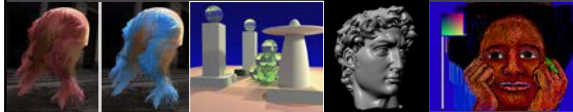


## Advanced Computer Graphics

CSE 163 [Spring 2017], Lecture 11

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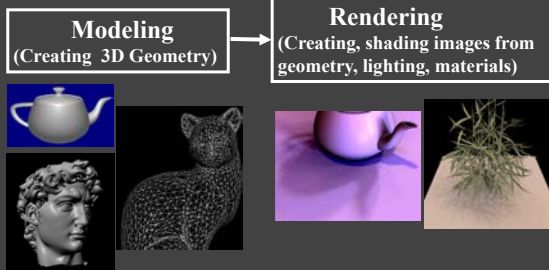


## To Do

- Assignment 2 due May 19
  - Should already be well on way.
  - Contact us for difficulties etc.
- This lecture on rendering, rendering equation. Pretty advanced theoretical material. Don't worry if a bit lost; not directly required on the homeworks.

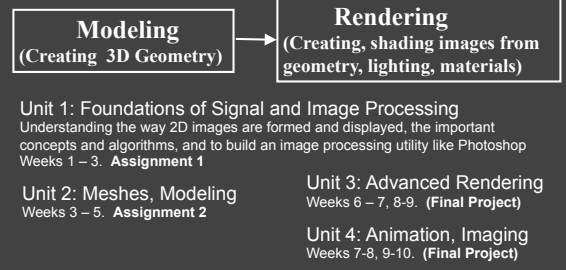
## Course Outline

- 3D Graphics Pipeline



## Course Outline

- 3D Graphics Pipeline



## Illumination Models

### Local Illumination

- Light directly from light sources to surface
- No shadows (cast shadows are a global effect)

### Global Illumination: multiple bounces (indirect light)

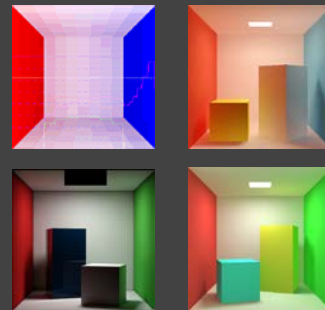
- Hard and soft shadows
- Reflections/refractions (already seen in ray tracing)
- Diffuse and glossy interreflections (radiosity, caustics)



Some images courtesy Henrik Wann Jensen

## Diffuse Interreflection

Diffuse interreflection, color bleeding [Cornell Box]



## Radiosity



## Caustics

Caustics: Focusing through specular surface



- Major research effort in 80s, 90s till today

## Overview of lecture

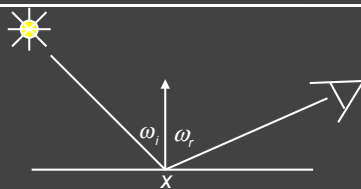
- Theory** for all global illumination methods (ray tracing, path tracing, radiosity)
- We derive **Rendering Equation** [Kajiya 86]
  - Major theoretical development in field
  - Unifying framework for all global illumination
- Discuss existing approaches as special cases

Fairly theoretical lecture (but important). Not well covered in textbooks (though see Eric Veach's thesis). See reading if you are interested.

## Outline

- Reflectance Equation**
- Global Illumination**
- Rendering Equation**
  - As a general Integral Equation and Operator
  - Approximations (Ray Tracing, Radiosity)
  - Surface Parameterization (Standard Form)

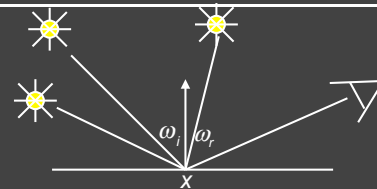
## Reflection Equation



$$L_r(x, \omega_r) = L_e(x, \omega_r) + L_i(x, \omega_i) f(x, \omega_i, \omega_r) (\omega_i \cdot n)$$

Reflected Light (Output Image)   Emission   Incident Light (from light source)   BRDF   Cosine of Incident angle

## Reflection Equation



Sum over all light sources

$$L_r(x, \omega_r) = L_e(x, \omega_r) + \sum L_i(x, \omega_i) f(x, \omega_i, \omega_r) (\omega_i \cdot n)$$

Reflected Light (Output Image)   Emission   Incident Light (from light source)   BRDF   Cosine of Incident angle

## Reflection Equation

Replace sum with integral

$$L_r(x, \omega_r) = L_e(x, \omega_r) + \int_{\Omega} L_i(x, \omega_i) f(x, \omega_i, \omega_r) \cos \theta_i d\omega_i$$

|                                   |          |                                          |      |                             |
|-----------------------------------|----------|------------------------------------------|------|-----------------------------|
| Reflected Light<br>(Output Image) | Emission | Incident<br>Light (from<br>light source) | BRDF | Cosine of<br>Incident angle |
|-----------------------------------|----------|------------------------------------------|------|-----------------------------|

