Advanced Computer Graphics

CSE 163 [Spring 2018], Lecture 4

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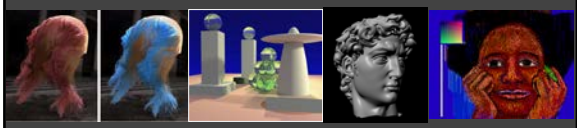


To Do

- Assignment 1, Due Apr 27.
  - Please START EARLY
  - This lecture completes all the material you need

Outline

- Implementation of digital filters
  - *Discrete convolution in spatial domain*
  - Basic image-processing operations
  - Antialiased shift and resize

Discrete Convolution

- Previously: Convolution as mult in freq domain
  - But need to convert digital image to and from to use that
  - Useful in some cases, but not for small filters
- Previously seen: Sinc as ideal low-pass filter
  - But has infinite spatial extent, exhibits spatial ringing
  - In general, use frequency ideas, but consider implementation issues as well
- Instead, use simple discrete convolution filters e.g.
  - Pixel gets sum of nearby pixels weighted by filter/mask

2	0	-7
5	4	9
1	-6	-2

Implementing Discrete Convolution

- Fill in each pixel new image convolving with old
  - Not really possible to implement it in place

$$I_{new}(a,b) = \sum_{x=a-width}^{a+width} \sum_{y=b-width}^{b+width} f(x-a, y-b) I_{old}(x,y)$$

  - More efficient for smaller kernels/filters f
- Normalization
  - If you don't want overall brightness change, entries of filter must sum to 1. You may need to normalize by dividing
- Integer arithmetic
  - Simpler and more efficient
  - In general, normalization outside, round to nearest int

Outline

- Implementation of digital filters
  - Discrete convolution in spatial domain
  - *Basic image-processing operations*
  - Antialiased shift and resize

Basic Image Processing (Assn 3.4)

- *Blur*
- Sharpen
- Edge Detection

All implemented using convolution with different filters

Blurring

- Used for softening appearance
- Convolve with gaussian filter
  - Same as mult. by gaussian in freq. domain, so reduces high-frequency content
  - Greater the spatial width, smaller the Fourier width, more blurring occurs and vice versa
- How to find blurring filter?

Blurring Filter

- In general, for symmetry f(u,v) = f(u) f(v)
  - You might want to have some fun with asymmetric filters
- We will use a Gaussian blur
  - Blur width sigma depends on kernel size n (3,5,7,11,13,19)

$$f(u) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left[\frac{-u^2}{2\sigma^2}\right] \qquad \sigma = floor(n/2)/2$$

Discrete Filtering, Normalization

- Gaussian is infinite
  - In practice, finite filter of size n (much less energy beyond 2 sigma or 3 sigma).
  - Must renormalize so entries add up to 1
- Simple practical approach
  - Take smallest values as 1 to scale others, round to integers
  - Normalize. E.g. for n = 3, sigma = ½

$$f(u,v) = \frac{1}{2\pi\sigma^2}\exp\left[-\frac{u^2+v^2}{2\sigma^2}\right] = \frac{2}{\pi}\exp\left[-2\left(u^2+v^2\right)\right]$$

$$\approx \begin{pmatrix} 0.012 & 0.09 & 0.012 \\ 0.09 & 0.64 & 0.09 \\ 0.012 & 0.09 & 0.012 \end{pmatrix} \approx \frac{1}{86}\begin{pmatrix} 1 & 7 & 1 \\ 7 & 54 & 7 \\ 1 & 7 & 1 \end{pmatrix}$$

Basic Image Processing

- Blur
- *Sharpen*
- Edge Detection

All implemented using convolution with different filters

Sharpening Filter

- Unlike blur, want to accentuate high frequencies
- Take differences with nearby pixels (rather than avg)

$$f(x,y) = \frac{1}{7}\begin{pmatrix} -1 & -2 & -1 \\ -2 & 19 & -2 \\ -1 & -2 & -1 \end{pmatrix}$$

Basic Image Processing

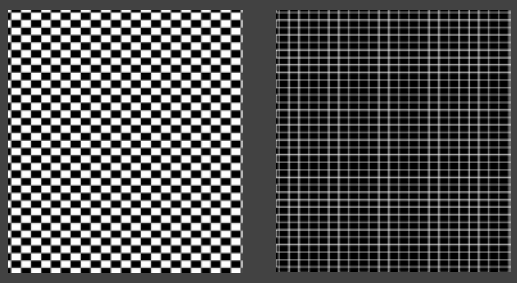
- Blur
- Sharpen
- *Edge Detection*

All implemented using convolution with different filters

Edge Detection

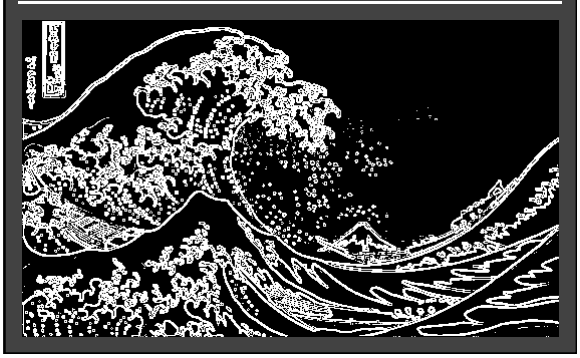
- Complicated topic: subject of many PhD theses
  - Including newest work at UCSD, Marr Prize ICCV 15
- Here, we present one approach (Sobel edge detector)
- Step 1: Convolution with gradient (Sobel) filter
  - Edges occur where image gradients are large
  - Separately for horizontal and vertical directions
- Step 2: Magnitude of gradient
  - Norm of horizontal and vertical gradients
- Step 3: Thresholding
  - Threshold to detect edges

