for more details and possible extra credit

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Ray Scene Intersection

Intersection FindIntersection(Ray ray, Scene scene)

```
{
  min_t = infinity
  min_primitive = NULL
  For each primitive in scene {
    t = Intersect(ray, primitive);
    if (t > 0 && t < min_t) then
      min_primitive = primitive
      min_t = t
  }
  return Intersect(min_t, min_primitive)
}
```



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Outline

- Camera Ray Casting (choosing ray directions)
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Transformed Objects

- E.g. transform sphere into ellipsoid
- Could develop routine to trace ellipsoid (compute parameters after transformation)
- May be useful for triangles, since triangle after transformation is still a triangle in any case
- But can also use original optimized routines

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Ray-Tracing Transformed Objects

- We have an optimized ray-sphere test
- But we want to ray trace an ellipsoid...
- Solution: Ellipsoid transforms sphere
- Apply inverse transform to ray, use ray-sphere
 - Allows for instancing (traffic jam of cars)
 - Same idea for other primitives

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Transformed Objects

- Consider a general 4x4 transform M
 - Will need to implement matrix stacks like in OpenGL
- Apply inverse transform M^{-1} to ray
 - Locations stored and transform in homogeneous coordinates
 - Vectors (ray directions) have homogeneous coordinate set to 0 [so there is no action because of translations]
- Do standard ray-surface intersection as modified
- Transform intersection back to actual coordinates
 - Intersection point p transforms as Mp
 - Distance to intersection if used may need recalculation
 - Normals n transform as $M^{-T}n$. Do all this before lighting

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Outline

- Camera Ray Casting (choosing ray directions)
- Ray-object intersections
- Ray-tracing transformed objects
- *Lighting calculations*
- Recursive ray tracing

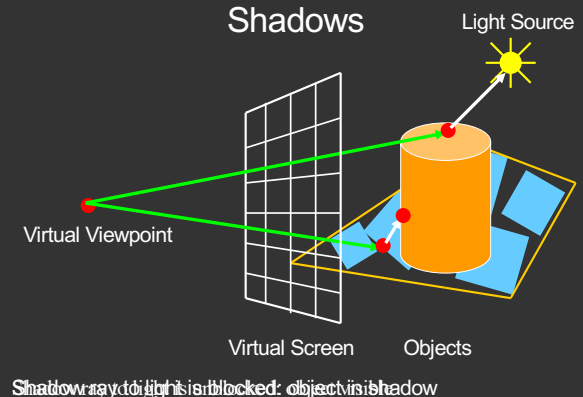
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Outline in Code

```
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{
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    for (int i = 0; i < height; i++)
        for (int j = 0; j < width; j++) {
            Ray ray = RayThruPixel (cam, i, j);
            Intersection hit = Intersect (ray, scene);
            image[i][j] = FindColor (hit);
        }
    return image;
}
```

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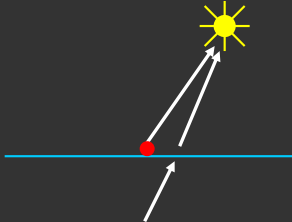
Shadows



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Shadows: Numerical Issues

- Numerical inaccuracy may cause intersection to be below surface (effect exaggerated in figure)
- Causing surface to incorrectly shadow itself
- Move a little towards light before shooting shadow ray



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Lighting Model

- Similar to OpenGL
 - Lighting model parameters (global)
 - Ambient r g b
 - Attenuation const linear quadratic
- $$L = \frac{L_0}{const + lin * d + quad * d^2}$$
- Per light model parameters
 - Directional light (direction, RGB parameters)
 - Point light (location, RGB parameters)
 - Some differences from HW 2 syntax

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Material Model

- Diffuse reflectance (r g b)
- Specular reflectance (r g b)
- Shininess s
- Emission (r g b)
- All as in OpenGL

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Shading Model

$$I = K_a + K_e + \sum_{i=1}^n V_i L_i (K_d \max(I_i \cdot n, 0) + K_s (\max(h_i \cdot n, 0))^s)$$

- Global ambient term, emission from material
- For each light, diffuse specular terms
- Note visibility/shadowing for each light (not in OpenGL)
- Evaluated per pixel per light (not per vertex)

