Advanced Computer Graphics

CSE 163 [Spring 2018], Lecture 11

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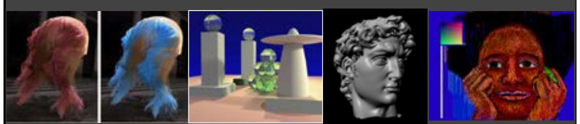


To Do

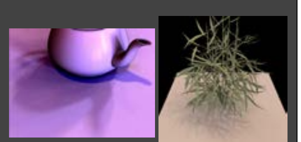
- Assignment 2 due May 18
  - Should already be well on way.
  - Contact us for difficulties etc.

  - This lecture on rendering, rendering equation. Pretty advanced theoretical material. Don't worry if a bit lost; not directly required on the homeworks.

Course Outline

- 3D Graphics Pipeline

Modeling (Creating 3D Geometry) → **Rendering** (Creating, shading images from geometry, lighting, materials)



Course Outline

- 3D Graphics Pipeline

Modeling (Creating 3D Geometry) → **Rendering** (Creating, shading images from geometry, lighting, materials)

Unit 1: Foundations of Signal and Image Processing
Understanding the way 2D images are formed and displayed, the important concepts and algorithms, and to build an image processing utility like Photoshop
Weeks 1 – 3. **Assignment 1**

Unit 2: Meshes, Modeling
Weeks 3 – 5. **Assignment 2**

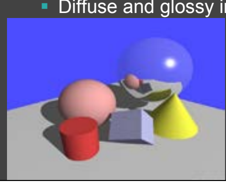
Unit 3: Advanced Rendering
Weeks 6 – 7, 8-9. **(Final Project)**

Unit 4: Animation, Imaging
Weeks 7-8, 9-10. **(Final Project)**

Illumination Models

Local Illumination
- Light directly from light sources to surface
- No shadows (cast shadows are a global effect)

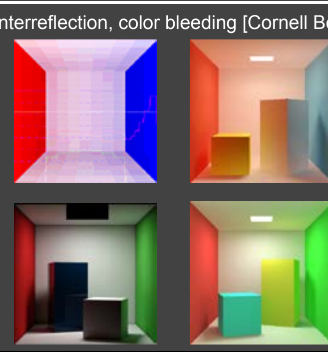
Global Illumination: multiple bounces (indirect light)
- Hard and soft shadows
- Reflections/refractions (already seen in ray tracing)
- Diffuse and glossy interreflections (radiosity, caustics)



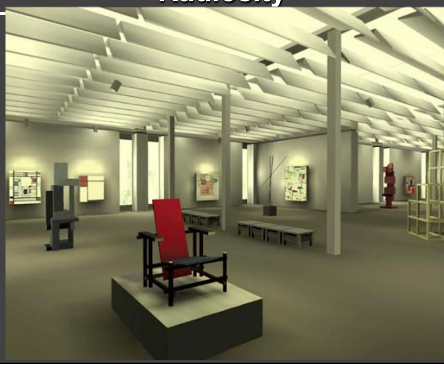
Some images courtesy Henrik Wann Jensen

Diffuse Interreflection

Diffuse interreflection, color bleeding [Cornell Box]

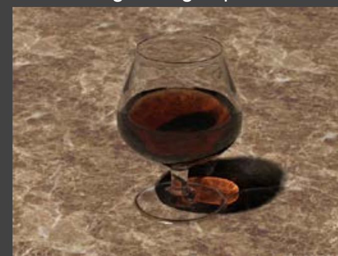


Radiosity



Caustics

Caustics: Focusing through specular surface



- Major research effort in 80s, 90s till today

Overview of lecture

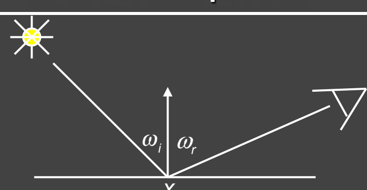
- ***Theory*** for all global illumination methods (ray tracing, path tracing, radiosity)
- We derive ***Rendering Equation*** [Kajiya 86]
  - Major theoretical development in field
  - Unifying framework for all global illumination
- Discuss existing approaches as special cases

Fairly theoretical lecture (but important). Not well covered in textbooks (though see Eric Veach's thesis). See reading if you are interested.

