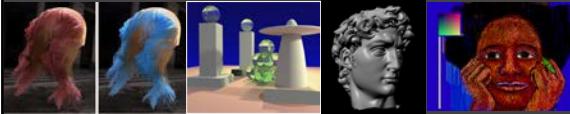


## Advanced Computer Graphics

CSE 163 [Spring 2018], Lecture 8

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### To Do

- Assignment 1, Due Apr 27
  - Any last minute issues or difficulties?
- Assignment 2 due May 18
  - Please START EARLY. Can do most after this week
  - Contact us for difficulties, help finding partners etc.

### Outline

- *Basic assignment overview*
- *Detailed discussion of mesh simplification*
- Progressive meshes
- Quadric error metrics

### Assignment Overview

- Implement complete system for mesh simplification
- Plus progressive meshes
- Possibly challenging assignment: start very early and proceed in incremental fashion
- Choice of data structure for meshes is the key (read the assignment)
- This involves fairly recent work. No one answer
  - Think about the best way of proceeding, use creativity

### Mesh Viewer (3.1)

Deliberately, no skeleton code for assignment

- Think about and implement full system from scratch

First step: Mesh viewer

- Read meshes (in simple OFF file format), view them
- Should be able to reuse some code from 167 etc.
  - Please ask instructor or TA if stuck, need some help
- Shading: must average face normals per vertex (this may give you a start in implementing a mesh data structure)
- Debugging modes for shading (color each triangle separately with an individual color)

Software Design

- Define mesh class with display method etc.
- Use C++ STL data structures where appropriate (see assn)

### Mesh Connectivity (3.2)

Build up mesh connectivity data structure

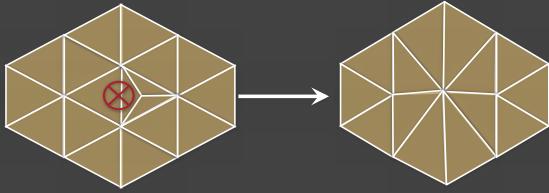
- Input is vertices and faces from input file

Goal is to do edge collapses in constant time

- No iteration over whole mesh
- Most of mesh unchanged
- Important questions for your data structure to answer: “What vertices neighbor my current vertex?” and “What faces neighbor my current vertex”
- Think about updating your data structure. Collapsing an edge may require more than just the edge itself. You must update every vertex or face that has changed

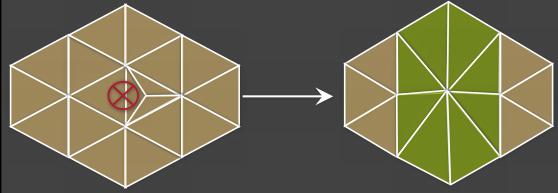
### Mesh Decimation (edge collapse)

- Can you handle this correctly and efficiently? Debugging examples in testpatch and plane (do these first)

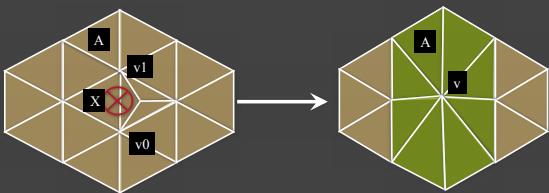


### Mesh Decimation (edge collapse)

- Can you handle this correctly and efficiently? Debugging examples in testpatch and plane (do these first)



### Mesh Decimation (edge collapse 3.3)

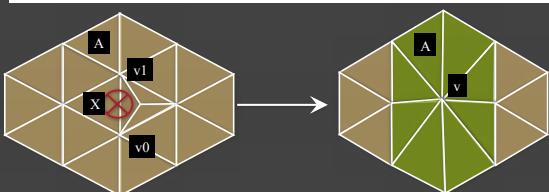


- Create new vertex v (based on appropriate rule)
- Find all faces/edges neighbor vertex v1** (such as A)
- Change them to use v instead of v1. Do the same for v0
- Depend on data structure, you need to fix all faces, edges

### Mesh Data Structure Hints

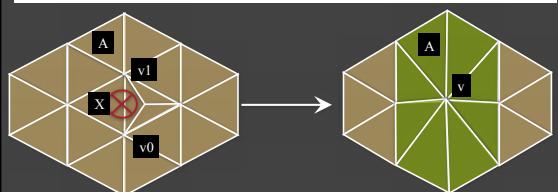
- Simplest (I think): Faces store constituent vertices [indexed face set as in OFF], vertices store adjacent faces (how do you create vertex-face adjacency?)
- To simplify, first create new vertex v. Adjacent faces are those adjacent to v0 or v1
- For each of those faces, update to point to v instead of v0 or v1

### Mesh Decimation (edge collapse 3.3)



- Create new vertex v (based on appropriate rule *like average*)
- Find all faces that neighbor vertex v1** (such as A)
  - Simple use of vertex to face adjacency
- Change them to use v instead of v1. Do the same for v0

