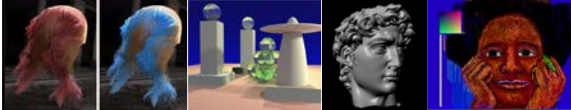


Advanced Computer Graphics

CSE 163 [Spring 2017], Lecture 3

Ravi Ramamoorthi

<http://www.cs.ucsd.edu/~ravr>



To Do

- Sign up for Piazza
- Assignment 1, Due Apr 28.
 - Anyone need help finding partners?
 - Any issues with skeleton code?
 - Please START EARLY

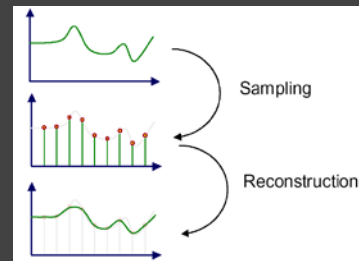
Outline

- *Basic ideas of sampling, reconstruction, aliasing*
- Signal processing and Fourier analysis
- Implementation of digital filters (second part of homework): next lecture
- Section 14.10 of FvDFH 2nd edition (should read)
 - Readings: Chapter 13 (color) and 14.10

Some slides courtesy Tom Funkhouser

Sampling and Reconstruction

- An image is a 2D array of samples
- Discrete samples from real-world continuous signal



Sampling and Reconstruction

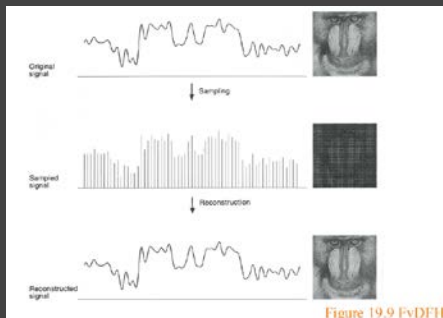
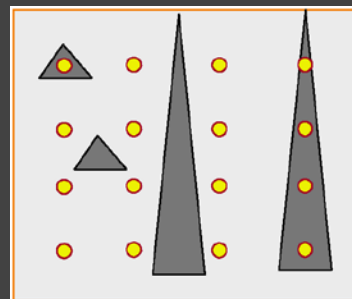


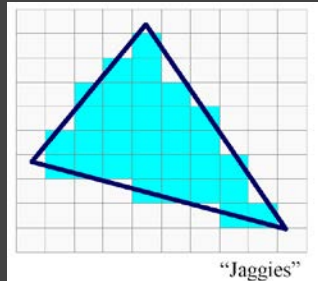
Figure 19.9 FvDFH

(Spatial) Aliasing



(Spatial) Aliasing

- Jaggies probably biggest aliasing problem



Sampling and Aliasing

- Artifacts due to undersampling or poor reconstruction
- Formally, high frequencies masquerading as low
- E.g. high frequency line as low freq jaggies

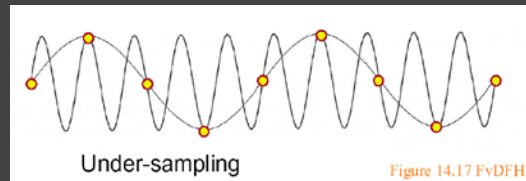
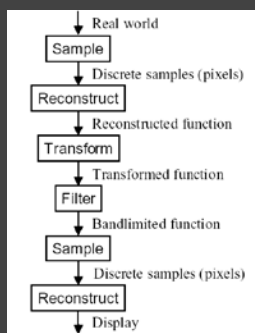


Image Processing pipeline



Outline

- Basic ideas of sampling, reconstruction, aliasing
- *Signal processing and Fourier analysis*
- Implementation of digital filters (second part of homework): next lecture
- Section 14.10 of FvDFH 2nd edition (should read)
 - Readings: Chapter 13 (color) and 14.10

Some slides courtesy Tom Funkhouser

Motivation

- Formal analysis of sampling and reconstruction
- Important theory (signal-processing) for graphics
- Also relevant in rendering, modeling, animation

Ideas

- Signal (function of time generally, here of space)
- Continuous: defined at all points; discrete: on a grid
- High frequency: rapid variation; Low Freq: slow variation
- Images are converting continuous to discrete. Do this sampling as best as possible.
- Signal processing theory tells us how best to do this
- Based on concept of frequency domain Fourier analysis

Sampling Theory

Analysis in the frequency (not spatial) domain

- Sum of sine waves, with possibly different offsets (phase)
- Each wave different frequency, amplitude