Outline

- *Reflectance Equation*
- *Global Illumination*
- *Rendering Equation*
- As a general Integral Equation and Operator
- Approximations (Ray Tracing, Radiosity)
- Surface Parameterization (Standard Form)

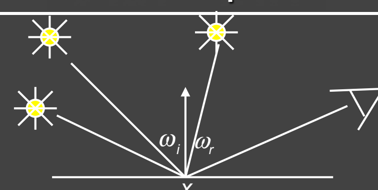
Reflection Equation



$$L_r(x,\omega_r) = L_e(x,\omega_r) + L_i(x,\omega_i) f(x,\omega_i,\omega_r)(\omega_i \bullet n)$$

Reflected Light (Output Image) | Emission | Incident Light (from light source) | BRDF | Cosine of Incident angle

Reflection Equation

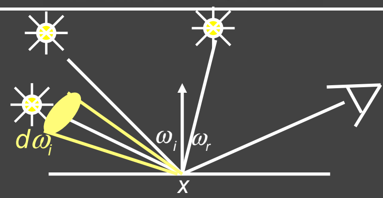


Sum over all light sources

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \sum L_i(x,\omega_i) f(x,\omega_i,\omega_r)(\omega_i \bullet n)$$

Reflected Light (Output Image) | Emission | Incident Light (from light source) | BRDF | Cosine of Incident angle

Reflection Equation



Replace sum with integral

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_{\Omega} L_i(x,\omega_i) f(x,\omega_i,\omega_r) \cos\theta_i \, d\omega_i$$

Reflected Light (Output Image) | Emission | Incident Light (from light source) | BRDF | Cosine of Incident angle

Environment Maps

- Light as a function of direction, from entire environment
- Captured by photographing a chrome steel or mirror sphere
- Accurate only for one point, but distant lighting same at other scene locations (typically use only one env. map)

Blinn and Newell 1976, Miller and Hoffman, 1984
Later, Greene 86, Cabral et al. 87

Environment Maps

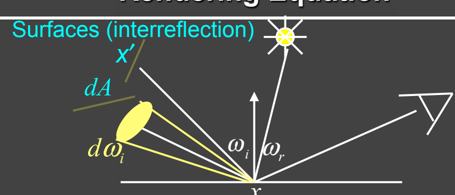
- Environment maps widely used as lighting representation
- Many modern methods deal with offline and real-time rendering with environment maps
- Image-based complex lighting + complex BRDFs

The Challenge

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_{\Omega} L_i(x,\omega_i) f(x,\omega_i,\omega_r) \cos\theta_i \, d\omega_i$$

- Computing reflectance equation requires knowing the incoming radiance from surfaces
- But determining incoming radiance requires knowing the reflected radiance from surfaces

Rendering Equation



$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_{\Omega} L_r(x',-\omega_i) f(x,\omega_i,\omega_r) \cos\theta_i \, d\omega_i$$

Reflected Light (Output Image) UNKNOWN | Emission KNOWN | Reflected Light UNKNOWN | BRDF KNOWN | Cosine of Incident angle KNOWN

Outline

- Reflectance Equation (review)
- Global Illumination
- Rendering Equation
- *As a general Integral Equation and Operator*
- *Approximations (Ray Tracing, Radiosity)*
- Surface Parameterization (Standard Form)

Rendering Equation (Kajiya 86)

Figure 6. A sample image. All objects are neutral grey. Color on the objects is due to caustics from the green glass balls and color bleeding from the base polygon.

Rendering Equation as Integral Equation

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_\Omega L_r(x', -\omega_i) \boxed{f(x,\omega_i,\omega_r)\cos\theta_i d\omega_i}$$

Reflected Light (Output Image)	Emission	Reflected Light	BRDF	Cosine of Incident angle
UNKNOWN	KNOWN	UNKNOWN	KNOWN	KNOWN