## Environment Maps

- Light as a function of direction, from entire environment
- Captured by photographing a chrome steel or mirror sphere
- Accurate only for one point, but distant lighting same at other scene locations (typically use only one env. map)

Blinn and Newell 1976, Miller and Hoffman, 1984  
Later, Greene 86, Cabral et al. 87

## Environment Maps

- Environment maps widely used as lighting representation
- Many modern methods deal with offline and real-time rendering with environment maps
- Image-based complex lighting + complex BRDFs

## The Challenge

$$L_r(x, \omega_r) = L_e(x, \omega_r) + \int_{\Omega} L_i(x, \omega_i) f(x, \omega_i, \omega_r) \cos \theta_i d\omega_i$$

- Computing reflectance equation requires knowing the incoming radiance from surfaces
- But determining incoming radiance requires knowing the reflected radiance from surfaces

## Rendering Equation

Surfaces (interreflection)

$$L_r(x, \omega_r) = L_e(x, \omega_r) + \int_{\Omega} L_r(x', -\omega_i) f(x, \omega_i, \omega_r) \cos \theta_i d\omega_i$$

|                                   |          |                    |       |                             |
|-----------------------------------|----------|--------------------|-------|-----------------------------|
| Reflected Light<br>(Output Image) | Emission | Reflected<br>Light | BRDF  | Cosine of<br>Incident angle |
| UNKNOWN                           | KNOWN    | UNKNOWN            | KNOWN | KNOWN                       |

## Outline

- Reflectance Equation (review)
- Global Illumination
- Rendering Equation
- As a general Integral Equation and Operator
- Approximations (Ray Tracing, Radiosity)
- Surface Parameterization (Standard Form)

## Rendering Equation (Kajiya 86)



Figure 6. A sample image. All objects are neutral grey. Color on the objects is due to caustics from the green glass balls and color bleeding from the base polygons.

## Rendering Equation as Integral Equation

$$L_r(x, \omega_r) = L_e(x, \omega_r) + \int_{\Omega} L_r(x', -\omega_i) f(x, \omega_i, \omega_r) \cos \theta_i d\omega_i$$

|                                   |          |                    |       |                             |
|-----------------------------------|----------|--------------------|-------|-----------------------------|
| Reflected Light<br>(Output Image) | Emission | Reflected<br>Light | BRDF  | Cosine of<br>Incident angle |
| UNKNOWN                           | KNOWN    | UNKNOWN            | KNOWN | KNOWN                       |

Is a Fredholm Integral Equation of second kind  
[extensively studied numerically] with canonical form

$$l(u) = e(u) + \int l(v) K(u, v) dv$$

Kernel of equation

## Linear Operator Theory

- Linear operators act on functions like matrices act on vectors or discrete representations

$$h(u) = (M \circ f)(u)$$

M is a linear operator.  
f and h are functions of u

- Basic linearity relations hold a and b are scalars  
f and g are functions

$$M \circ (af + bg) = a(M \circ f) + b(M \circ g)$$

- Examples include integration and differentiation

$$(K \circ f)(u) = \int k(u, v) f(v) dv$$

$$(D \circ f)(u) = \frac{\partial f}{\partial u}(u)$$

## Linear Operator Equation

$$l(u) = e(u) + \int l(v) K(u, v) dv$$

Kernel of equation  
Light Transport Operator

$$L = E + KL$$

Can be discretized to a simple matrix equation  
[or system of simultaneous linear equations]  
(L, E are vectors, K is the light transport matrix)

## Solving the Rendering Equation

- Too hard for analytic solution, numerical methods
- Approximations, that compute different terms, accuracies of the rendering equation
- Two basic approaches are ray tracing, radiosity. More formally, Monte Carlo and Finite Element
- Monte Carlo techniques sample light paths, form statistical estimate (example, path tracing)
- Finite Element methods discretize to matrix equation

