

Outline

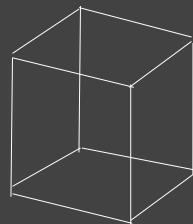
- Orthographic projection (simpler)
- Perspective projection, basic idea
- Derivation of gluPerspective (handout: glFrustum)
- Brief discussion of nonlinear mapping in z

Projections

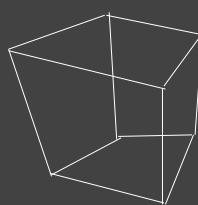
- To lower dimensional space (here 3D \rightarrow 2D)
- Preserve straight lines
- Trivial example: Drop one coordinate (Orthographic)

Orthographic Projection

- Characteristic: Parallel lines remain parallel
- Useful for technical drawings etc.



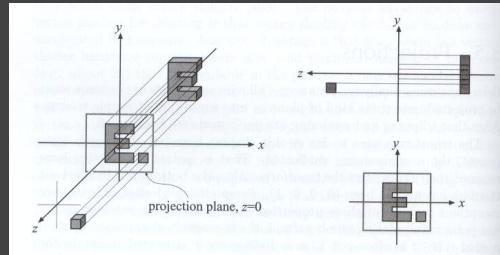
Orthographic



Perspective

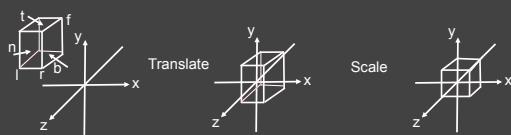
Example

- Simply project onto xy plane, drop z coordinate



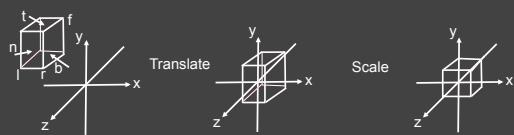
In general

- We have a cuboid that we want to map to the normalized or square cube from $[-1, +1]$ in all axes
- We have parameters of cuboid (l, r, t, b, n, f)



Orthographic Matrix

- First center cuboid by translating
- Then scale into unit cube

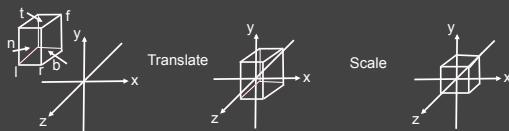


Transformation Matrix

$$M = \begin{pmatrix} \text{Scale} & \text{Translation (centering)} \\ \begin{pmatrix} \frac{2}{r-l} & 0 & 0 & 0 \\ 0 & \frac{2}{t-b} & 0 & 0 \\ 0 & 0 & \frac{2}{f-n} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} & \begin{pmatrix} 1 & 0 & 0 & -\frac{l+r}{2} \\ 0 & 1 & 0 & -\frac{t+b}{2} \\ 0 & 0 & 1 & -\frac{f+n}{2} \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{pmatrix}$$

Caveats

- Looking down $-z$, f and n are negative ($n > f$)
- OpenGL convention: positive n, f , negate internally



Final Result

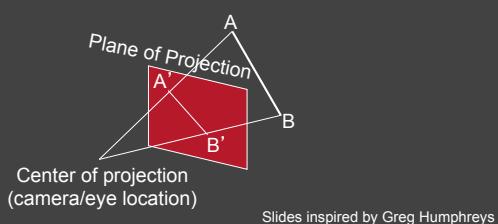
$$M = \begin{pmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & \frac{2}{f-n} & -\frac{f+n}{f-n} \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad gOrtho = \begin{pmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & \frac{-2}{f-n} & -\frac{f+n}{f-n} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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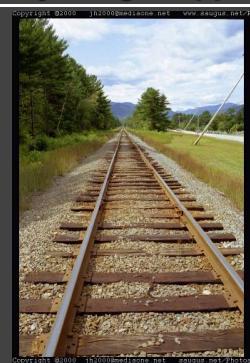
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Perspective Projection

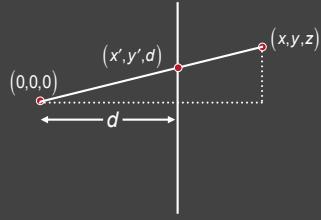
- Most common computer graphics, art, visual system
- Further objects are smaller (size, inverse distance)
- Parallel lines not parallel; converge to single point



Funny things happen...



Overhead View of Our Screen



Looks like we've got some nice similar triangles here?

$$\frac{x}{z} = \frac{x'}{d} \Rightarrow x' = \frac{d * x}{z} \quad \frac{y}{z} = \frac{y'}{d} \Rightarrow y' = \frac{d * y}{z}$$

In Matrices

- Note negation of z coord (focal plane $-d$)
- (Only) last row affected (no longer $0 \ 0 \ 0 \ 1$)
- w coord will no longer = 1. Must divide at end

$$P = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{d} & 0 \end{pmatrix}$$

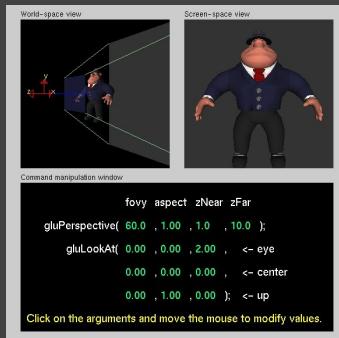
Verify

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{d} & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = ? \quad \begin{pmatrix} x \\ y \\ z \\ -\frac{z}{d} \end{pmatrix} = \begin{pmatrix} -\frac{d * x}{z} \\ -\frac{d * y}{z} \\ -d \\ 1 \end{pmatrix}$$