Is a Fredholm Integral Equation of second kind [extensively studied numerically] with canonical form

$$l(u) = e(u) + \int l(v) \boxed{K(u,v) dv}$$

Kernel of equation

Linear Operator Theory

- Linear operators act on functions like matrices act on vectors or discrete representations

$$h(u) = (M \circ f)(u)$$

M is a linear operator.
f and h are functions of u

- Basic linearity relations hold

a and b are scalars
f and g are functions

$$M \circ (af + bg) = a(M \circ f) + b(M \circ g)$$

- Examples include integration and differentiation

$$(K \circ f)(u) = \int k(u,v) f(v) dv$$

$$(D \circ f)(u) = \frac{\partial f}{\partial u}(u)$$

Linear Operator Equation

$$l(u) = e(u) + \int l(v) \boxed{K(u,v) dv}$$

Kernel of equation
Light Transport Operator

$$L = E + KL$$

Can be discretized to a simple matrix equation [or system of simultaneous linear equations] (L, E are vectors, K is the light transport matrix)

Solving the Rendering Equation

- Too hard for analytic solution, numerical methods
- Approximations, that compute different terms, accuracies of the rendering equation
- Two basic approaches are ray tracing, radiosity. More formally, Monte Carlo and Finite Element
- Monte Carlo techniques sample light paths, form statistical estimate (example, path tracing)
- Finite Element methods discretize to matrix equation

Solving the Rendering Equation

- General linear operator solution. Within raytracing:
- General class numerical ***Monte Carlo*** methods
- Approximate set of all paths of light in scene

$$L = E + KL$$

$$IL - KL = E$$

$$(I - K)L = E$$

$$L = (I - K)^{-1}E$$

Binomial Theorem

$$L = (I + K + K^2 + K^3 + ...)E$$

$$L = E + KE + K^2E + K^3E + ...$$

Term n corresponds to n bounces of light

Ray Tracing

$$L = E + KE + K^2E + K^3E + \ldots$$

- E: Emission directly From light sources
- KE: Direct Illumination on surfaces
- K^2E: Global Illumination (One bounce indirect) [Mirrors, Refraction]
- K^3E: (Two bounce indirect) [Caustics etc]

Ray Tracing

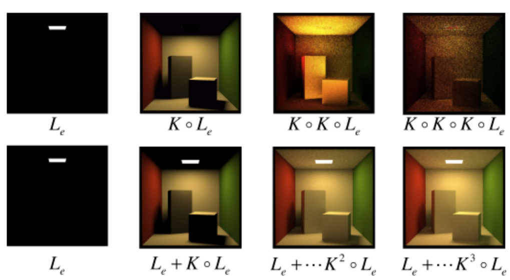
$$L = E + KE + K^2E + K^3E + \ldots$$

- E: Emission directly From light sources
- KE: Direct Illumination on surfaces

OpenGL Shading

- K^2E: Global Illumination (One bounce indirect) [Mirrors, Refraction]
- K^3E: (Two bounce indirect) [Caustics etc]

Successive Approximation



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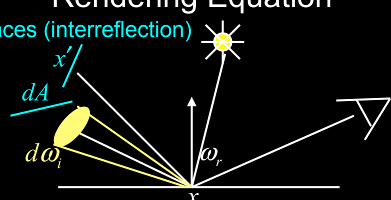
Rendering Equation

Surfaces (interreflection)



$\omega_i : x' - x$

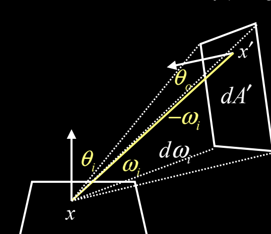
$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_\Omega L_r(x',-\omega_i) f(x,\omega_i,\omega_r) \cos\theta_i d\omega_i$$

Reflected Light (Output Image)	Emission	Reflected Light	BRDF	Cosine of Incident angle
UNKNOWN	KNOWN	UNKNOWN	KNOWN	KNOWN

Change of Variables

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_\Omega L_r(x',-\omega_i) f(x,\omega_i,\omega_r) \boxed{\cos\theta_i d\omega_i}$$

Integral over angles sometimes insufficient. Write integral in terms of surface radiance only (change of variables)



$$d\omega_i = \frac{dA' \cos\theta_o}{|x - x'|^2}$$

Change of Variables

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_\Omega L_r(x',-\omega_i) f(x,\omega_i,\omega_r) \boxed{\cos\theta_i d\omega_i}$$

Integral over angles sometimes insufficient. Write integral in terms of surface radiance only (change of variables)

