Advanced Computer Graphics

CSE 163 [Spring 2018], Lecture 9

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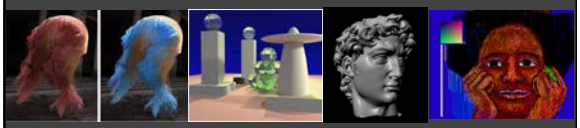


To Do

- Assignment 2 due May 18
  - Should already be well on way. This is last part
  - Contact us for difficulties etc.

Resources

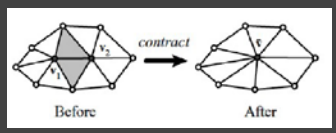
- Garland and Heckbert SIGGRAPH 97 paper
- Garland website, implementation notes (in thesis)
- Notes in this and previous lectures

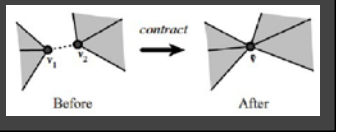
Surface Simplification: Goals (Garland)

- Efficiency (70000 to 100 faces in 15s in 1997)
- High quality, feature preserving (primary appearance emphasized rather than topology)
- Generality, non-manifold models, collapse disjoint regions

Simplifications

- Pair contractions in addition to edge collapses
- Previously connected regions may come together



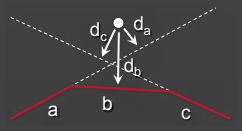


Algorithm Outline

- Restrict process to a set of *valid pairs*:
  - $(\mathbf{v}_i, \mathbf{v}_j)$ is an edge, or
  - $||\mathbf{v}_i - \mathbf{v}_j|| < t$, a threshold
    - $t = 0$ restricts to edge contraction
    - $t \gg 0$ can connect distant regions or yield $O(n^2)$ pairs
- Iteratively remove *best* pair and update valid pairs list:
  - Each vertex has a set with the pairs it belongs to:
    - $\mathbf{v}_i \rightarrow \text{Pairs}(\mathbf{v}_i)$
    - $(\mathbf{v}_i, \mathbf{v}_j) \rightarrow \bar{\mathbf{v}} \Rightarrow \text{Pairs}(\bar{\mathbf{v}}) = \text{Pairs}(\mathbf{v}_i) \cup \text{Pairs}(\mathbf{v}_j)$
- But how to choose *best* pair?

Quadric Error Metrics

- Based on point-to-plane distance
- Better quality than point-to-point



Background: Computing Planes

- Each triangle in mesh has associated plane

$$ax + by + cz + d = 0$$

- For a triangle, find its (normalized) normal using cross products

$$\vec{n} = \frac{AB \times AC}{|AB \times AC|} \qquad \vec{n} \bullet \vec{v} - \vec{A} \bullet \vec{n} = 0$$

- Plane equation?

$$\vec{n} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \qquad d = -\vec{A} \bullet \vec{n}$$

Survey

Quadric Error Metrics

- Sum of squared distances from vertex to planes:

$$\underset{\mathbf{v}}{\Delta} = \sum_{\mathbf{p}} Dist(\mathbf{v},\mathbf{p})^2$$

$$\mathbf{v} = \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}, \quad \mathbf{p} = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$$

$$Dist(\mathbf{v},\mathbf{p}) = ax + by + cz + d = \mathbf{p}^T\mathbf{v}$$

Quadric Error Metrics

$$\begin{aligned} \Delta &= \sum_{\mathbf{p}} (\mathbf{p}^T\mathbf{v})^2 \\ &= \sum_{\mathbf{p}} \mathbf{v}^T\mathbf{p}\mathbf{p}^T\mathbf{v} \\ &= \mathbf{v}^T\left(\sum_{\mathbf{p}} \mathbf{p}\mathbf{p}^T\right)\mathbf{v} \\ &= \mathbf{v}^T\mathbf{Q}\mathbf{v} \end{aligned}$$

- Common mathematical trick: quadratic form = symmetric matrix Q multiplied twice by a vector

Quadric Error Metrics

- Simply a 4x4 symmetric matrix
- Storage efficient: 10 floating point numbers per vertex
- Initially, error is 0 for all vertices

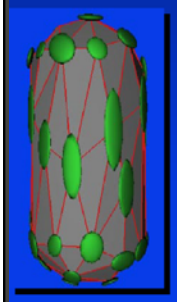
$$\mathbf{v}^T\mathbf{Q}\mathbf{v} = \begin{bmatrix} x & y & z & 1 \end{bmatrix} \begin{bmatrix} q_{11} & q_{12} & q_{13} & q_{14} \\ q_{12} & q_{22} & q_{23} & q_{24} \\ q_{13} & q_{23} & q_{33} & q_{34} \\ q_{14} & q_{24} & q_{34} & q_{44} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Quadric Error Metrics

- 2nd degree polynomial in x, y and z
- Level surface ($v^T Qv = k$) is a quadric surface
  - Ellipsoid, paraboloid, hyperboloid, plane etc.

$$v^T Qv = q_{11}x^2 + 2q_{12}xy + 2q_{13}xz + 2q_{14}x + q_{22}y^2 + 2q_{23}yz + 2q_{24}y + q_{33}z^2 + 2q_{34}z + q_{44}$$

But What Are These Quadrics Really Doing?