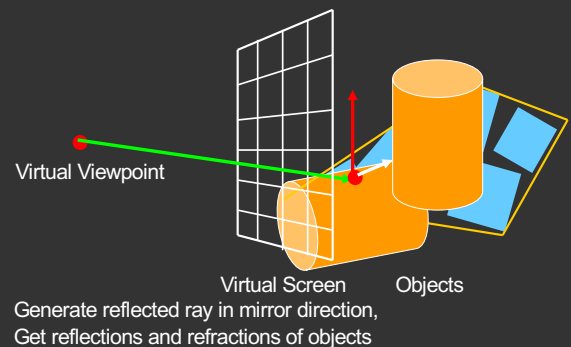
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Outline

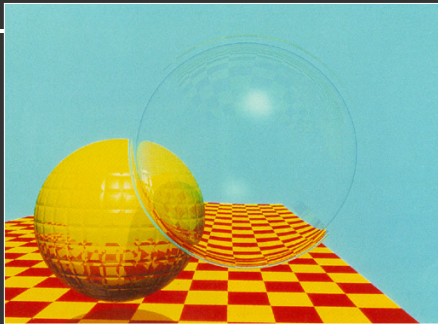
- Camera Ray Casting (choosing ray directions)
- Ray-object intersections
- Ray-tracing transformed objects
- Lighting calculations
- *Recursive ray tracing*

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Mirror Reflections/Refractions



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Turner Whitted 1980

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Basic idea

For each pixel

- Trace Primary Eye Ray, find intersection
- Trace Secondary Shadow Ray(s) to all light(s)
 - Color = Visible ? Illumination Model : 0 ;
- Trace Reflected Ray
 - Color += reflectivity * Color of reflected ray

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Recursive Shading Model

$$I = K_d + K_r + \sum_{i=1}^n V_i L_i (K_d \max(|I_i \cdot n|, 0) + K_s (\max(h_i \cdot n, 0))^s) + K_t I_t + K_r I_r$$

- Highlighted terms are recursive specularities [mirror reflections] and transmission (latter is extra credit)
- Trace secondary rays for mirror reflections and refractions, include contribution in lighting model
- GetColor calls RayTrace recursively (the I values in equation above of secondary rays are obtained by recursive calls)

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Problems with Recursion

- Reflection rays may be traced forever
- Generally, set maximum recursion depth
- Same for transmitted rays (take refraction into account)

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Effects needed for Realism

- (Soft) Shadows
- Reflections (Mirrors and Glossy)
- Transparency (Water, Glass)
- Interreflections (Color Bleeding)
- Complex Illumination (Natural, Area Light)
- Realistic Materials (Velvet, Paints, Glass)

Discussed in this lecture so far

Not discussed but possible with distribution ray tracing

Hard (but not impossible) with ray tracing; radiosity methods

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Some basic add ons

- Area light sources and soft shadows: break into grid of n x n point lights
 - Use jittering: Randomize direction of shadow ray within small box for given light source direction
 - Jittering also useful for antialiasing shadows when shooting primary rays
- More complex reflectance models
 - Simply update shading model
 - But at present, we can handle only mirror global illumination calculations

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Acceleration

Testing each object for each ray is slow

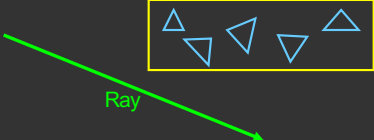
- Fewer Rays
 - Adaptive sampling, depth control
- Generalized Rays
 - Beam tracing, cone tracing, pencil tracing etc.
- Faster Intersections
 - Optimized Ray-Object Intersections
 - ***Fewer Intersections***

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Acceleration Structures

Bounding boxes (possibly hierarchical)

If no intersection bounding box, needn't check objects



Bounding Box

Ray

Spatial Hierarchies (Oct-trees, kd trees, BSP trees)

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Bounding Volume Hierarchies 1

- Build hierarchy of bounding volumes
 - Bounding volume of interior node contains all children

The diagram illustrates the construction of a Bounding Volume Hierarchy (BVH) for a set of 2D objects. On the left, a tree structure shows the hierarchy: a root node 1 (green circle) contains nodes 2 (blue circle) and 3 (blue circle). Node 2 contains nodes A (yellow circle) and B (yellow triangle). Node 3 contains nodes D (yellow triangle), E (yellow circle), and F (yellow rectangle). On the right, a 2D scene shows the objects A, B, C, D, E, and F. A green dashed box labeled 1 encloses all objects. A blue dashed box labeled 2 encloses objects A and B. A blue dashed box labeled 3 encloses objects D, E, and F.