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_{\text{all } x' \text{ visible to } x} L_r(x',-\omega_i) f(x,\omega_i,\omega_r) \boxed{\frac{\cos\theta_i \cos\theta_o}{|x-x'|^2} dA'}$$

$$d\omega_i = \frac{dA' \cos\theta_o}{|x-x'|^2}$$

$$G(x,x') = G(x',x) = \frac{\cos\theta_i \cos\theta_o}{|x-x'|^2}$$

Rendering Equation: Standard Form

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_\Omega L_r(x',-\omega_i) f(x,\omega_i,\omega_r) \boxed{\cos\theta_i d\omega_i}$$

Integral over angles sometimes insufficient. Write integral in terms of surface radiance only (change of variables)

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_{\text{all } x' \text{ visible to } x} L_r(x',-\omega_i) f(x,\omega_i,\omega_r) \boxed{\frac{\cos\theta_i \cos\theta_o}{|x-x'|^2} dA'}$$

Domain integral awkward. Introduce binary visibility fn V

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_{\text{all surfaces } x'} L_r(x',-\omega_i) f(x,\omega_i,\omega_r) G(x,x') V(x,x')\, dA'$$

$$d\omega_i = \frac{dA' \cos\theta_o}{|x-x'|^2}$$

Same as equation 2.52 Cohen Wallace. It swaps primed
And unprimed, omits angular args of BRDF, - sign.
Same as equation above 19.3 in Shirley, except he has
no emission, slightly diff. notation

$$G(x,x') = G(x',x) = \frac{\cos\theta_i \cos\theta_o}{|x-x'|^2}$$

Radiosity Equation

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_{\text{all surfaces } x'} L_r(x',-\omega_i) f(x,\omega_i,\omega_r) G(x,x') V(x,x')\, dA'$$

Drop angular dependence (diffuse Lambertian surfaces)

$$L_r(x) = L_e(x) + f(x) \int_S L_r(x') G(x,x') V(x,x')\, dA'$$

Change variables to radiosity (B) and albedo (ρ)

$$B(x) = E(x) + \rho(x) \int_S B(x') \frac{G(x,x')V(x,x')}{\pi} dA'$$

Expresses conservation of light energy at all points in space

Same as equation 2.54 in Cohen Wallace handout (read sec 2.6.3)
Ignore factors of π which can be absorbed.

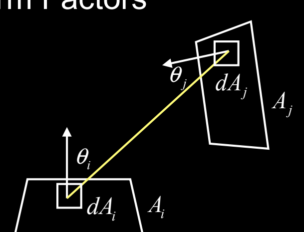
Discretization and Form Factors

$$B(x) = E(x) + \rho(x) \int_S B(x') \frac{G(x,x')V(x,x')}{\pi} dA'$$

$$B_i = E_i + \rho_i \sum_j B_j F_{j\to i} \frac{A_j}{A_i}$$

F is the ***form factor.*** It is dimensionless and is the fraction of energy leaving the entirety of patch j (*multiply by area of* j to get total energy) that arrives anywhere in the entirety of patch i (*divide by area of i* to get energy per unit area or radiosity).

Form Factors



$$A_i F_{i\to j} = A_j F_{j\to i} = \iint \frac{G(x,x')V(x,x')}{\pi} dA_i dA_j$$

$$G(x,x') = G(x',x) = \frac{\cos\theta_i \cos\theta_o}{|x-x'|^2}$$

Matrix Equation

$$B_i = E_i + \rho_i \sum_j B_j F_{j\to i} \frac{A_j}{A_i}$$

$$A_i F_{i\to j} = A_j F_{j\to i} = \iint \frac{G(x,x')V(x,x')}{\pi} dA_i dA_j$$

$$B_i = E_i + \rho_i \sum_j B_j F_{i\to j}$$

$$B_i - \rho_i \sum_j B_j F_{i\to j} = E_i$$

$$\sum_j M_{ij} B_j = E_i \qquad MB = E \qquad M_{ij} = I_{ij} - \rho_i F_{i\to j}$$

Summary

- ***Theory*** for all global illumination methods (ray tracing, path tracing, radiosity)
- We derive ***Rendering Equation*** [Kajiya 86]
  - Major theoretical development in field
  - Unifying framework for all global illumination
- Discuss existing approaches as special cases