Edge Detection



Edge Detection

Edge Detection



Details

- Step 1: Convolution with gradient (Sobel) filter
  - Edges occur where image gradients are large
  - Separately for horizontal and vertical directions

$$f_{horiz}(x,y) = \begin{pmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{pmatrix} \quad f_{vert}(x,y) = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{pmatrix}$$

- Step 2: Magnitude of gradient
  - Norm of horizontal and vertical gradients

$$G = \sqrt{|G_x|^2 + |G_y|^2}$$

- Step 3: Thresholding

Outline

- Implementation of digital filters
  - Discrete convolution in spatial domain
  - Basic image-processing operations
  - *Antialiased shift and resize (Assn 3.5, brief)*

Antialiased Shift

Shift image based on (fractional) s_x and s_y
- Check for integers, treat separately
- Otherwise convolve/resample with kernel/filter h:
- *In this part, no discrete kernel or mask; continuous*

$$u = x - s_x \qquad v = y - s_y$$

$$I(x,y) = \sum_{u',v'} h(u'-u, v'-v) I(u',v')$$

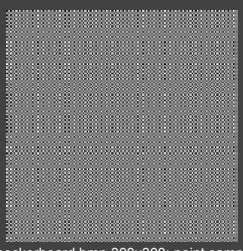
Antialiased Scale Magnification

Magnify image (scale s or γ > 1)
- Interpolate between orig. samples to evaluate frac vals
- Do so by convolving/resampling with kernel/filter:
- *Treat the two image dimensions independently (diff scales)*

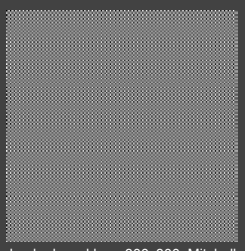
$$u = \frac{x}{\gamma}$$

$$I(x) = \sum_{u'=x/\gamma-\text{width}}^{x/\gamma+\text{width}} h(u'-u) I(u')$$

Antialiased Scale Minification



checkerboard.bmp 300x300: point sample



checkerboard.bmp 300x300: Mitchell

Antialiased Scale Minification

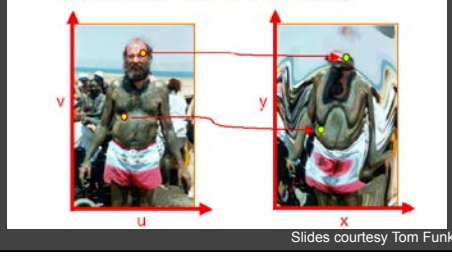
Minify (reduce size of) image

- Similar in some ways to mipmapping for texture maps
- We use *fat* pixels of size 1/γ, with new size γ*orig size (γ is scale factor < 1).
- Each fat pixel must integrate over corresponding region in original image using the filter kernel.

$$u = \frac{x}{\gamma} \qquad I(x) = \sum_{u'=u-\text{width}/\gamma}^{u+\text{width}/\gamma} h(\gamma(u'-u))I(u') = \sum_{u'=u-\text{width}/\gamma}^{u+\text{width}/\gamma} h(\gamma u'-x)I(u')$$

Bonus and Details: Image Warping

- Define transformation
  - Describe the destination (x,y) for every location (u,v) in the source (or vice-versa, if invertible)



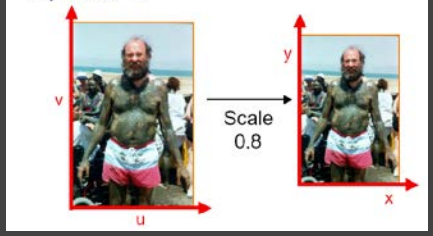
Slides courtesy Tom Funkhouser

A note on notation

- This segment uses (u,v) for warped location in the source image (or old coordinates) and (u', v') for integer coordinates, and (x,y) for new coordinates
- Most of the homework assignment uses (x,y) for old integer coordinates and (a,b) for new coordinates. The warped location is not written explicitly, but is implicit in the evaluation of the filter