### Mesh Decimation (edge collapse 2.3)

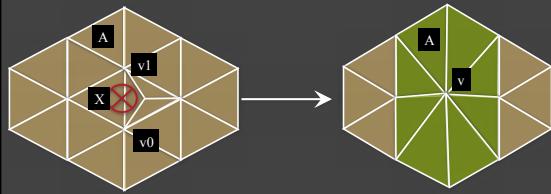


- Find faces neighboring edge v0-v1** (such as X)
- Remove from mesh
  - This may involve updating face/vertex adjacency relationships etc.
  - E.g. what is adjacency for v (faces adjacent to vertex?)
  - Are other vertices affected in terms of adjacent faces?
- Worry about triangle fins (extra credit, not discussed)

## Mesh Data Structure Hints

- With indexed face set plus vertex to face adjacency, removing a face should just work (remember to delete face from vertex adjacency lists)
- In general, winged edge, half-edge may be (slightly) more efficient, but also harder to implement
- Ultimately, your choice and work out the details
- Good luck!!

## Mesh Decimation (edge collapse 3.3)



- Find faces neighboring edge  $v_0-v_1$  (such as  $X$ )
  - Union of adjacent faces to vertex  $v_0$  and vertex  $v_1$
- Update adjacency lists
  - For all vertices, remove that face from their adjacency list
- Remove face from mesh

## Implementation

- Tricky
- When you remove something, need to update appropriately
- Work out on paper first (e.g. indexed face set plus adjacent faces for each vertex)
- Depends on choice of data structure (pick easy to do)
- Start with simple debugging cases (make sure not just that it looks right, but all adjacencies remain correct)

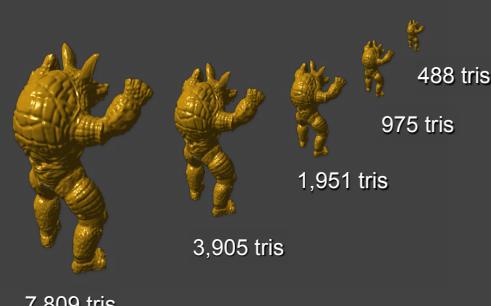
## Outline

- Basic assignment overview
- Detailed discussion of mesh simplification
- Progressive meshes*
- Quadric error metrics

## Successive Edge Collapses

- We have discussed one edge collapse, how to do that
- In practice, sequence of edge collapses applied
- Order etc. based on some metric (later in lecture)
- So, we gradually reduce complexity of model
- Progressive meshes is opposite: gradually increase complexity

## Appearance Preserving

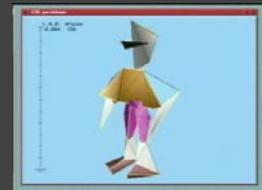


Caltech & Stanford Graphics Labs and Jonathan Cohen

## Progressive Meshes (3.5)

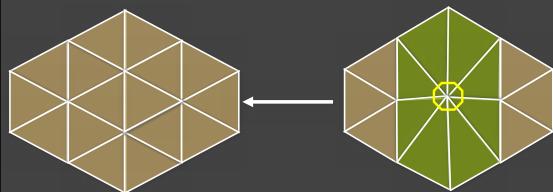
- Write edge collapses to file
- Read in file and invert order
- Key idea is **vertex-split** (opposite of edge-collapse)
- Include some control to make model coarser/finer
- E.g. Hoppe geomorph demo

## GeoMorph



## Vertex splits

- Can you handle this correctly and efficiently? Debugging examples in testpatch and plane (do these first)

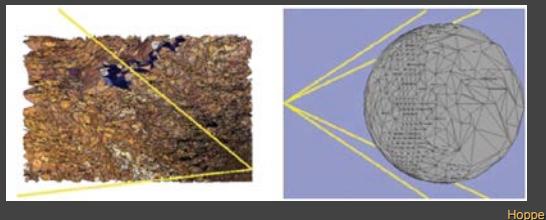


## Implementation

- Tricky
- What info do you need to add something?
- Work out on paper first (e.g. indexed face set plus adjacent faces for each vertex)
- Start with simple debugging cases (make sure not just that it looks right, but all adjacencies remain correct)

## View-Dependent Simplification

- Simplify dynamically according to viewpoint
  - Visibility
  - Silhouettes
  - Lighting



## Outline

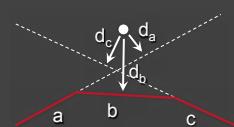
- Basic assignment overview
- Detailed discussion of mesh simplification
- Progressive meshes
- *Quadric error metrics*

## Quadratic Error Metrics

- Garland & Heckbert, SIGGRAPH 97
- Greedy decimation algorithm
- Pair collapse (allow edge + non-edge collapses)
- Quadratic error metrics:
  - Evaluate potential collapses
  - Determine optimal new vertex locations

## Quadratic Error Metrics

- Based on point-to-plane distance
- Better quality than point-to-point



## Quadratic Error Metrics

$$\begin{aligned}\Delta &= \sum_p (\mathbf{p}^\top \mathbf{v})^2 \\ &= \sum_p \mathbf{v}^\top \mathbf{p} \mathbf{p}^\top \mathbf{v} \\ &= \mathbf{v}^\top \left( \sum_p \mathbf{p} \mathbf{p}^\top \right) \mathbf{v} \\ &= \mathbf{v}^\top \mathbf{Q} \mathbf{v}\end{aligned}$$