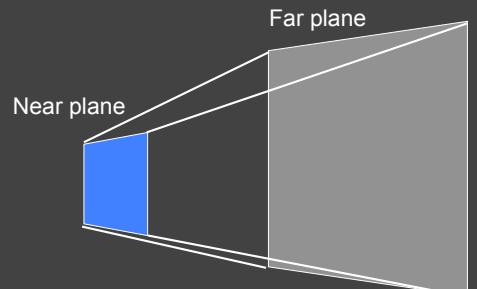
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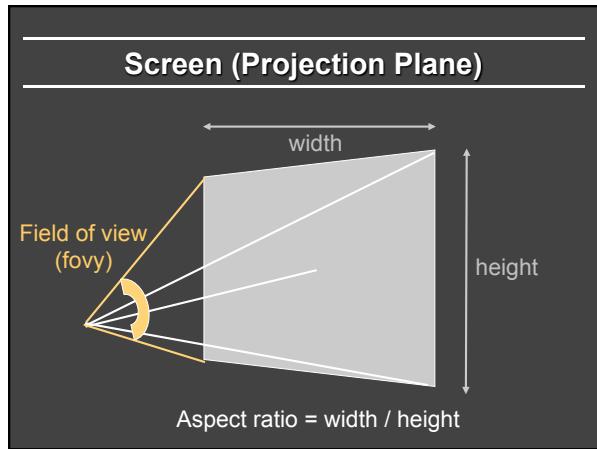
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Remember projection tutorial



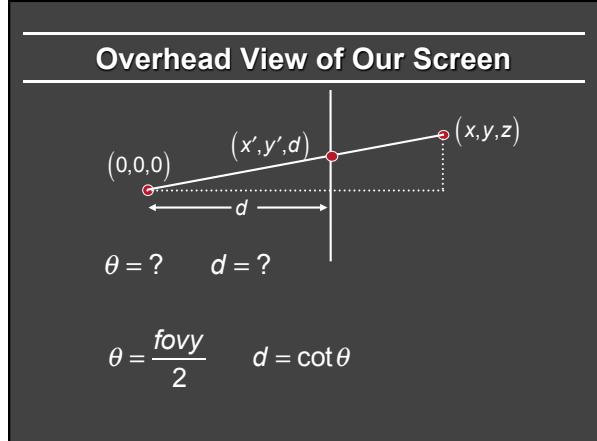
Viewing Frustum





gluPerspective

- gluPerspective(fovy, aspect, zNear > 0, zFar > 0)
- Fovy, aspect control fov in x, y directions
- zNear, zFar control viewing frustum



In Matrices

- Simplest form:

$$P = \begin{pmatrix} \frac{1}{\text{aspect}} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{d} & 0 \end{pmatrix}$$

- Aspect ratio taken into account
- Homogeneous, simpler to multiply through by d
- Must map z vals based on near, far planes (not yet)

In Matrices

$$P = \begin{pmatrix} \frac{1}{\text{aspect}} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{d} & 0 \end{pmatrix} \rightarrow \begin{pmatrix} \frac{d}{\text{aspect}} & 0 & 0 & 0 \\ 0 & d & 0 & 0 \\ 0 & 0 & A & B \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

- A and B selected to map n and f to -1, +1 respectively

Z mapping derivation

$$\begin{pmatrix} A & B \\ -1 & 0 \end{pmatrix} \begin{pmatrix} z \\ 1 \end{pmatrix} = ? \quad \begin{pmatrix} Az + B \\ -z \end{pmatrix} = -A - \frac{B}{z}$$

- Simultaneous equations?

$$-A + \frac{B}{n} = -1 \quad A = -\frac{f+n}{f-n}$$

$$-A + \frac{B}{f} = +1 \quad B = -\frac{2fn}{f-n}$$

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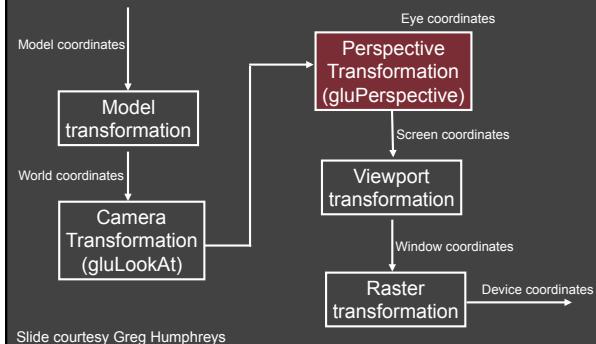
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Mapping of Z is nonlinear

$$\begin{pmatrix} Az + B \\ -z \end{pmatrix} = -A - \frac{B}{z}$$

- Many mappings proposed: all have nonlinearities
- Advantage: handles range of depths (10cm – 100m)
- Disadvantage: depth resolution not uniform
- More close to near plane, less further away
- Common mistake: set near = 0, far = infinity. Don't do this. Can't set near = 0; lose depth resolution.
- We discuss this more in review session

Summary: The Whole Viewing Pipeline



Slide courtesy Greg Humphreys