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Bounding Volume Hierarchies 2

- Use hierarchy to accelerate ray intersections
 - Intersect node contents only if hit bounding volume

The diagram illustrates a Bounding Volume Hierarchy (BVH) for ray intersection acceleration. On the left, a tree structure shows nodes 1, 2, 3, and C. Node 1 is the root, containing nodes 2, C, and 3. Node 2 contains nodes A and B. Node 3 contains nodes D, E, and F. Node C is a leaf. The right side shows a 3D scene with objects A, B, C, D, E, and F, each with a bounding volume. A ray is shown intersecting the bounding volume of node C, which is highlighted with a red dashed box.

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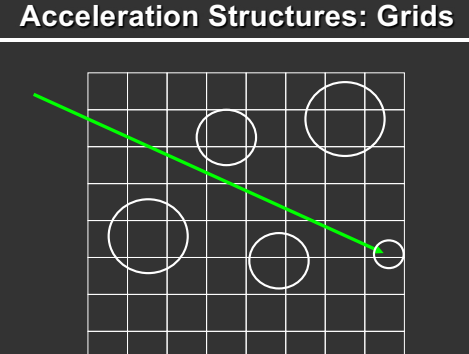
Bounding Volume Hierarchies 3

- Sort hits & detect early termination

```
FindIntersection(Ray ray, Node node)
{
    // Find intersections with child node bounding volumes
    ...
    // Sort intersections front to back
    ...
    // Process intersections (checking for early termination)
    min_t = infinity;
    for each intersected child i {
        if (min_t < bv_t[i]) break;
        shape_t = FindIntersection(ray, child);
        if (shape_t < min_t) { min_t = shape_t; }
    }
    return min_t;
}
```

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Acceleration Structures: Grids



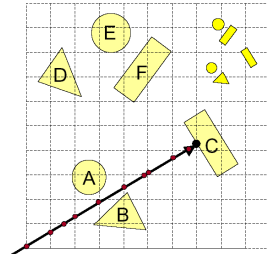
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Uniform Grid: Problems

- Potential problem:
 - How choose suitable grid resolution?

Too little benefit
if grid is too coarse

Too much cost
if grid is too fine

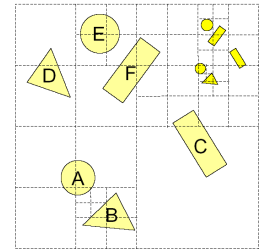


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Octree

- Construct adaptive grid over scene
 - Recursively subdivide box-shaped cells into 8 octants
 - Index primitives by overlaps with cells

Generally fewer cells

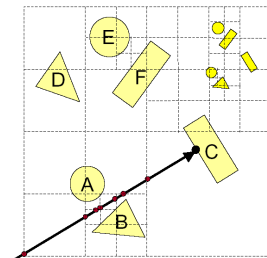


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Octree traversal

- Trace rays through neighbor cells
 - Fewer cells
 - More complex neighbor finding

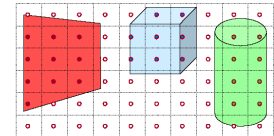
Trade-off fewer cells for
more expensive traversal



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Other Accelerations

- Screen space coherence
 - Check last hit first
 - Beam tracing
 - Pencil tracing
 - Cone tracing
- Memory coherence
 - Large scenes
- Parallelism
 - Ray casting is "embarrassingly parallelizable"
- etc.



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CAPE Evaluations

- Fill out now, can be done on phone
- Enthusiasm important to future offerings
- Comments useful to future years
- Some key innovations: modern OpenGL, GLSL; feedback servers (including code), UCSD online, ...
- Separately, please also evaluate the TAs

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Ray Tracing Acceleration Structures

- Bounding Volume Hierarchies (BVH)
- Uniform Spatial Subdivision (Grids)
- Binary Space Partitioning (BSP Trees)
 - Axis-aligned often for ray tracing: kd-trees
- Conceptually simple, implementation a bit tricky
 - Lecture relatively high level: Start early, go to section
 - Remember that acceleration a small part of grade

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Math of 2D Bounding Box Test

- Can you find a t in range

$$t > 0$$

$$t_{xmin} \leq t \leq t_{xmax}$$

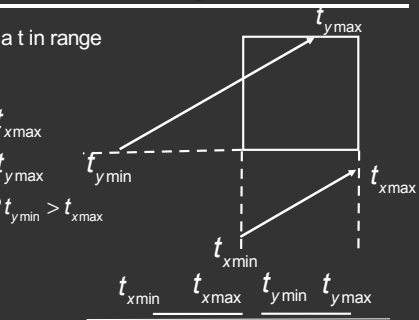
$$t_{ymin} \leq t \leq t_{ymax}$$

$$\text{if } t_{xmin} > t_{ymax} \text{ OR } t_{ymin} > t_{xmax}$$

return false;

else

return true;