## Solving the Rendering Equation

- General linear operator solution. Within raytracing:
- General class numerical **Monte Carlo** methods
- Approximate set of all paths of light in scene

$$L = E + KL$$

$$IL - KL = E$$

$$(I - K)L = E$$

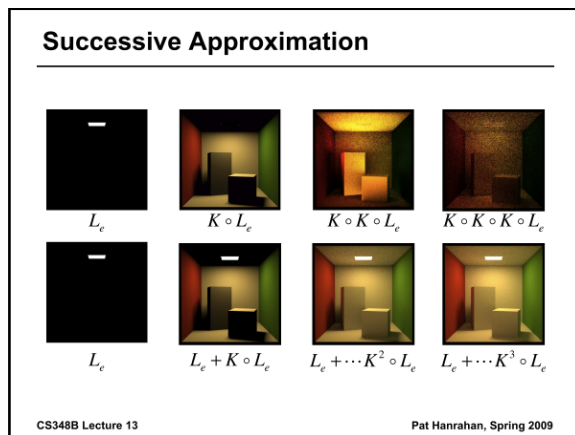
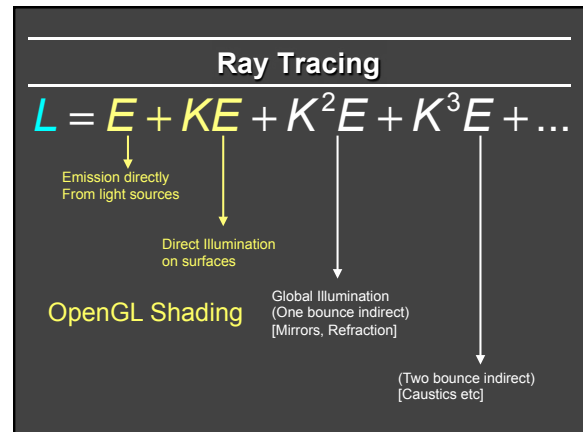
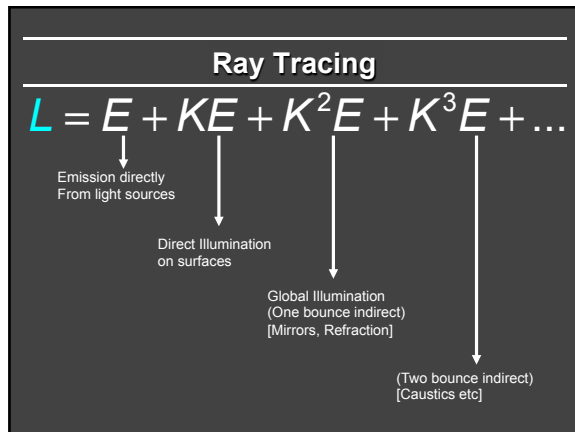
$$L = (I - K)^{-1}E$$

Binomial Theorem

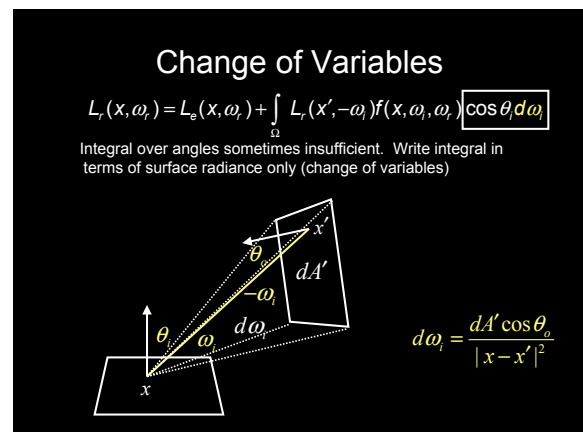
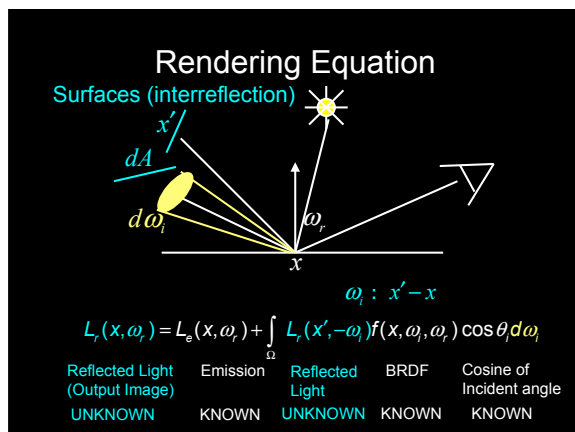
$$L = (I + K + K^2 + K^3 + \dots)E$$

$$L = E + KE + K^2E + K^3E + \dots$$

Term n corresponds to n bounces of light



- ### Outline
- Reflectance Equation (review)
  - Global Illumination
  - Rendering Equation
  - As a general Integral Equation and Operator
  - Approximations (Ray Tracing, Radiosity)
  - *Surface Parameterization (Standard Form)*



## Change of Variables

$$L_r(x, \omega_r) = L_e(x, \omega_r) + \int_{\Omega} L_r(x', -\omega_l) f(x, \omega_l, \omega_r) \cos \theta_l d\omega_l$$