- Common mathematical trick: quadratic form = symmetric matrix  $\mathbf{Q}$  multiplied twice by a vector
- Initially, distance to all planes 0, net is 0 for all verts

## Background: Computing Planes

- Each triangle in mesh has associated plane
 
$$ax + by + cz + d = 0$$
- For a triangle, find its (normalized) normal using cross products
 
$$\vec{n} = \frac{\vec{AB} \times \vec{AC}}{|\vec{AB} \times \vec{AC}|} \quad \vec{n} \cdot \vec{v} - \vec{A} \cdot \vec{n} = 0$$
- Plane equation?
 
$$\vec{n} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \quad d = -\vec{A} \cdot \vec{n}$$

## Quadratic Error Metrics

- Sum of squared distances from vertex to planes:

$$\begin{aligned}\Delta &= \sum_p \text{Dist}(\mathbf{v}, \mathbf{p})^2 \\ \mathbf{v} &= \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}, \quad \mathbf{p} = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} \\ \text{Dist}(\mathbf{v}, \mathbf{p}) &= ax + by + cz + d = \mathbf{p}^\top \mathbf{v}\end{aligned}$$

## Quadratic Error Metrics

- Sum of squared distances from vertex to planes:

## Using Quadratics

- Approximate error of edge collapses
  - Each vertex  $v$  has associated quadric  $\mathbf{Q}$
  - Error of collapsing  $v_1$  and  $v_2$  to  $v'$  is  $v'^\top \mathbf{Q}_1 v' + v'^\top \mathbf{Q}_2 v'$
  - Quadric for new vertex  $v'$  is  $\mathbf{Q}' = \mathbf{Q}_1 + \mathbf{Q}_2$

## Using Quadrics

- Find optimal location  $v'$  after collapse:

$$\mathbf{Q}' = \begin{bmatrix} q_{11} & q_{12} & q_{13} & q_{14} \\ q_{12} & q_{22} & q_{23} & q_{24} \\ q_{13} & q_{23} & q_{33} & q_{34} \\ q_{14} & q_{24} & q_{34} & q_{44} \end{bmatrix}$$

$$\min_{v'} v'^T \mathbf{Q}' v': \frac{\partial}{\partial x} = \frac{\partial}{\partial y} = \frac{\partial}{\partial z} = 0$$

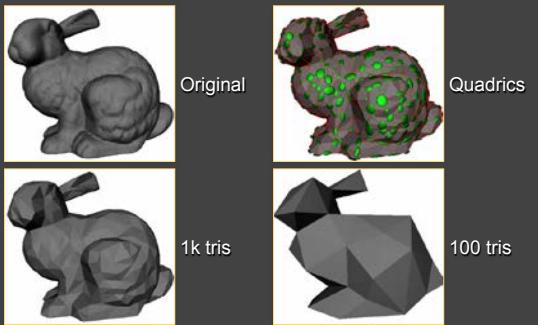
## Using Quadrics

- Find optimal location  $v'$  after collapse:

$$\begin{bmatrix} q_{11} & q_{12} & q_{13} & q_{14} \\ q_{12} & q_{22} & q_{23} & q_{24} \\ q_{13} & q_{23} & q_{33} & q_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{v}' = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

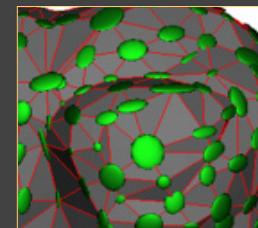
$$\mathbf{v}' = \begin{bmatrix} q_{11} & q_{12} & q_{13} & q_{14} \\ q_{12} & q_{22} & q_{23} & q_{24} \\ q_{13} & q_{23} & q_{33} & q_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

## Results

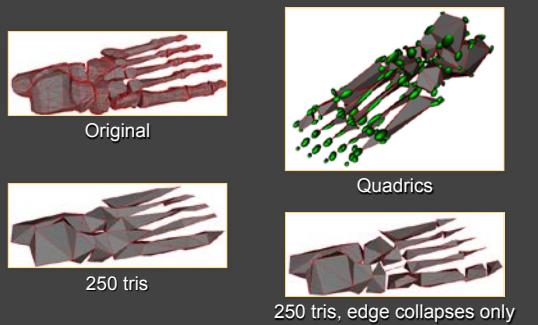


## Quadric Visualization

- Ellipsoids: iso-error surfaces
- Smaller ellipsoid = greater error for a given motion
- Lower error for motion parallel to surface
- Lower error in flat regions than at corners
- Elongated in “cylindrical” regions



## Results



## Summary

- First, implement basic mesh simplification on one edge
- Helps to have right data structure
  - Tricky since needs to be efficient and properly update
- Then, implement quadric error metrics
  - Tricky; we will spend most of another lecture on this
  - Put edge collapses in priority queue
  - Problem is that when you do one, you have to update all the neighbors as well (just as for standard edge collapse)
  - And re-insert in queue (use appropriate data structure)