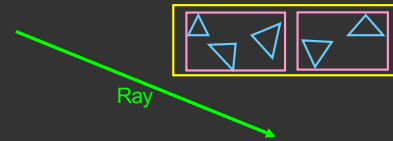


No intersection if x and y ranges don't overlap

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Bounding Box Test

- Ray-Intersection is simple coordinate check
- Intricacies with test, see book
- Hierarchical Bounding Boxes



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Hierarchical Bounding Box Test

- If ray hits root box
 - Intersect left subtree
 - Intersect right subtree
 - Merge intersections (find closest one)
- Standard hierarchical traversal
 - But caveat, since bounding boxes may overlap
- At leaf nodes, must intersect objects

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Creating Bounding Volume Hierarchy

```
function bvh-node::create (object array A, int AXIS)
    N = A.length();
    if (N == 1) {left = A[0]; right = NULL; bbox = bound(A[0]);}
    else if (N == 2) {
        left = A[0]; right = A[1];
        bbox = combine(bound(A[0]), bound(A[1]));
    }
    else
        Find midpoint m of bounding box of A along AXIS
        Partition A into lists of size k and N-k around m
        left = new bvh-node (A[0...k], (AXIS+1) mod 3);
        right = new bvh-node (A[k+1...N-1], (AXIS+1) mod 3);
        bbox = combine (left -> bbox, right -> bbox);
```

From page 305 of book

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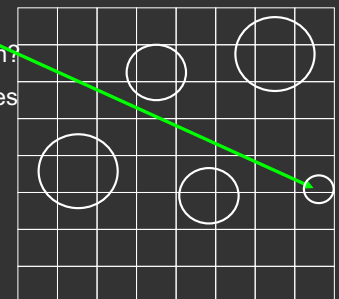
Uniform Spatial Subdivision

- Different idea: Divide space rather than objects
- In BVH, each object is in one of two sibling nodes
 - A point in space may be inside both nodes
- In spatial subdivision, each space point in one node
 - But object may lie in multiple spatial nodes
- Simplest is uniform grid (have seen this already)
- Challenge is keeping all objects within cell
- And in traversing the grid

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Traversal of Grid High Level

- Next Intersect Pt?
- Irreg. samp. pattern?
- But regular in planes
- Fast algo. possible
- (more on board)



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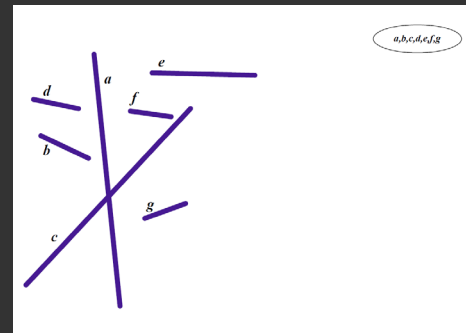
BSP Trees

- Used for visibility and ray tracing
 - Book considers only axis-aligned splits for ray tracing
 - Sometimes called kd-tree for axis aligned
- Split space (binary space partition) along planes
- Fast queries and back-to-front (painter's) traversal
- Construction is conceptually simple
 - Select a plane as root of the sub-tree
 - Split into two children along this root
 - Random polygon for splitting plane (may need to split polygons that intersect it)

BSP slides courtesy Prof. O'Brien

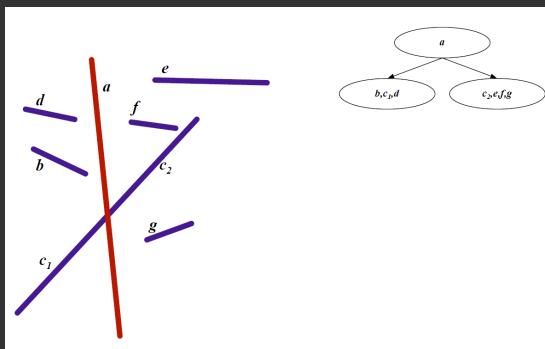
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Initial State