Integral over angles sometimes insufficient. Write integral in terms of surface radiance only (change of variables)

$$L_r(x, \omega_r) = L_e(x, \omega_r) + \int_{\text{all } x' \text{ visible to } x} L_r(x', -\omega_l) f(x, \omega_l, \omega_r) \frac{\cos \theta_l \cos \theta_o}{|x - x'|^2} dA'$$

$$d\omega_l = \frac{dA' \cos \theta_o}{|x - x'|^2}$$

$$G(x, x') = G(x', x) = \frac{\cos \theta_l \cos \theta_o}{|x - x'|^2}$$

## Rendering Equation: Standard Form

$$L_r(x, \omega_r) = L_e(x, \omega_r) + \int_{\Omega} L_r(x', -\omega_l) f(x, \omega_l, \omega_r) \cos \theta_l d\omega_l$$

Integral over angles sometimes insufficient. Write integral in terms of surface radiance only (change of variables)

$$L_r(x, \omega_r) = L_e(x, \omega_r) + \int_{\text{all } x' \text{ visible to } x} L_r(x', -\omega_l) f(x, \omega_l, \omega_r) \frac{\cos \theta_l \cos \theta_o}{|x - x'|^2} dA'$$

Domain integral awkward. Introduce binary visibility fn V

$$L_r(x, \omega_r) = L_e(x, \omega_r) + \int_{\text{all surfaces } x'} L_r(x', -\omega_l) f(x, \omega_l, \omega_r) G(x, x') V(x, x') dA'$$

Same as equation 2.52 Cohen Wallace. It swaps primed and unprimed, omits angular args of BRDF, - sign.

Same as equation above 19.3 in Shirley, except he has no emission, slightly diff. notation

$$d\omega_l = \frac{dA' \cos \theta_o}{|x - x'|^2}$$

$$G(x, x') = G(x', x) = \frac{\cos \theta_l \cos \theta_o}{|x - x'|^2}$$

## Radiosity Equation

$$L_r(x, \omega_r) = L_e(x, \omega_r) + \int_{\text{all surfaces } x'} L_r(x', -\omega_l) f(x, \omega_l, \omega_r) G(x, x') V(x, x') dA'$$

Drop angular dependence (diffuse Lambertian surfaces)

$$L_r(x) = L_e(x) + f(x) \int_{\Omega} L_r(x') G(x, x') V(x, x') dA'$$

Change variables to radiosity (B) and albedo ( $\rho$ )

$$B(x) = E(x) + \rho(x) \int_{\Omega} B(x') \frac{G(x, x') V(x, x')}{\pi} dA'$$

Expresses conservation of light energy at all points in space

Same as equation 2.54 in Cohen Wallace handout (read sec 2.6.3) Ignore factors of  $\pi$  which can be absorbed.

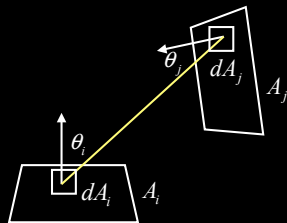
## Discretization and Form Factors

$$B(x) = E(x) + \rho(x) \int_{\Omega} B(x') \frac{G(x, x') V(x, x')}{\pi} dA'$$

$$B_i = E_i + \rho_i \sum_j B_j F_{j \rightarrow i} \frac{A_j}{A_i}$$

F is the **form factor**. It is dimensionless and is the fraction of energy leaving the entirety of patch j (multiply by area of j to get total energy) that arrives anywhere in the entirety of patch i (divide by area of i to get energy per unit area or radiosity).

## Form Factors



$$A_i F_{i \rightarrow j} = A_j F_{j \rightarrow i} = \iint \frac{G(x, x') V(x, x')}{\pi} dA_i dA_j$$

$$G(x, x') = G(x', x) = \frac{\cos \theta_l \cos \theta_o}{|x - x'|^2}$$

## Matrix Equation

$$B_i = E_i + \rho_i \sum_j B_j F_{j \rightarrow i} \frac{A_j}{A_i}$$

$$A_i F_{i \rightarrow j} = A_j F_{j \rightarrow i} = \iint \frac{G(x, x') V(x, x')}{\pi} dA_i dA_j$$

$$B_i = E_i + \rho_i \sum_j B_j F_{j \rightarrow i}$$

$$B_i - \rho_i \sum_j B_j F_{j \rightarrow i} = E_i$$

$$\sum_j M_{ij} B_j = E_i \quad MB = E \quad M_{ij} = I_{ij} - \rho_i F_{i \rightarrow j}$$

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## Summary

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- **Theory** for all global illumination methods (ray tracing, path tracing, radiosity)
- We derive **Rendering Equation** [Kajiya 86]
  - Major theoretical development in field
  - Unifying framework for all global illumination
- Discuss existing approaches as special cases