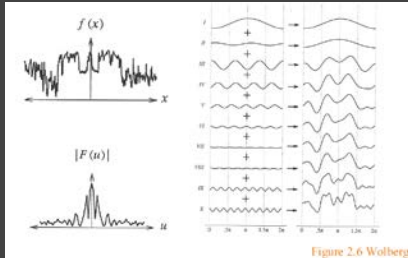


Figure 2.6 Wolberg

Fourier Transform

- Tool for converting from spatial to frequency domain
- Or vice versa
- One of most important mathematical ideas
- Computational algorithm: Fast Fourier Transform
 - One of 10 great algorithms scientific computing
 - Makes Fourier processing possible (images etc.)
 - Not discussed here, but look up if interested

Fourier Transform

- Simple case, function sum of sines, cosines

$$f(x) = \sum_{u=-\infty}^{+\infty} F(u)e^{2\pi iux}$$

$$F(u) = \int_0^1 f(x)e^{-2\pi iux} dx$$

- Continuous infinite case

$$\text{Forward Transform: } F(u) = \int_{-\infty}^{\infty} f(x)e^{-2\pi iux} dx$$

$$\text{Inverse Transform: } f(x) = \int_{-\infty}^{+\infty} F(u)e^{2\pi iux} du$$

Fourier Transform

- Simple case, function sum of sines, cosines

$$f(x) = \sum_{u=-\infty}^{+\infty} F(u)e^{2\pi iux}$$

$$F(u) = \int_0^1 f(x)e^{-2\pi iux} dx$$

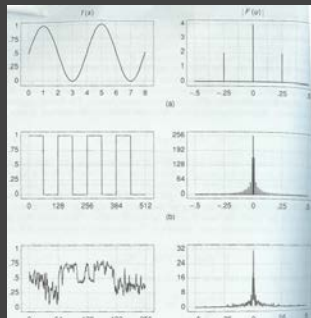
- Discrete case

$$F(u) = \sum_{x=0}^{x=N-1} f(x) [\cos(2\pi ux / N) - i \sin(2\pi ux / N)], \quad 0 \leq u \leq N-1$$

$$f(x) = \frac{1}{N} \sum_{u=0}^{u=N-1} F(u) [\cos(2\pi ux / N) + i \sin(2\pi ux / N)], \quad 0 \leq x \leq N-1$$

Fourier Transform: Examples 1

Single sine curve
(+constant DC term)



$$f(x) = \sum_{u=-\infty}^{+\infty} F(u)e^{2\pi iux}$$

$$F(u) = \int_0^1 f(x)e^{-2\pi iux} dx$$

Fourier Transform Examples 2

$$\text{Forward Transform: } F(u) = \int_{-\infty}^{\infty} f(x)e^{-2\pi iux} dx$$

$$\text{Inverse Transform: } f(x) = \int_{-\infty}^{+\infty} F(u)e^{2\pi iux} du$$

- Common examples

$f(x)$	$F(u)$
$\delta(x - x_0)$	$e^{-2\pi iux_0}$
1	$\delta(u)$
e^{-ax^2}	$\sqrt{\frac{\pi}{a}} e^{-\pi^2 u^2 / a}$

Fourier Transform Properties

Forward Transform: $F(u) = \int_{-\infty}^{\infty} f(x)e^{-2\pi iux} dx$

Inverse Transform: $f(x) = \int_{-\infty}^{\infty} F(u)e^{2\pi iux} du$

Common properties

▪ Linearity: $F(af(x) + bg(x)) = aF(f(x)) + bF(g(x))$

▪ Derivatives: [integrate by parts] $F(f'(x)) = \int_{-\infty}^{\infty} f'(x)e^{-2\pi iux} dx = 2\pi iuF(u)$

2D Fourier Transform

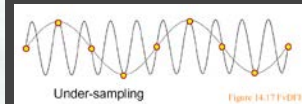
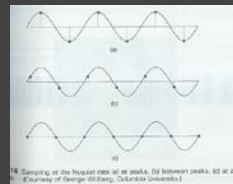
Forward Transform: $F(u,v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y)e^{-2\pi iux} e^{-2\pi ivy} dx dy$

Convolution (next)

Inverse Transform: $f(x,y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(u,v)e^{2\pi iux} e^{2\pi ivy} du dv$

Sampling Theorem, Bandlimiting

- A signal can be reconstructed from its samples, if the original signal has no frequencies above half the sampling frequency – Shannon
- The minimum sampling rate for a bandlimited function is called the Nyquist rate



Sampling Theorem, Bandlimiting

- A signal can be reconstructed from its samples, if the original signal has no frequencies above half the sampling frequency – Shannon
- The minimum sampling rate for a bandlimited function is called the Nyquist rate
- A signal is bandlimited if the highest frequency is bounded. This frequency is called the bandwidth
- In general, when we transform, we want to filter to bandlimit before sampling, to avoid aliasing