Almost always ellipsoids

- When **Q** is positive definite

Characterize error at vertex

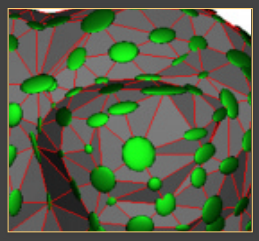
- Vertex at center of each ellipsoid
- Move it anywhere on ellipsoid with constant error

Capture local shape of surface

- Stretch in least curved direction

Quadric Visualization

- Ellipsoids: iso-error surfaces
- Smaller ellipsoid = greater error for a given motion
- Lower error for motion parallel to surface
- Lower error in flat regions than at corners
- Elongated in "cylindrical" regions



Using Quadrics

Approximate error of edge collapses

- Each vertex v has associated quadric Q
- Error of collapsing v_1 and v_2 to v' is $v'^T Q_1 v' + v'^T Q_2 v'$
- Quadric for new vertex v' is $Q' = Q_1 + Q_2$

Using Quadrics

- Find optimal location v' after collapse:

$$\mathbf{Q}' = \begin{bmatrix} q_{11} & q_{12} & q_{13} & q_{14} \\ q_{12} & q_{22} & q_{23} & q_{24} \\ q_{13} & q_{23} & q_{33} & q_{34} \\ q_{14} & q_{24} & q_{34} & q_{44} \end{bmatrix}$$

$$\min_{\mathbf{v}'} \mathbf{v}'^T \mathbf{Q}' \mathbf{v}' : \quad \frac{\partial}{\partial x} = \frac{\partial}{\partial y} = \frac{\partial}{\partial z} = 0$$

Using Quadrics

- Find optimal location v' after collapse:

$$\begin{bmatrix} q_{11} & q_{12} & q_{13} & q_{14} \\ q_{12} & q_{22} & q_{23} & q_{24} \\ q_{13} & q_{23} & q_{33} & q_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{v}' = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\mathbf{v}' = \begin{bmatrix} q_{11} & q_{12} & q_{13} & q_{14} \\ q_{12} & q_{22} & q_{23} & q_{24} \\ q_{13} & q_{23} & q_{33} & q_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

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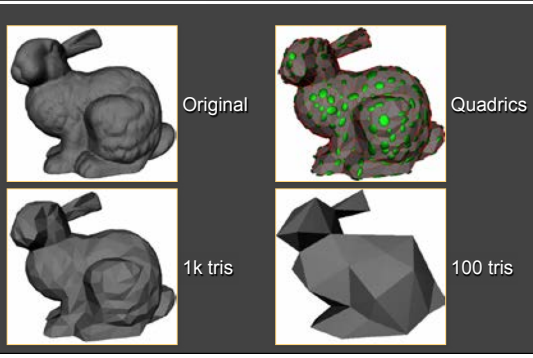
Algorithm Summary

- Compute the Q matrices for all the initial vertices
- Select all valid pairs
- Compute the optimal contraction target v for each valid pair. The error $v^T(Q_1+Q_2)v$ of this target vertex becomes the *cost* of contracting that pair
- Place all pairs in a heap keyed on cost with minimum cost pair on the top
- Iteratively remove the least cost pair, contract this pair, and update the costs of all valid pairs of interest

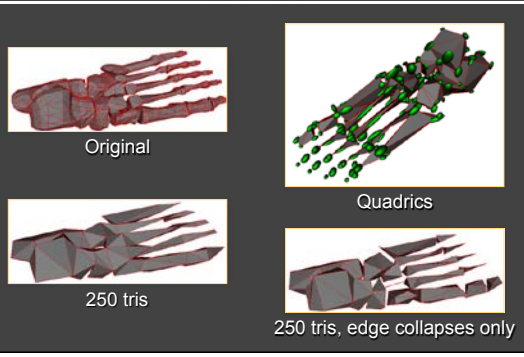
Final Algorithm

- Compute Q_i for all vertices $\mathbf{v}_i$
- Determine valid pairs
- Compute optimal contraction target and associated quadric error for each pair
- Place all pairs in a heap, ordered by smallest error
- Repeat
  - Get least error pair $(\mathbf{v}_i, \mathbf{v}_j)$ from heap
  - Contract pair (move edges to $\bar{\mathbf{v}}$, remove degenerate planes)
  - Update cost for all pairs involving $\mathbf{v}_i$ and $\mathbf{v}_j$
- Until done.

Results



Results



Additional Details

- Preserving boundaries/discontinuities (weight quadrics by appropriate penalty factors)
- Preventing mesh inversion (flipping of orientation): compare normal of neighboring faces, before after
- Has been modified for many other applications
  - E.g. in silhouettes, want to make sure volume always increases, never decreases
  - Take color and texture into account (followup paper)
- See paper, other more recent works for details

Implementation Tips

- Incremental, test, debug, simple cases
- Find good data structure for heap etc.
- May help to visualize error quadrics if possible
- Challenging, but hopefully rewarding assignment

Questions?

- Issues or questions?
- All the material for assignment covered
- Start early, work hard

- Brief discussion of survey