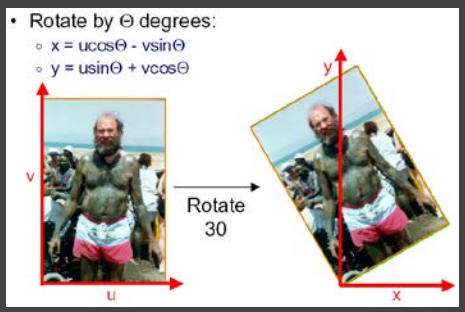
Example Mappings

- Scale by *factor*:
  - x = *factor* * u
  - y = *factor* * v



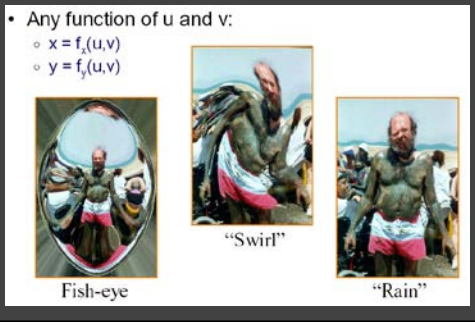
Example Mappings

- Rotate by Θ degrees:
  - x = ucosΘ - vsinΘ
  - y = usinΘ + vcosΘ



Example Mappings

- Any function of u and v:
  - x = f_x(u,v)
  - y = f_y(u,v)

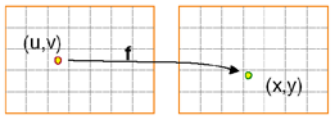


Forward Warping/Mapping

- Iterate over source, sending pixels to destination

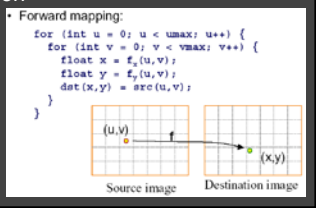
• Forward mapping:

```
for (int u = 0; u < umax; u++) {
  for (int v = 0; v < vmax; v++) {
    float x = f_x(u,v);
    float y = f_y(u,v);
    dst(x,y) = src(u,v);
  }
}
```



Forward Warping: Problems

- Iterate over source, sending pixels to destination
- Same source pixel map to multiple dest pixels
- Some dest pixels have no corresponding source
- Holes in reconstruction
- Must splat etc.

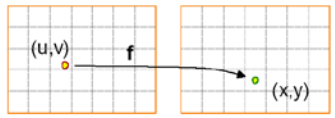


Inverse Warping/Mapping

- Iterate destination, finding pixels from source

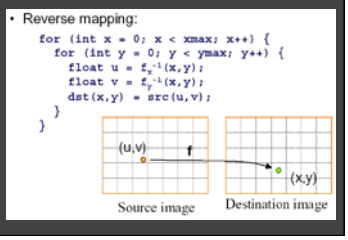
• Reverse mapping:

```
for (int x = 0; x < xmax; x++) {
  for (int y = 0; y < ymax; y++) {
    float u = f_x^-1(x,y);
    float v = f_y^-1(x,y);
    dst(x,y) = src(u,v);
  }
}
```



Inverse Warping/Mapping

- Iterate over dest, finding pixels from source
- ***Non-integer evaluation source image, resample***
- May oversample source
- But no holes
- Simpler, better than forward mapping

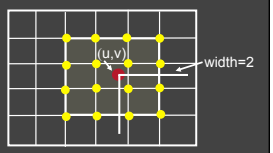


Filtering or Resampling

Compute weighted sum of pixel neighborhood

- Weights are normalized values of kernel function
- Equivalent to convolution at samples with kernel
- Find good ***(normalized)*** filters h using earlier ideas

```
s=0 ;
for (u ' = u-width ; u ' <= u+width; u ' ++)
   for (v'  = v-width ; v ' <= v+width; v ' ++)
     s += h (u ' - u, v' - v) src (u ' , v ') ;
```

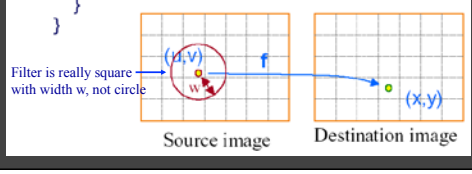


Inverse Warping/Mapping

- Iterate destination, finding pixels from source

• Reverse mapping:

```
for (int x = 0; x < xmax; x++) {
  for (int y = 0; y < ymax; y++) {
    float u = f_x^-1(x,y);
    float v = f_y^-1(x,y);
    dst(x,y) = resample_src(u,v,w);
  }
}
```



Filters for Assignment

Implement 3 filters (for anti-aliased shift, resize)

- Nearest neighbor or point sampling
- Hat filter (linear or triangle)

$$h(u) = 1 - |u|$$

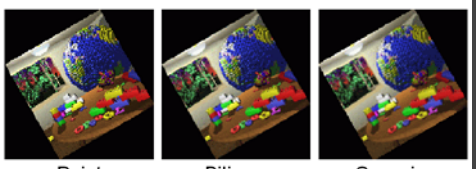
- Mitchell cubic filter (form in assigments). This is a good finite filter that approximates ideal sinc without ringing or infinite width. Alternative is gaussian

Construct 2D filters by multiplying 1D filters

$$h(u,v) = h(u)h(v)$$

Filtering Methods Comparison

- Trade-offs
  - Aliasing versus blurring
  - Computation speed



Point Bilinear Gaussian