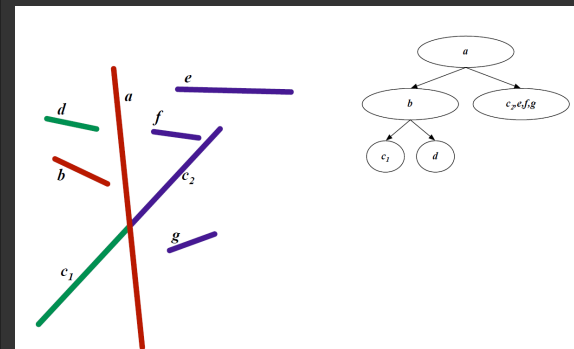
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First Split



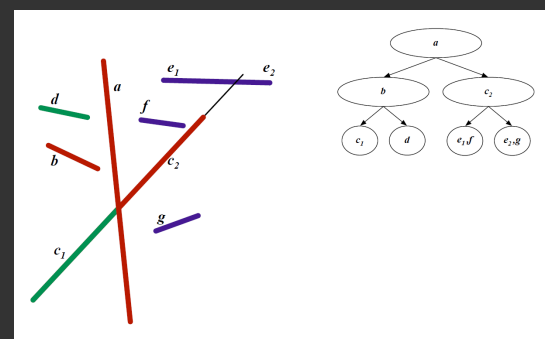
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Second Split



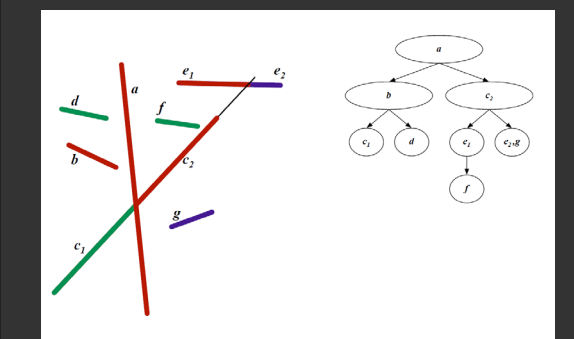
64

Third Split



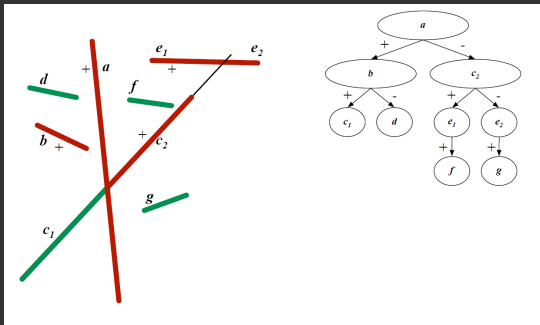
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Fourth Split



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Final BSP Tree



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BSP Trees Cont'd

- Continue splitting until leaf nodes
- Visibility traversal in order
 - Child one
 - Root
 - Child two
- Child one chosen based on viewpoint
 - Same side of sub-tree as viewpoint
- BSP tree built once, used for all viewpoints
 - More details in book
- 168 lectures (UCSD online) more detail re acceln

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Interactive Raytracing

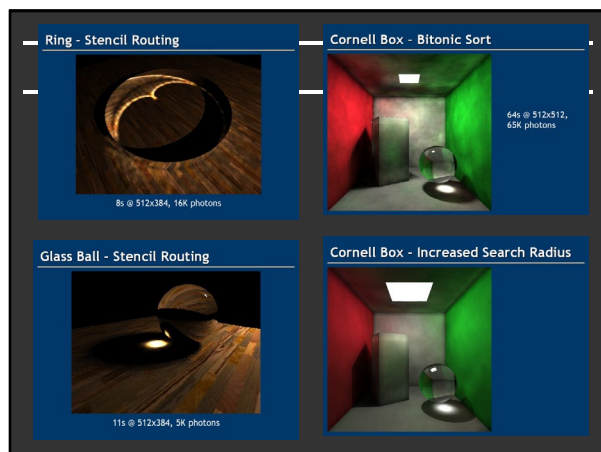
- Ray tracing historically slow
- Now viable alternative for complex scenes
 - Key is sublinear complexity with acceleration; need not process all triangles in scene
- Allows many effects hard in hardware
- Today graphics hardware and software (NVIDIA Optix 5, RTX chips claim 10G rays per second).
- Tiger Demo: [Video](#)

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Raytracing on Graphics Hardware

- Modern Programmable Hardware general streaming architecture
 - Can map various elements of ray tracing
 - Kernels like eye rays, intersect etc.
 - In vertex or fragment programs
 - Convergence between hardware, ray tracing
- [Purcell et al. 2002, 2003]
- <http://graphics.stanford.edu/papers/photongfx>

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