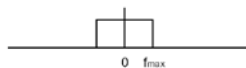
Antialiasing

- Sample at higher rate
 - Not always possible
 - Real world: lines have infinitely high frequencies, can't sample at high enough resolution
- Prefilter to bandlimit signal
 - Low-pass filtering (blurring)
 - Trade blurriness for aliasing

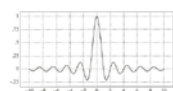
Ideal bandlimiting filter

- Formal derivation is homework exercise

Frequency domain



Spatial domain



$$\text{Sinc}(x) = \frac{\sin \pi x}{\pi x}$$

Figure 4.5 Wolberg

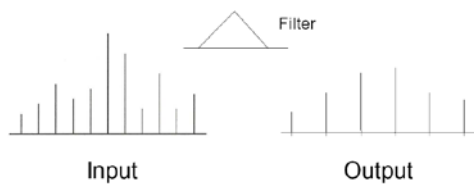
Outline

- Basic ideas of sampling, reconstruction, aliasing
- *Signal processing and Fourier analysis*
 - Convolution
- Implementation of digital filters (second part of homework): next lecture
- Section 14.10 of FvDFH 2nd edition (should read)
 - Readings: Chapter 13 (color) and 14.10

Some slides courtesy Tom Funkhouser

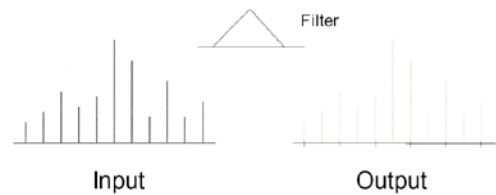
Convolution 1

- Spatial domain: output pixel is weighted sum of pixels in neighborhood of input image
 - Pattern of weights is the “filter”



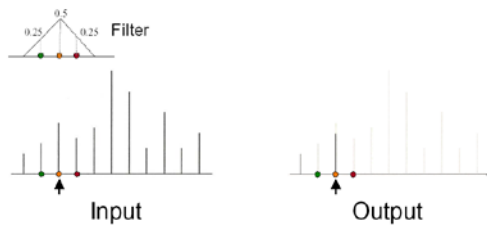
Convolution 2

- Example 1:



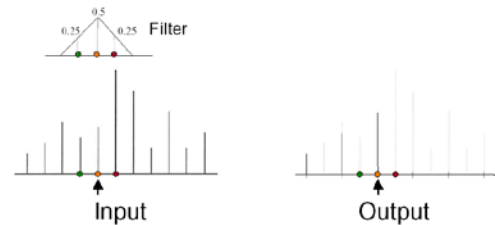
Convolution 3

- Example 1:



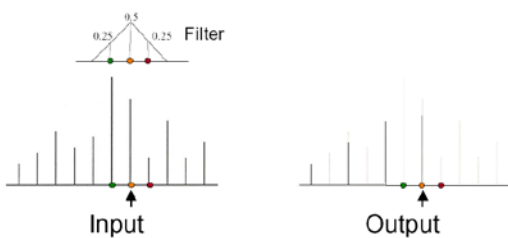
Convolution 4

- Example 1:



Convolution 5

- Example 1:



Convolution in Frequency Domain

Forward Transform: $F(u) = \int_{-\infty}^{\infty} f(x)e^{-2\pi iux} dx$

Inverse Transform: $f(x) = \int_{-\infty}^{\infty} F(u)e^{2\pi iux} du$

- Convolution (f is signal ; g is filter [or vice versa])

$$h(y) = \int_{-\infty}^{\infty} f(x)g(y-x)dx = \int_{-\infty}^{\infty} g(x)f(y-x)dx$$

$$h = f * g \text{ or } f \otimes g$$

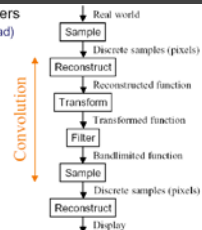
- Fourier analysis (frequency domain multiplication) $H(u) = F(u)G(u)$

Practical Image Processing

- Discrete convolution (in spatial domain) with filters for various digital signal processing operations
- Easy to analyze, understand effects in frequency domain
 - E.g. blurring or bandlimiting by convolving with low pass filter

Finite low-pass filters

- Point sampling (bad)
- Triangle filter
- Gaussian filter



Outline

- Basic ideas of sampling, reconstruction, aliasing
- Signal processing and Fourier analysis
- *Implementation of digital filters (second part of homework): next lecture*
- Section 14.10 of FvDFH 2nd edition (should read)
 - Readings: Chapter 13 (color) and 14.10

Some slides courtesy Tom Funkhouser